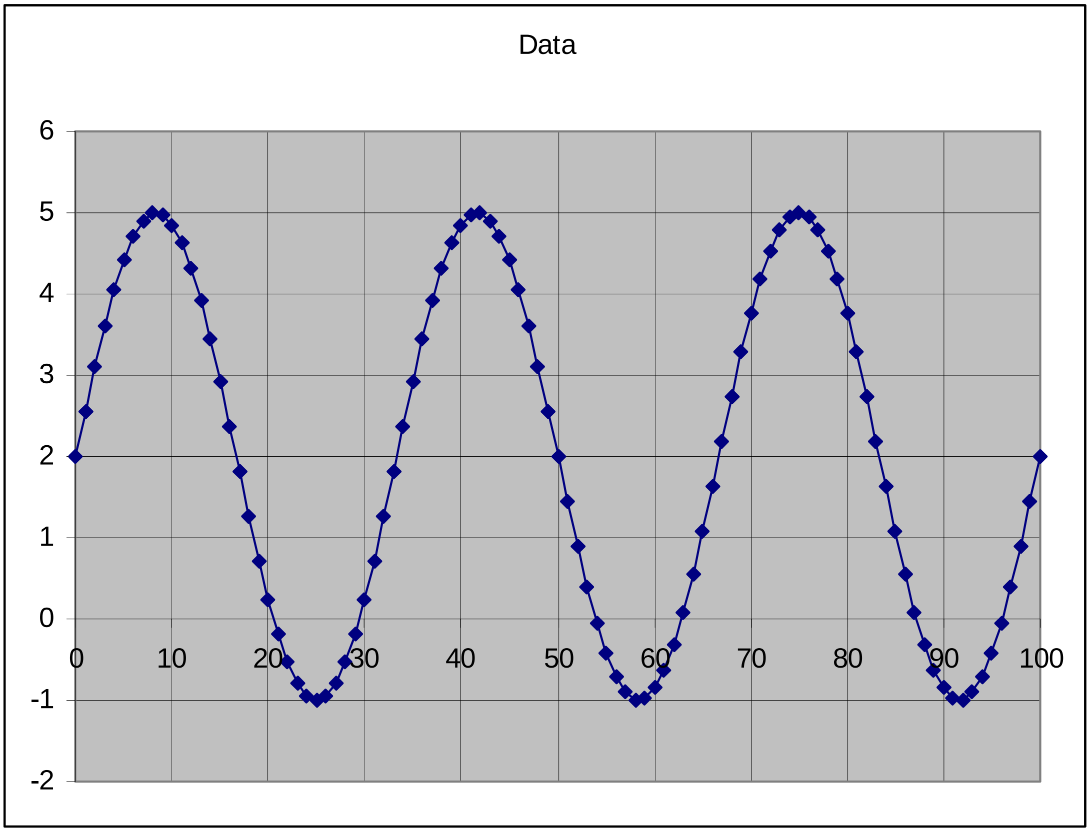
Spring 2004
4) Sinusoids (20 points)

The following data was created using Excel.



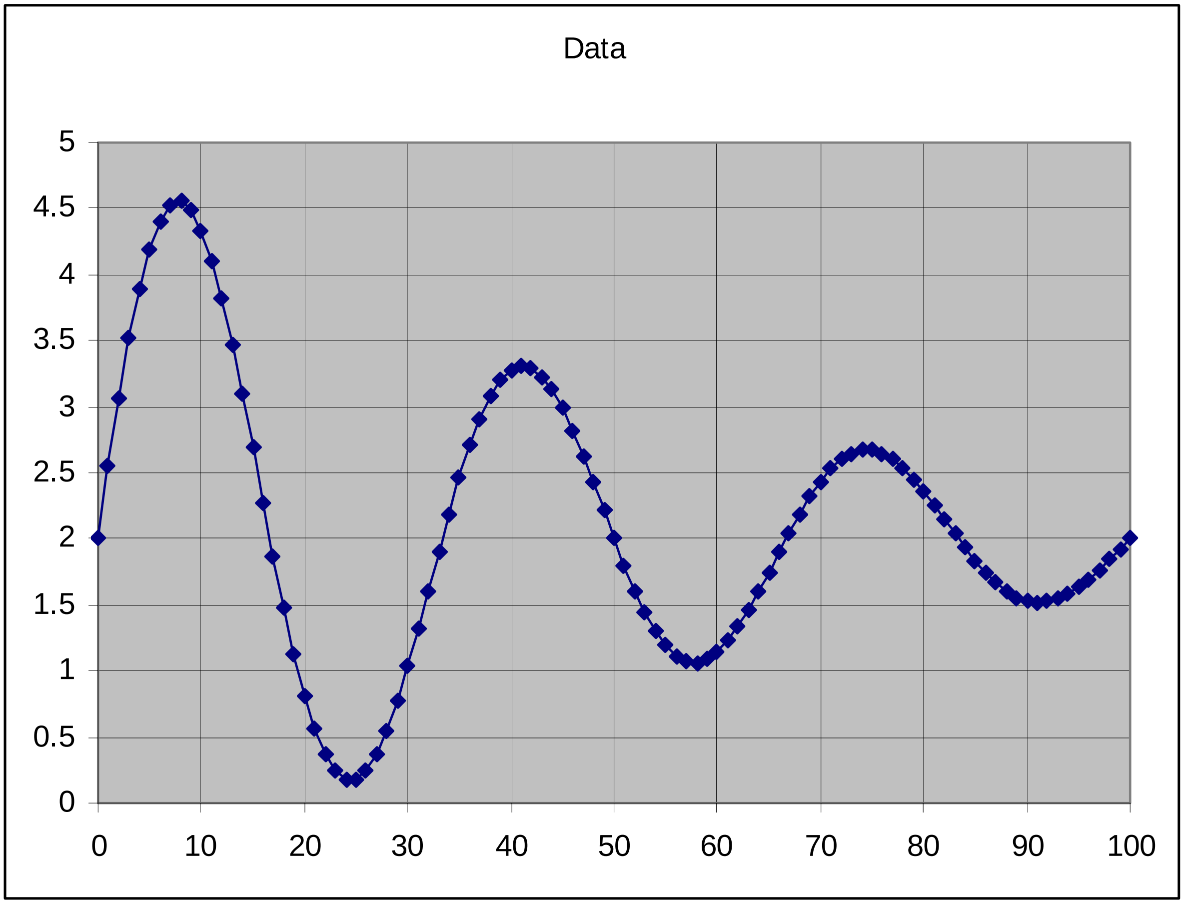
a. Assuming the horizontal scale is in seconds, find the frequency and period of this signal. Include units. *(4 points)*

b. What is the DC offset of this signal? Include units. *(2 points)*

c. What is the phase of the signal? Include units. *(2 points)*

d. Write the mathematical expression for this signal and its offset. In general, this is given by $X = X_o + X_1 \sin(\omega t + \phi_o)$ which accounts for its frequency and phase shift. *(2 points)*

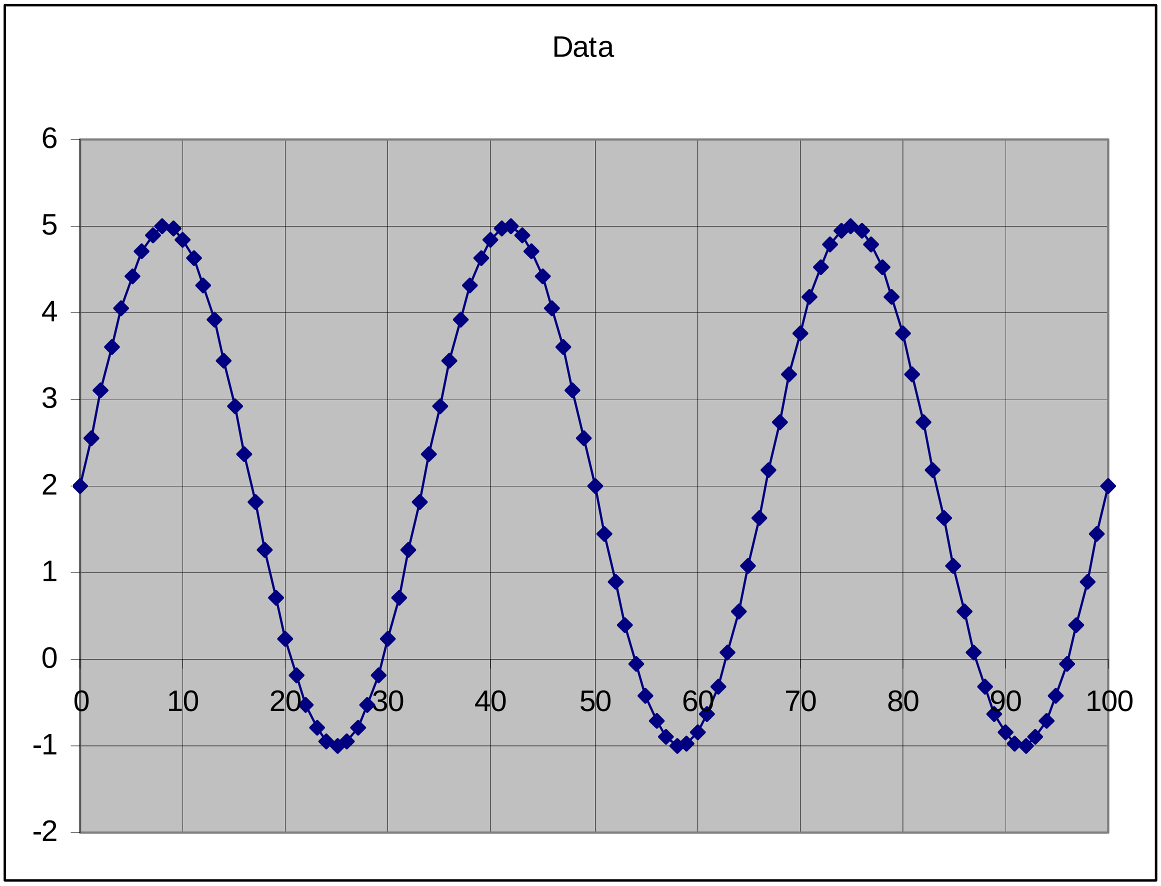
The same data with damping looks like:



e. Find the damping constant α for this data. Include units. *(6 points)*

f. Write the mathematical expression for this data $X = X_o + X_1 \sin(\omega t + \phi_o)e^{-\alpha t}$
Your units must be consistent. *(4 points)*

Spring 2004 solution
4) Sinusoids (20 points) [Both A and B]
The following data was created using Excel.



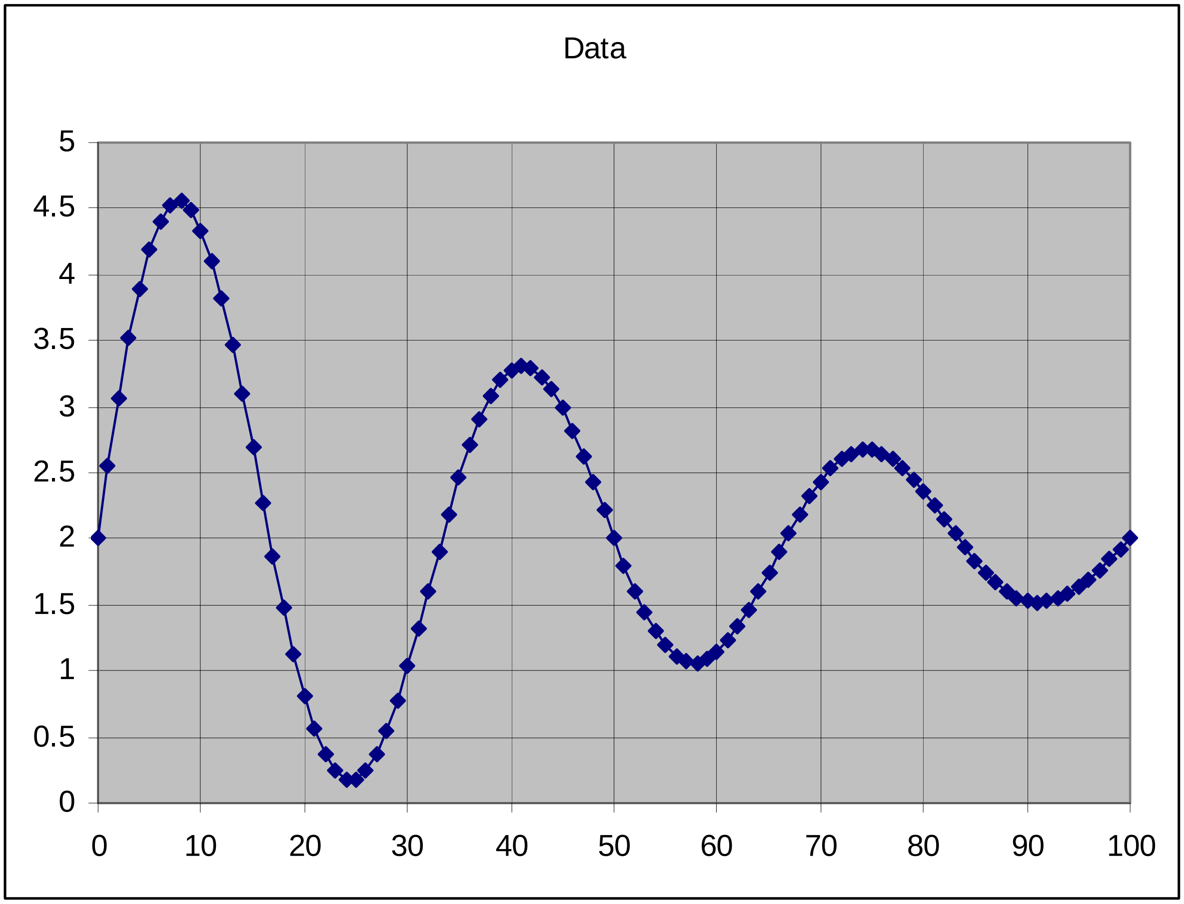
g. Assuming the horizontal scale is in seconds, find the frequency and period of this signal. Include units. *(4 points)*
T=33s – 0 ***T=33s***
f = 1/33s ***f = 0.03 Hz***

h. What is the DC offset of this signal? Include units. *(2 points)*
VDC = 2V

i. What is the phase of the signal? Include units. *(2 points)*
phase = 0 degrees or 0 radians

j. Write the mathematical expression for this signal and its offset. In general, this is given by $X = X_o + X_1 \sin(\omega t + \phi_o)$ which accounts for its frequency and phase shift. *(2 points)*
$\omega = 2\pi f$ *= 2*3.24*0.03 = 0.1885 rad/sec* ***X = 2V + 3V sin (0.1885 t)***

The same data with damping looks like:



k. Find the damping constant α for this data. Include units. *(6 points)*

VDC = 2V Therefore, let V=2V be the zero point. V0=(7s, 4.6-2) = (7,2.6)
V1=(74s,2.7-2) =(74,0.7)
V1 = V0 $e^{-\alpha(t1-t0)}$ *0.7 = 2.6* $e^{-\alpha(74-7)}$ $-\alpha(67)=\ln(0.7/2.6)$ $\boldsymbol{\alpha}$***=0.0196/s***

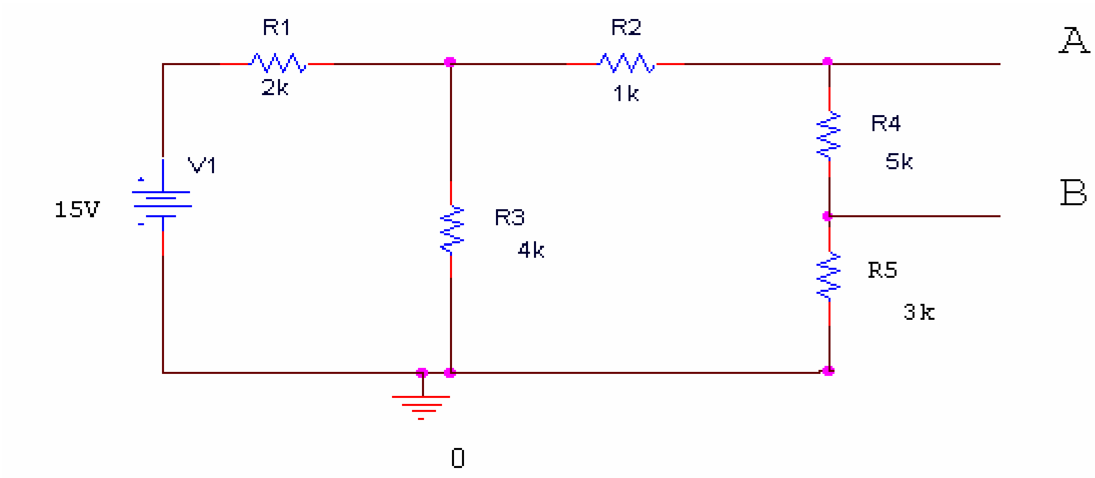
l. Write the mathematical expression for this data $X = X_o + X_1 \sin(\omega t + \phi_o)e^{-\alpha t}$
Your units must be consistent. *(4 points)*

From part d we know that:
VDC = 2V Amplitude = 3V f=0.03Hz w=2π*f=0.1885 rad/sec*

X = 2V + 3V sin (0.1885 t) e $^{-0.0196\,t}$

Spring 2003

2. Thevenin circuits (20 points)



a) (6 points) Find the Thevenin voltage (Voc) of the circuit assuming the load will be connected between A and B.

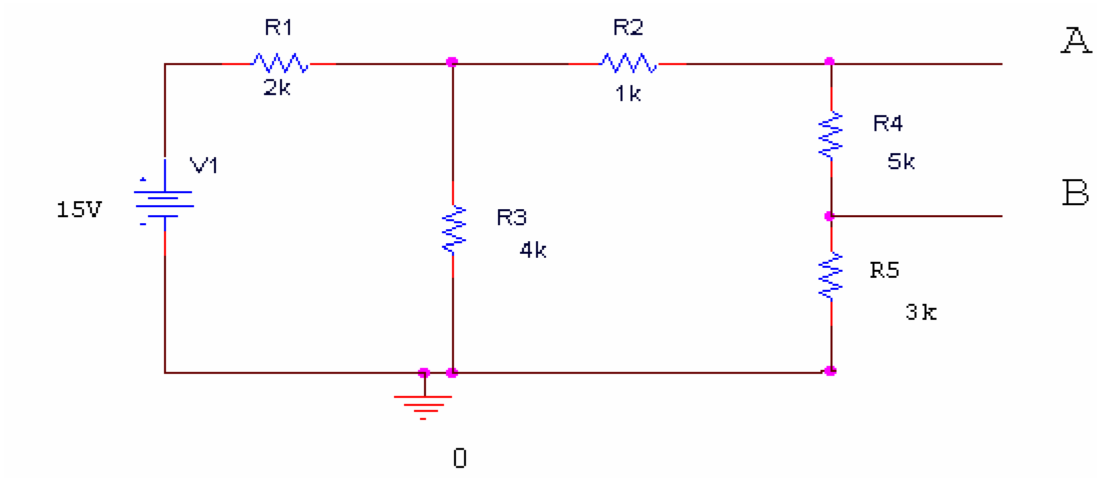
b) (6 points) Find the Thevenin resistance.

c) (4 points) Draw the Thevinen equivalent circuit with a load of 1K ohms.

d) (4 points) Find the voltage between A and B for this circuit with a load of 1K ohms.

Spring 2003 solution

2. Thevenin circuits (20 points)



a) (6 points) Find the Thevenin voltage (Vth) of the circuit assuming the load will be connected between A and B.

R245 = R2+R4+R5 = 1k+5k+3k = 9k
R2345 = R3 // R245 = (9kx4k)/(9k+4k) = 2.77k
V2345 = V1(R2345)/(R1+R2345) = 15x2.77/(2+2.77) = 8.71 volts
V4 = V2345xR4/(R2+R4+R5) = (8.71x5k)/9k = 4.84 volts

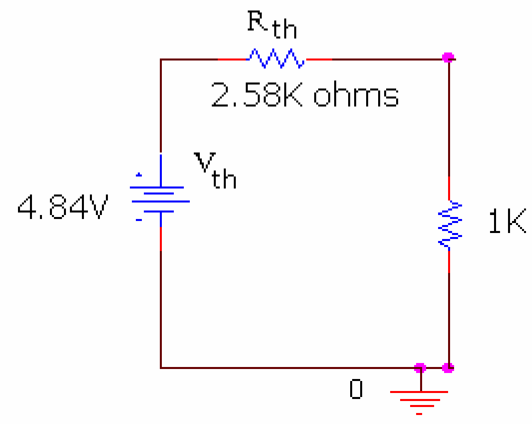
Vth = 4.84 volts

b) (6 points) Find the Thevenin resistance.

R13 = R1//R3=(R1xR3)/(R1+R3) = (2kx4k)/(2k+4k) = 1.33k
R1235 = R13 + R2 + R5 = 1.33K + 1K + 3k = 5.33 K
R12345 = R1235//R4 = (5.33x5)/(5.33+5) = 2.58K ohms

Rth = 2.58 K ohms

c) (4 points) Draw the Thevinen equivalent circuit with a load of 1K ohms.



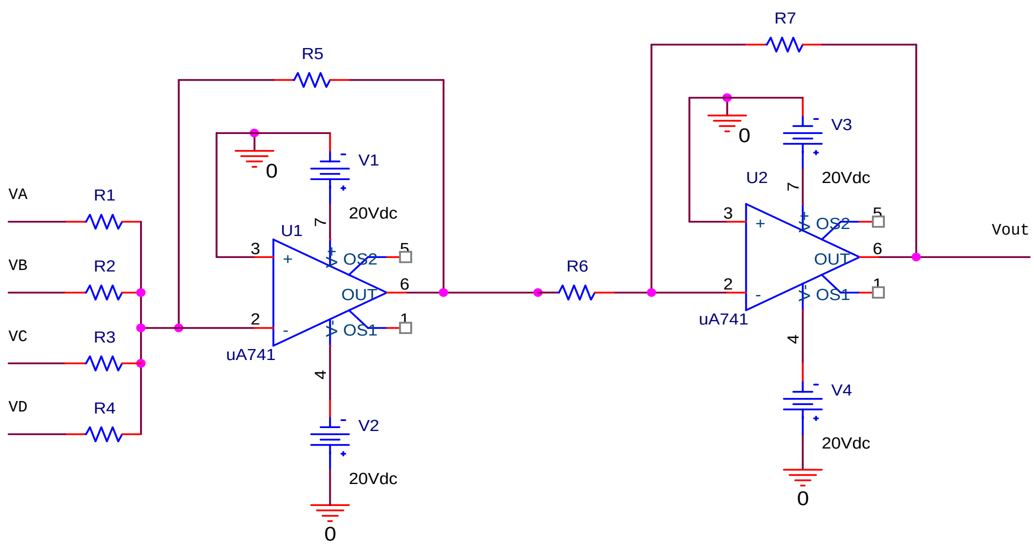
d) (4 points) Find the voltage between A and B for this circuit with a load of 1K ohms.

VAB = (Vthx1K)/(1K+Rth) = (4.84x1k)/(1K+2.58K) = 1.35 volts

<u>VAB = 1.35 volts</u>

Fall 2003
Question 4 -- Digital-to-Analog Converter (20 points)

The circuit below converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word DCBA in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic of "one" and 0 Volts for a logic "zero" and that R5 = 5KΩ, R6 = 2KΩ and R7 = 30KΩ.



a) Select values for R1, R2, R3, and R4 so that the output voltage will be the decimal equivalent of DCBA. For example, if DCBA=1010, or equivalently VD=VB=5 V, VA=VC=0 V, then Vout = 10 V. The circuit should work for **all** possible DCBA combinations. *(12 points)*

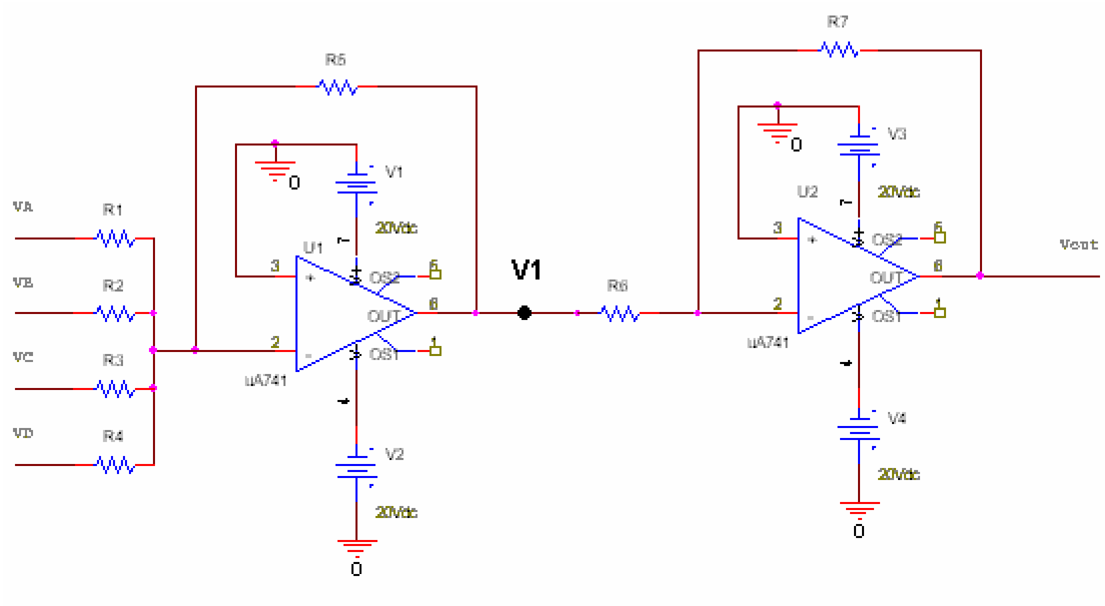
R1 =　　　　R2 =　　　　R3 =　　　　R4 =

b) Show that the circuit correctly converts binary input 0111 to an AC voltage. What decimal number does 0111 represent? *(4 points)*

c) Explain one way you could modify the values of the resistors in this circuit so that the output voltage gives 4*N (rather than N), when N is the digital number at the input . For example, when the input is DCBA=1010, then Vout = 4*10 V or 40V. You can modify any of the resistors R1-R7.*(4 points)*

Fall 2003 Solution
Question 4 -- Digital-to-Analog Converter (20 points)

The circuit below converts digital signals into analog signals. This circuit produces an analog output voltage equal to the binary word DCBA in terms of the four inputs. Please assume that the input voltage levels for this circuit is 5 Volts for a logic of "one" and 0 Volts for a logic "zero" and that R5 = 5KΩ, R6 = 2KΩ and R7 = 30KΩ.



a) Select values for R1, R2, R3, and R4 so that the output voltage will be the decimal equivalent of DCBA. For example, if DCBA=1010, or equivalently VD=VB=5 V, VA=VC=0 V, then Vout = 10 V. The circuit should work for **all** possible DCBA combinations. *(12 points)*

V1=(-R5)[(VA/R1)+(VB/R2)+(VC/R3)+(VD/R4)]   *Vout = (-R7/R6)V1*

*Vout=(R5*R7/R6)[(VA/R1)+(VB/R2)+(VC/R3)+(VD/R4)]*

*(R5*R7/R6)=(5K*30K)/2K=75K*

Vout	*VA*	*VB*	*VC*	*VD*	*plug in*	*solve*
1	*5V*	*0*	*0*	*0*	*75K(5/R1)=1*	*R1=375K*
2	*0*	*5V*	*0*	*0*	*75K(5/R2)=2*	*R2=187.5K*
4	*0*	*0*	*5V*	*0*	*75K(5/R3)=4*	*R3=93.75K*
8	*0*	*0*	*0*	*5V*	*75K(5/R4)=8*	*R4=46.875K*

R1 = 375K   ***R2 = 187.5K***   ***R3 = 93.75K***   ***R4 = 46.875K***

b) Show that the circuit correctly converts binary input 0111 to an AC voltage. What decimal number does 0111 represent? *(4 points)*

*Vout=(R5*R7/R6)[(VA/R1)+(VB/R2)+(VC/R3)+(VD/R4)]*
Vout=(75K)[(5/375K)+(5/187.5K)+(5/93.75K)+(0/46.875K)]
Vout = 1+2+4=7 volts

0111 = ***7 decimal***

c) Explain one way you could modify the values of the resistors in this circuit so that the output voltage gives 4*N (rather than N), when N is the digital number at the input . For example, when the input is DCBA=1010, then Vout = 4*10 V or 40V. You can modify any of the resistors R1-R7.*(4 points)*

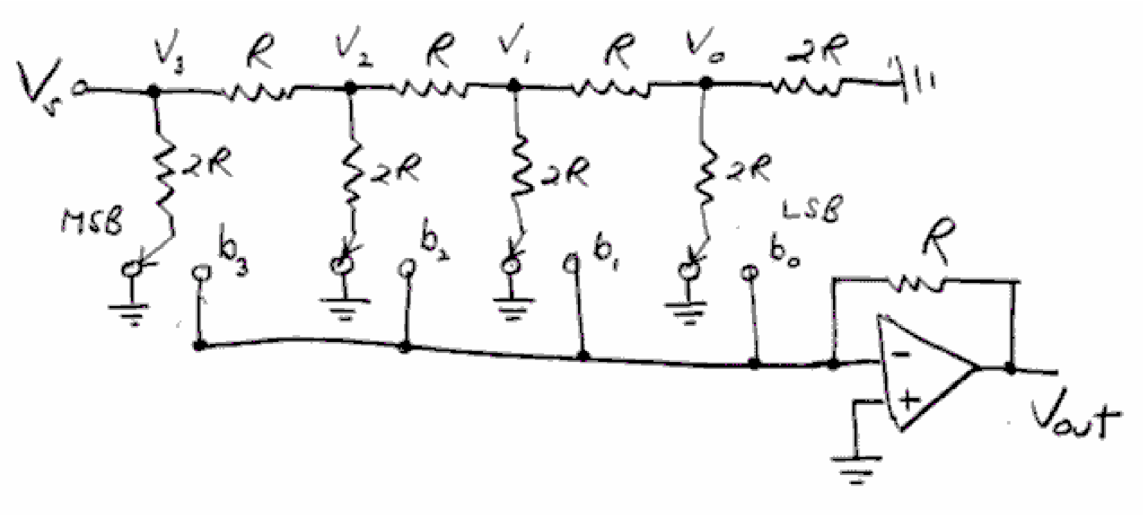
The easiest way to do this is to modify one of the gain resistors:
*R5 = 4*R5 = 20K*
or
*R7 = 4*R7=120K*
or
R6 = R6/4 = 1.25K

You could also modify the four input resistors:
R1 = R1/4 =93.75 and R2 = R2/4=46.875K and
R3 =R3/4= 23.47 and R4 = R4/4= 11.73K

A combination which changes the gain would work also:
*R5 = R5*2=10K and R6 = R6/2 = 2.5K*

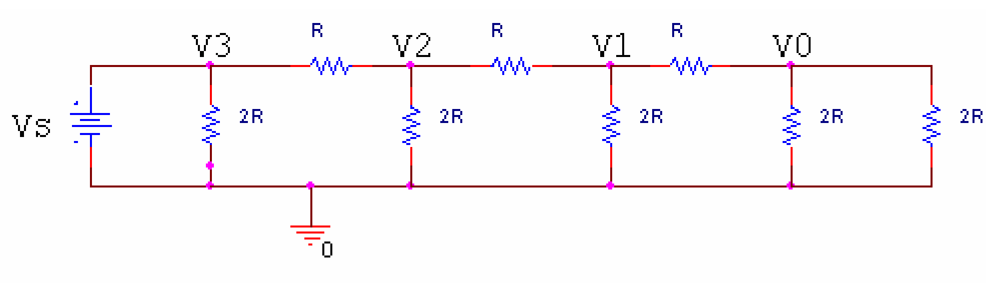
Spring 2004
Question 4 -- Digital-to-Analog Converter (20 points)

For a computer or other digital device to interface with external analog circuits and devices, a digital-to-analog converter (DAC) is required. The most common DAC is a R-2R resistor ladder network, which requires only two precision resistor values R and 2R. A 4-bit R-2R resistor ladder network is shown below:



The digital input to the DAC is a 4-bit binary number represented by bits b0, b1, b2 and b3, where b0 is the least significant bit (LSB) and b3 is the most significant bit (MSB). Each bit in the circuit controls a switch between ground and the *inverting* input op amp. When a bit is 1, the corresponding switch is connected to the op-amp; when a bit is zero, the corresponding switch is connected to ground.

a) If we assume this is an ideal op-amp, we can analyze the voltage levels in the circuit by removing the op amp. Below is a picture of the circuit when all bits are zero. Use the simplified circuit below to determine the voltage levels at V0, V1, V2 and V3 in terms of the source voltage, Vs. *(8 points)*



b) If we assume that closing switches has negligible effect on the voltage levels we found in part a), what will be the output of the DAC circuit, Vout (in terms of Vs), when the input (b3 b2 b1 b0) is: *(10 points)*

0001:

0010:

0100:

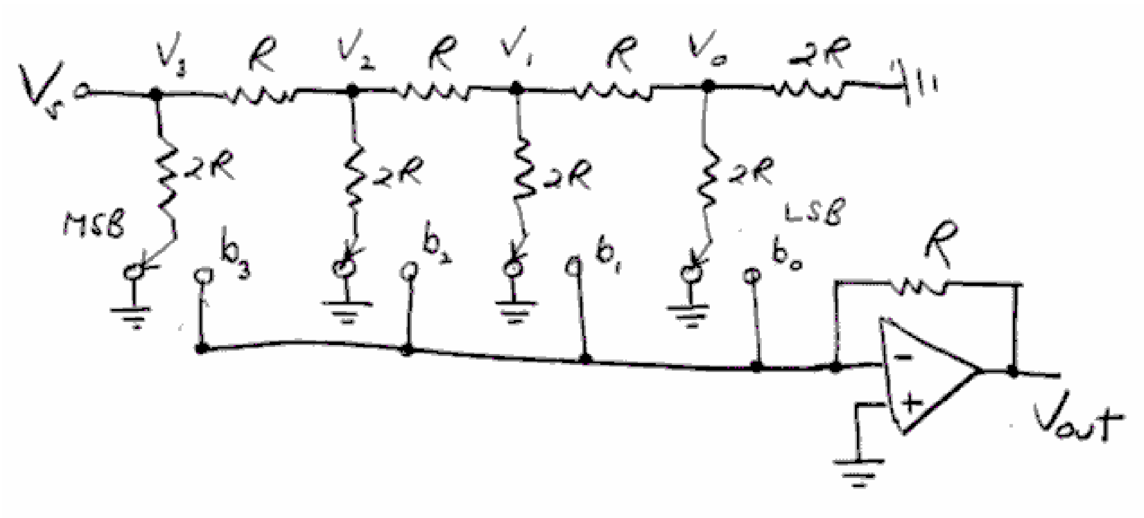
1000:

1111: [Hint: Use principal of superposition.]

c) In terms of Vs, what is the range of the analog output for a 4 bit binary input? (ie. If the input ranges from 0000 to 1111, what is the output range?) *(2 points)*

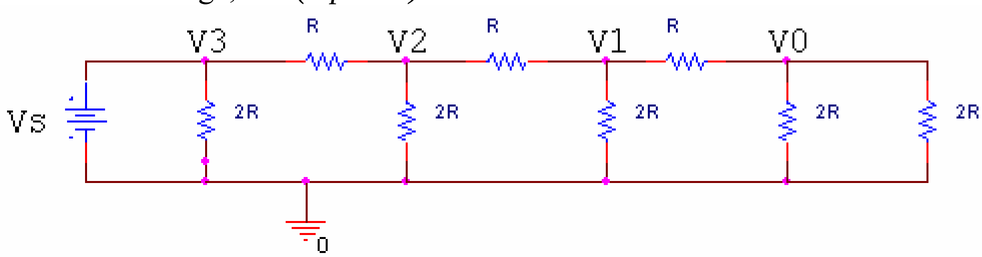
Spring 2004 solution
Question 4 -- Digital-to-Analog Converter (20 points)

For a computer or other digital device to interface with external analog circuits and devices, a digital-to-analog converter (DAC) is required. The most common DAC is a R-2R resistor ladder network, which requires only two precision resistor values R and 2R. A 4-bit R-2R resistor ladder network is shown below:

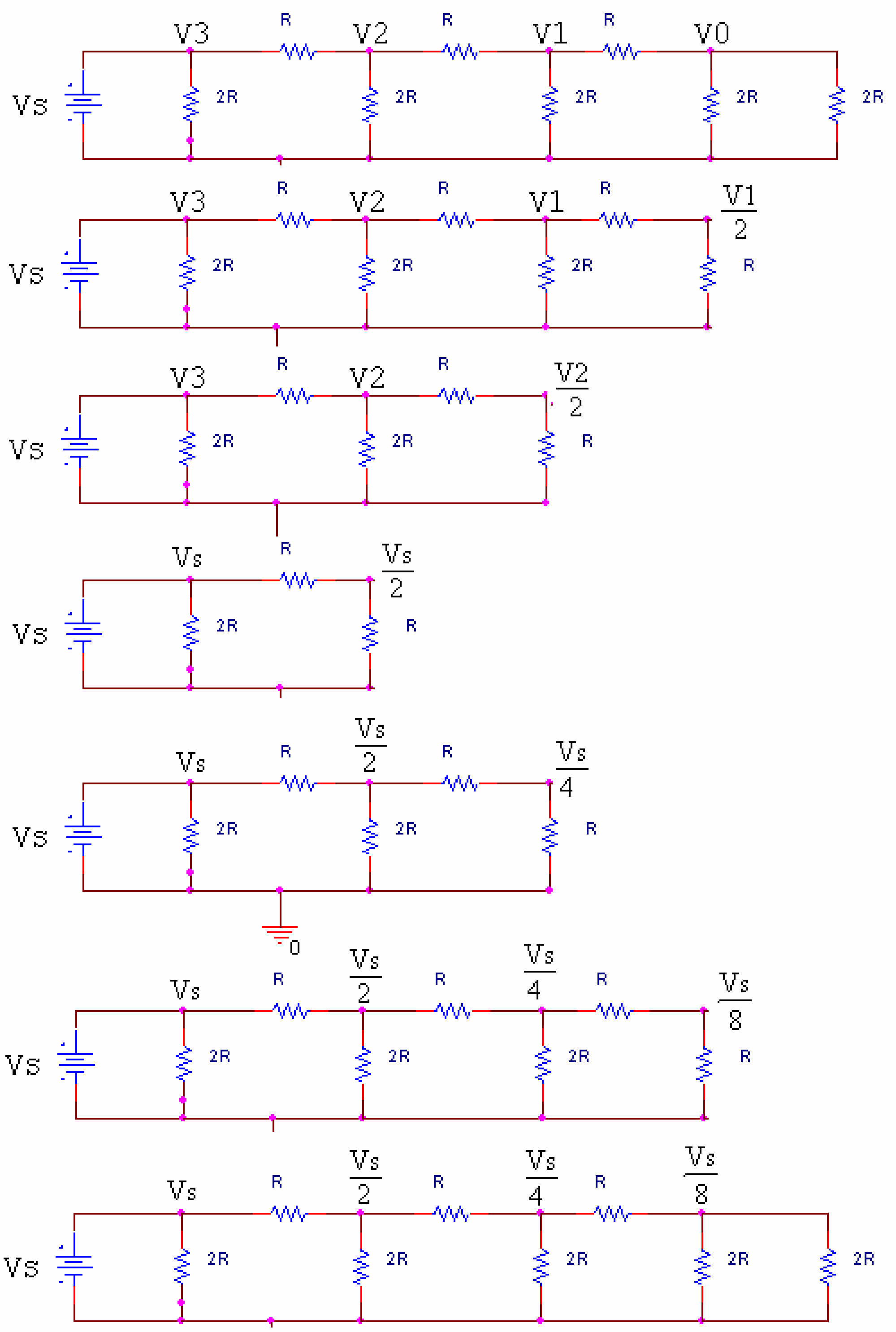


The digital input to the DAC is a 4-bit binary number represented by bits b0, b1, b2 and b3, where b0 is the least significant bit (LSB) and b3 is the most significant bit (MSB). Each bit in the circuit controls a switch between ground and the *inverting* input op amp. When a bit is 1, the corresponding switch is connected to the op-amp; when a bit is zero, the corresponding switch is connected to ground.

a) If we assume this is an ideal op-amp, we can analyze the voltage levels in the circuit by removing the op amp. Below is a picture of the circuit when all bits are zero. Use the simplified circuit below to determine the voltage levels at V0, V1, V2 and V3 in terms of the source voltage, Vs. *(8 points)*



*See following page for circuit analysis. When you combine 2 2R resistors in parallel, you get (2R*2R)/(2R+2R) = R. If you do this to the two 2R resistors in parallel at V0, you get a voltage divider that divides V1 in half. If you add the 2 1R resistors and combine them in parallel with the 2R resistor at V1, you get another voltage divider that divides V2 in half. You can continue this process until you get the voltage divider that divides V3 into half to get V2. You know that V3 is Vs. Therefore, V2 is Vs/2. You can then apply the relationships in reverse to get all the voltages.*



V3 R V2 R V1 R V0
Vs 2R 2R 2R 2R 2R
V3 R V2 R V1 R V1/2
Vs 2R 2R 2R R
V3 R V2 R V2/2
Vs 2R 2R R
Vs R Vs/2
Vs 2R R
Vs R Vs/2 R Vs/4
Vs 2R 2R R
0
Vs R Vs/2 R Vs/4 R Vs/8
Vs 2R 2R 2R R
Vs R Vs/2 R Vs/4 R Vs/8
Vs 2R 2R 2R 2R 2R

b) If we assume that closing switches has negligible effect on the voltage levels we found in part a), what will be the output of the DAC circuit, Vout (in terms of Vs), when the input (b3 b2 b1 b0) is: *(10 points)*

This assumption can be made because the op-amp is ideal and it tries to keep the voltages at the inputs the same. The positive input is grounded, therefore, the negative input is ground and the input circuit does not change regardless of the position of the switches. The value of Vout is determined by the inverting op-amp, which is acting on the input voltage of the corresponding bit with an input resistance of 2R and a feedback resistance or R.

0001: *Vout = -(R/2R)V0 = (-1/2)(Vs/8)* ***Vout = -(1/16)Vs***

0010: *Vout = -(R/2R)V1 = (-1/2)(Vs/4)* ***Vout = -(1/8)Vs***

0100: *Vout = -(R/2R)V2 = (-1/2)(Vs/2)* ***Vout = -(1/4)Vs***

1000: *Vout = -(R/2R)V3 = (-1/2)(Vs)* ***Vout = -(1/2)Vs***

1111: [Hint: Use principal of superposition.]
Vout = -(1/16)Vs + -(1/8)Vs + -(1/4)Vs + -(1/2) Vs
Vout = -(1/16+2/16+4/15+8/16)Vs

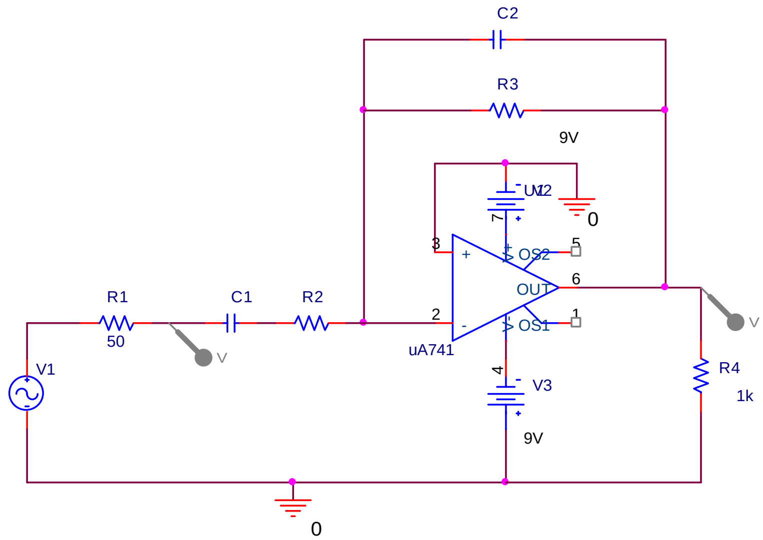
Vout = -(15/16)Vs

c) In terms of Vs, what is the range of the analog output for a 4 bit binary input? (ie. If the input ranges from 0000 to 1111, what is the output range?) *(2 points)*

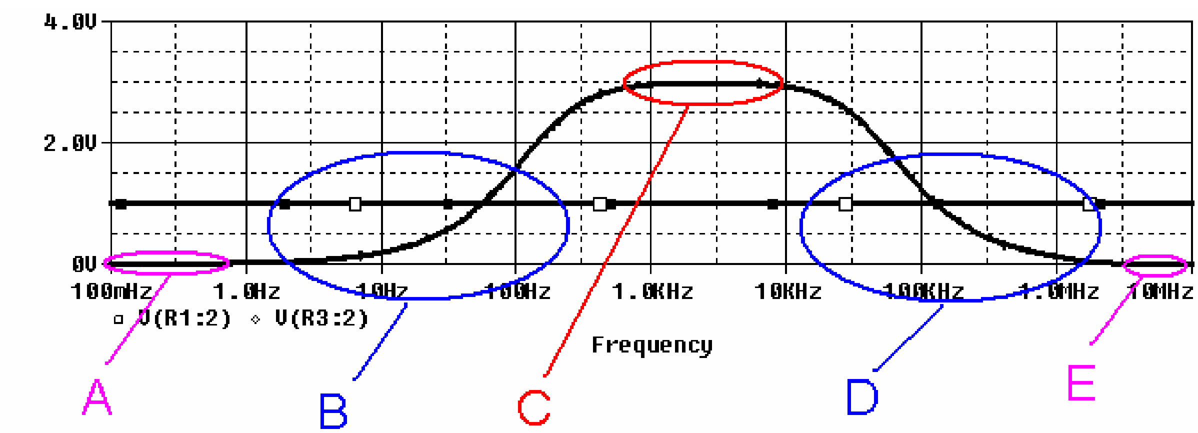
The range of outputs is from 0 volts to –(15/16)Vs.

Spring 2004
4. Op-Amps (25 points)

Here is a combined differentiator/integrator similar to the one you implemented in experiment 8. Let C1=0.01μF, R2=100K ohms, C2= 0.01 nF, and R3=300K ohms.



a. Below is an AC sweep for the above circuit.



i) identify the input and the output traces.*(2 points)*

ii) If you built this circuit in the studio, in which of the circled regions would the output look like the following? *(2 points each) [Total=8 points]*

a reasonable integration of the input?

a reasonable differentiation of the input?

an amplified inversion of the input?

disappear into the noise?

b. What are the general equations for the following: (Give specific values based on the components in the circuit.)

i. The circuit when it is acting as an ideal integrator *(3 points)*

ii. The circuit when it is acting as an ideal differentiator *(3 points)*

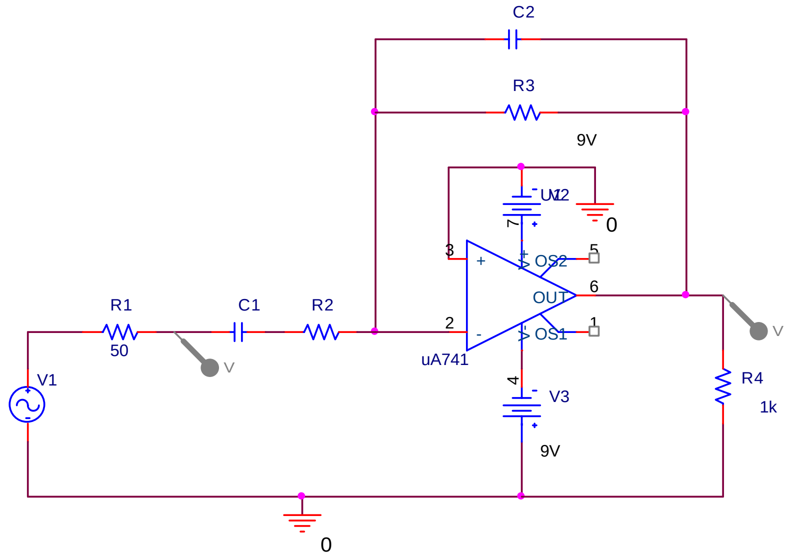
iii. The circuit when it is acting as an inverting amplifier *(3 points)*

cA. Sketch the AC sweep of an integrator that integrates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. *(6 points)*

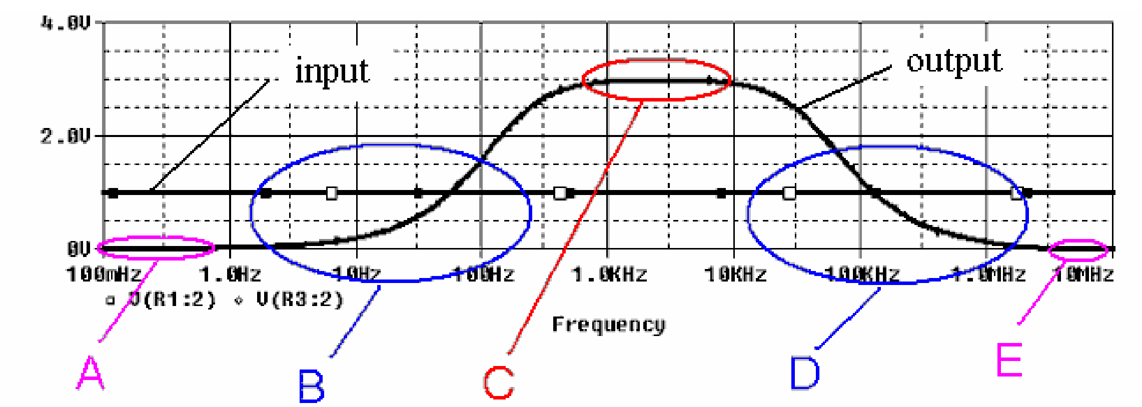
cB. Sketch the AC sweep of a differentiator that differentiates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. *(6 points)*

Spring 2004 solution
4. Op-Amps (25 points) *(Test A Only)*

Here is a combined differentiator/integrator similar to the one you implemented in experiment 8. Let C1=0.01µF, R2=100K ohms, C2= 0.01 nF, and R3=300K ohms.



b. Below is an AC sweep for the above circuit



i) Identify the input and the output traces.*(2 points) (Test A)*

ii) If you built this circuit in the studio, in which of the circled regions would the output look like the following? *(2 points each) [Total=8 points]*

a reasonable integration of the input? ***D***

a reasonable differentiation of the input? ***B***

an amplified inversion of the input? ***C***

disappear into the noise? ***A,E***

b. What are the general equations for the following: (Give specific values based on the components in the circuit.)

i. The circuit when it is acting as an ideal integrator *(3 points)*

$$v_{out} = \frac{-1}{R_{in}C_f}\int v_{in}dt = \frac{-1}{(100K)(0.01n)}\int v_{in}dt = -1Meg\int v_{in}dt$$

(Note how the large gain compensates for 1/ω in the integration. ω is large, its inverse is small, so a large gain is needed to make the signal recognizable.)

ii. The circuit when it is acting as an ideal differentiator *(3 points)*

$$v_{out} = -R_f C_{in}\frac{dv_{in}}{dt} = -(300K)(0.01\mu)\frac{dv_{in}}{dt} = -3m\frac{dv_{in}}{dt}$$

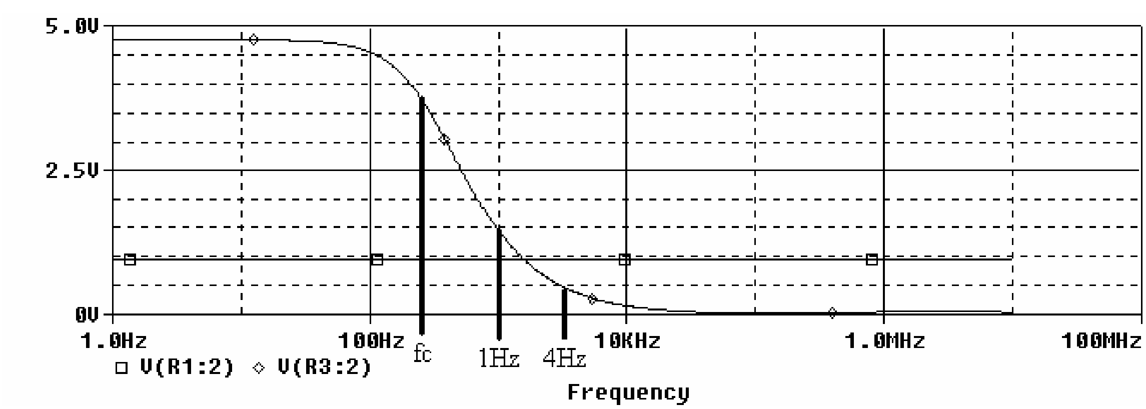
(Note how the small gain compensates for ω in the differentiation. ω is large, so the gain must be small or the op-amp will saturate.)

iii. The circuit when it is acting as an inverting amplifier *(3 points)*

$$v_{out} = \frac{-R_f}{R_{in}}v_{in} = \frac{-300K}{100K}v_{in} = -3v_{in}$$

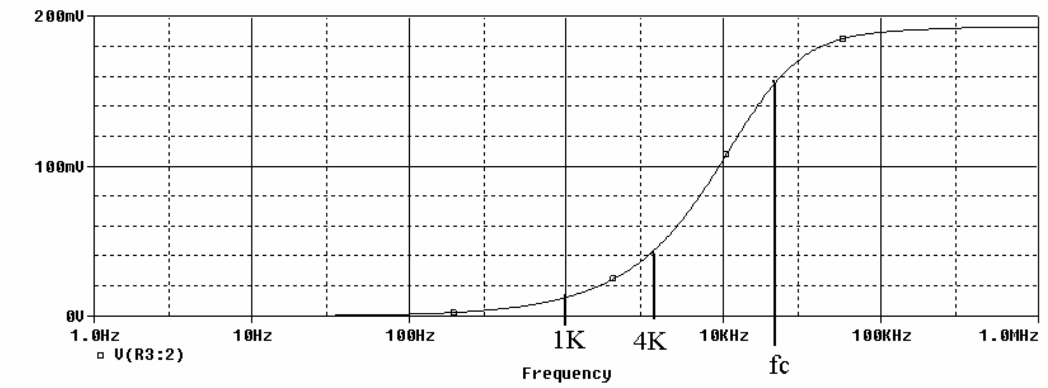
(Note that the input amplitude is 1V and at C, where the circuit is an inverting amplifier, the output amplitude is 3)

cA. Sketch the AC sweep of an integrator that integrates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. *(6 points)*



The corner frequency in this case is about 200 Hertz. Note that answers may vary. Corner frequencies should be at or below 500Hz.

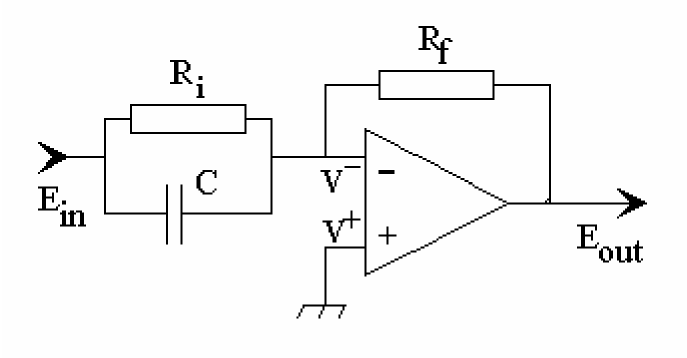
cB. Sketch the AC sweep of a differentiator that differentiates between 1K and 4K hertz. Give a ballpark estimate of the corner frequency. Mark 1K, 4K and the corner frequency on the sketch. Justify your decisions. *(6 points)*



The corner frequency in this case is about 20K Hertz. Note that answers may vary. Corner frequencies should be at or above 8K Hz

Fall 2002 Solution
4. Op-amp Analysis (20 points)

The circuit below shows an op-amp differentiator which has been modified by the addition of a resistor in parallel with the input capacitor.



a) What are the two rules for op-amp analysis (2 pnts)?

b) Using these two rules you have stated above, find the transfer function for the above circuit H(jω) = Eout/Ein. You must show your work! (10 pnts)

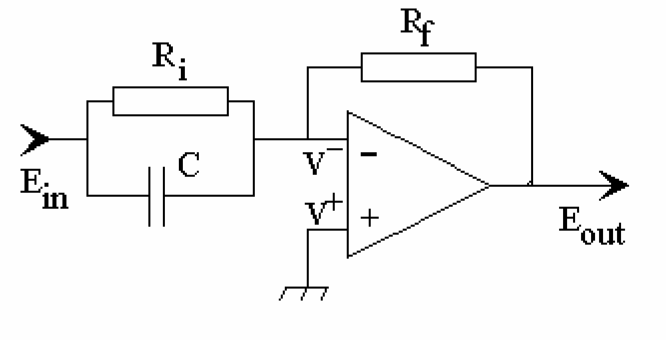
c) Use the transfer function from b) to find how the circuit behaves at low frequencies. Give the function in terms of ω (2 pnts), the magnitude (1 pnt) and the phase (1 pnt).

d) Use the transfer function from b) to find how the circuit behaves at high frequencies. Give the function in terms of ω (2 pnts), the magnitude (1 pnt) and the phase (1 pnt).

Extra credit (1 pnt): Is this a good differentiator? Why or why not?

Fall 2002 Solution
4. Op-amp Analysis (20 points)

The circuit below shows an op-amp differentiator which has been modified by the addition of a resistor in parallel with the input capacitor.



a) What are the two rules for op-amp analysis (2 pnts)?

1) $V^+ = V^-$ 2) $I^+ = I^- = 0$

b) Using these two rules you have stated above, find the transfer function for the above circuit H(jω) = Eout/Ein. You must show your work! (10 pnts)

$$Zin = \frac{R_i\left(\frac{1}{j\omega C}\right)}{R_i + \left(\frac{1}{j\omega C}\right)} = \frac{R_i}{j\omega R_i C + 1} \qquad I = \frac{E_{in} - V^-}{Zin} = \frac{V^- - E_{out}}{R_f} \qquad V^- = V^+ = 0 \qquad \frac{E_{in}}{Zin} = \frac{-E_{out}}{R_f}$$

$$\frac{E_{out}}{E_{in}} = -\frac{R_f}{Zin} = -\frac{R_f(j\omega R_i C + 1)}{R_i} \qquad H(j\omega) = -\frac{j\omega R_i R_f C + R_f}{R_i}$$

c) Use the transfer function from b) to find how the circuit behaves at low frequencies. Give the function in terms of ω (2 pnts), the magnitude (1 pnt) and the phase (1 pnt).

$$H_{LO} = -\frac{R_f}{R_i} \qquad |H_{LO}| = \frac{R_f}{R_i} \qquad \angle H_{LO} = \pi \text{ or } -\pi$$

d) Use the transfer function from b) to find how the circuit behaves at high frequencies. Give the function in terms of ω (2 pnts), the magnitude (1 pnt) and the phase (1 pnt).

$$H_{HI} = -\frac{j\omega R_i R_f C}{R_i} = -j\omega R_f C \qquad |H_{HI}| = \infty \qquad \angle H_{HI} = -\frac{\pi}{2}$$

Extra credit (1 pnt): Is this a good differentiator? Why or why not?

This is not a good differentiator because according to its transfer function, it is supposed to differentiate at high frequencies. However, at high frequencies, the output approaches infinity. Therefore, it would only differentiate when its magnitude was approaching infinity, which is somewhat useless. It would also likely saturate the op-amp before it did anything useful.