

ENGR4300
Fall 2005
Test 1B

Name.....solution.....

Section_____

Question 1 (25 points)_____

Question 2 (20 points) _____

Question 3 (30 points)_____

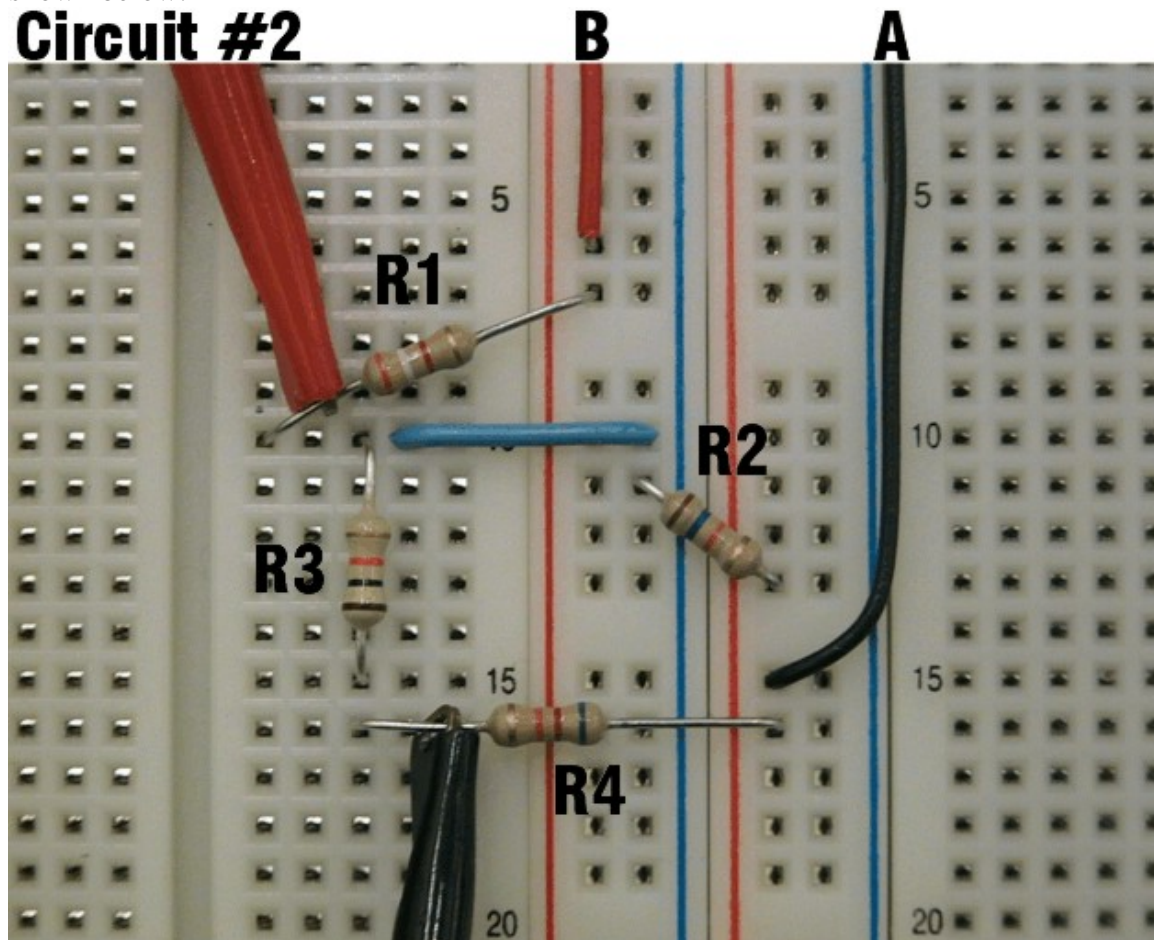
Question 4 (25 points)_____

Total (100 points): _____

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Question 1 – Resistive Circuits (25 points)

A circuit is constructed on the same type of proto board issued to each team in EI, as shown below:



(a) The resistors have the following color codes. Write the corresponding values and tolerance to the right of each (2 points):

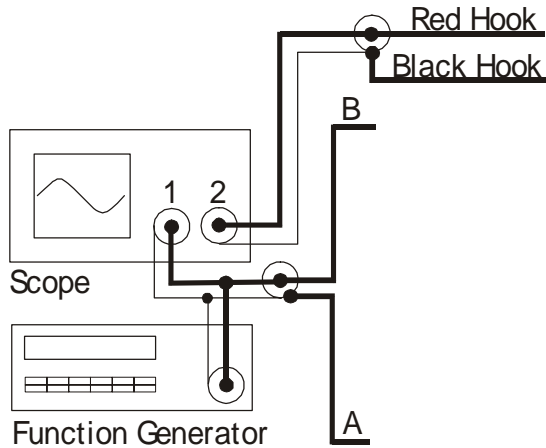
R1 = Orange White Red Gold = $3.9\text{kohm} \pm 5\%$ (or $3.9 \pm 0.195\text{ kohm}$)

R2 = Brown Blue Orange Gold = $16\text{kohm} \pm 5\%$

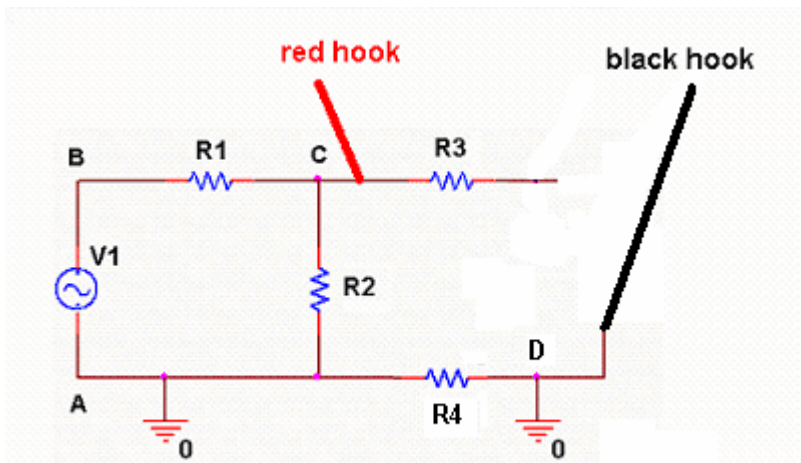
R3 = Brown Black Orange Gold = $10\text{kohm} \pm 5\%$

R4 = Blue Red Orange Gold = $62\text{kohm} \pm 5\%$

An AC signal is applied across points A and B in the circuit by connection to a function generator and this signal is also monitored on channel 1 of an oscilloscope, while a signal of interest is monitored on channel 2 of the scope using the e-z hooks (mini-grabber) shown in the photo.



(b) In the figure above, draw the schematic of the circuit shown (do not simplify), label R1, R2, R3, and R4, and show connections of the four wires going to our instruments to the appropriate points on your schematic. (6 points)

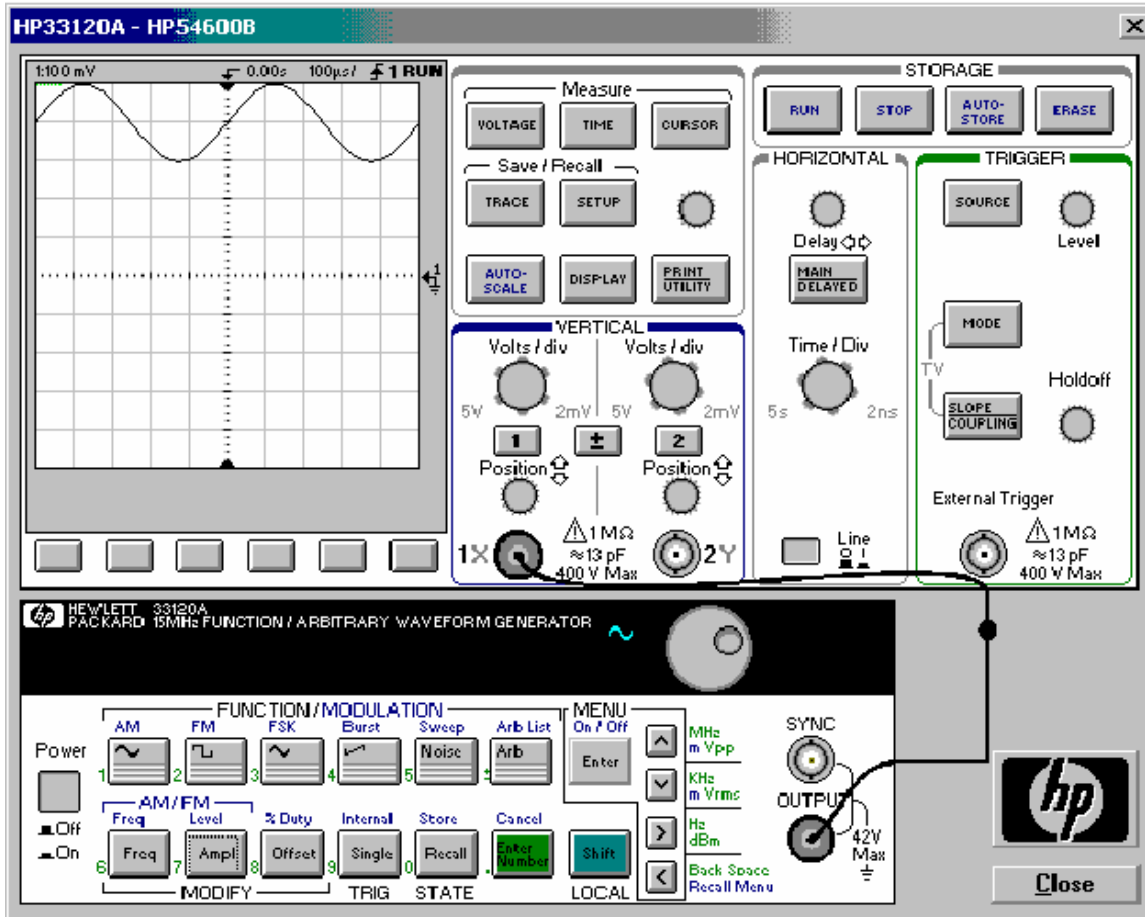


R4 is grounded on both sides, so it is not part of circuit. R3 has no current flowing through it, so it is also not part of the circuit. This is just a voltage divider using R1 and R2. $R1=3.9k$ $R2=16k$

$$V_c = V1(16k)/(3.9k+16k) = .804 V1$$

$$AMPL = 100mV * .804 = 80.4mV$$

$$DC\ offset = 400mV * .804 = 321.6mV$$



(c) Given the signal displayed on the scope above, write the expression for our input signal in the form:

_____ + _____ sin(_____ * t + _____)
and give the units with *each* value above (4 points).

$$f = 1/500\mu = 2000 \quad \omega = 2\pi(2000) = 12566$$

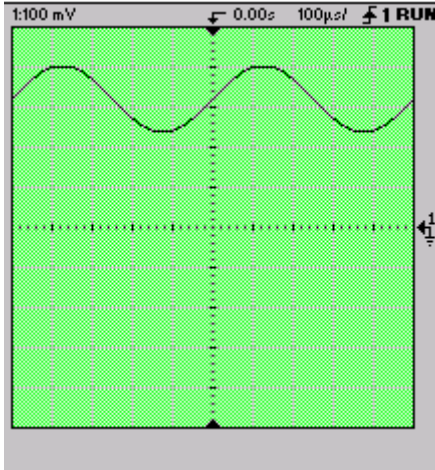
There is no phase because the scope triggers at 0.
 $400mV + 100mV \sin(12566 \text{ rad/sec} * t + 0 \text{ rad})$

(d) What is the frequency in Hz of the above signal? (2 points) _____

$2k \text{ Hz}$.

(e) On the scope display shown above, draw the voltage that would be measured on channel 2 of the scope if connected as shown previously (4 points).
(see calculations with circuit)

amplitude = 80mV DC offset = 322mV



(f) What is the average voltage applied across A-B in our circuit? (1 point) _____

400mV

(g) Find the average current flowing in resistors R1 & R2 (4 points).

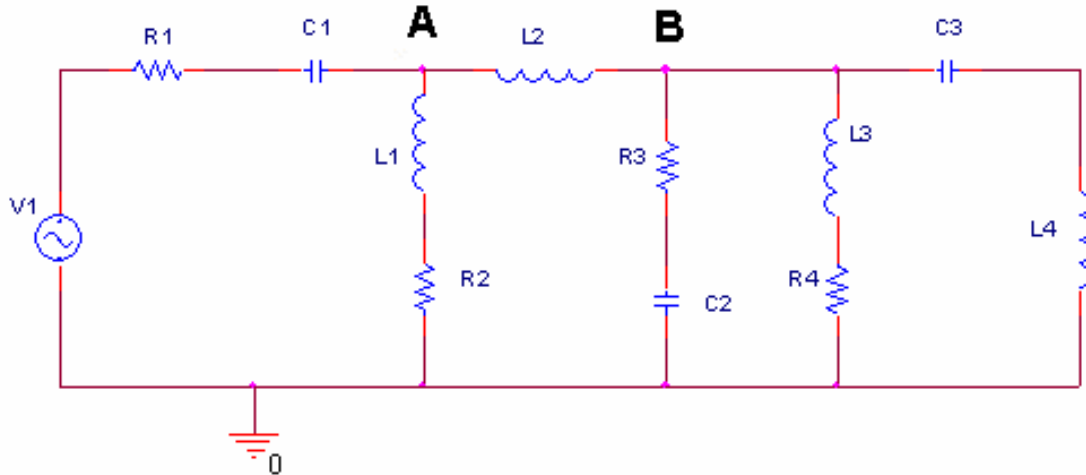
$$VR1 = 400 - 322 = 78mV \quad IR1 = IR2 = 78m / 3.9k = 0.0204mA$$

$$I_{R1} = 20.4\mu A$$

$$I_{R2} = 20.4\mu A$$

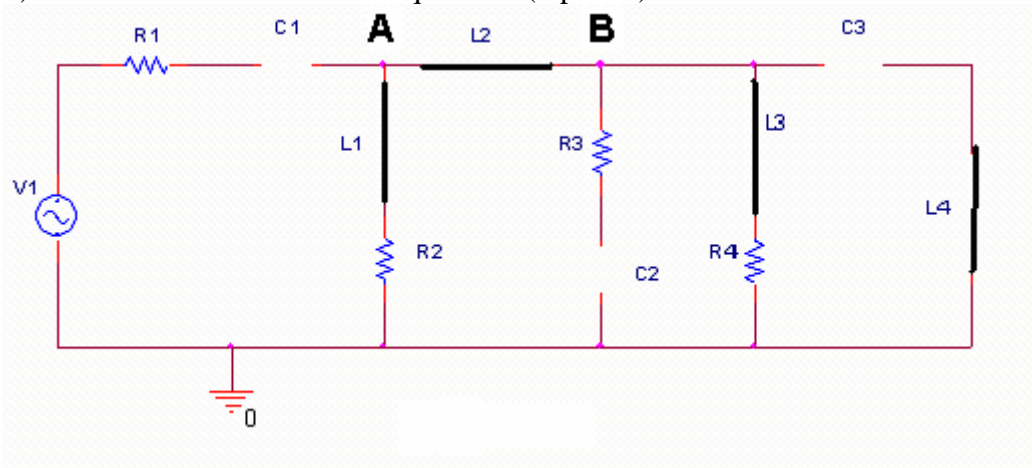
Question 2 – Filters (20 points)

You are given the following circuit. The input at V1 has the following properties:
 $V_{AMPL} = 200\text{mV}$, $FREQ = 1\text{KHz}$, $V_{OFF} = 0\text{V}$



A. The behavior of this circuit at low frequencies

1) Redraw the circuit at low frequencies (4 points)



2) What is the amplitude of the voltage at point A at low frequencies? (2 points)

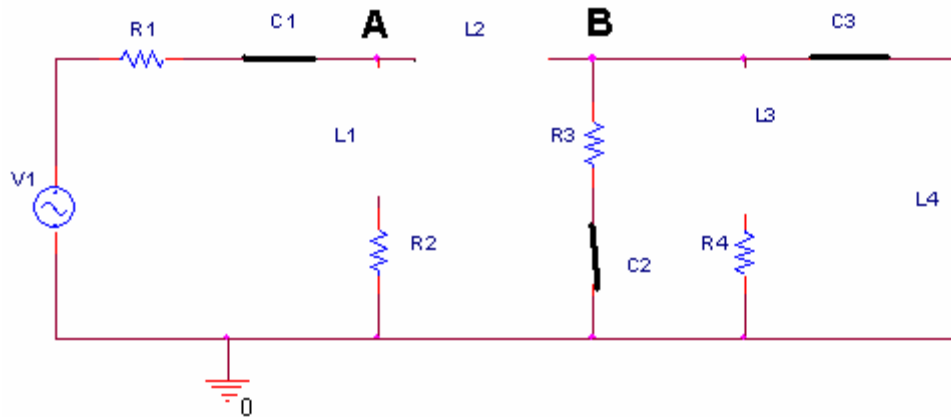
$$V_A = 0V$$

3) What is the amplitude of the voltage at point B at low frequencies? (2 points)

$$V_B = 0V$$

B. The behavior of the circuit at high frequencies

1) Redraw the circuit at high frequencies (4 points)



2) What is the amplitude of the voltage at point A at high frequencies? (2 points)

$$V_A = 200mV$$

3) What is the amplitude of the voltage at point B at high frequencies? (2 points)

$$V_B = 0V$$

C. Is this a filter?

1) What type of filter could this be at point A (circle one)? (2 points)

Low Pass

High Pass

Neither

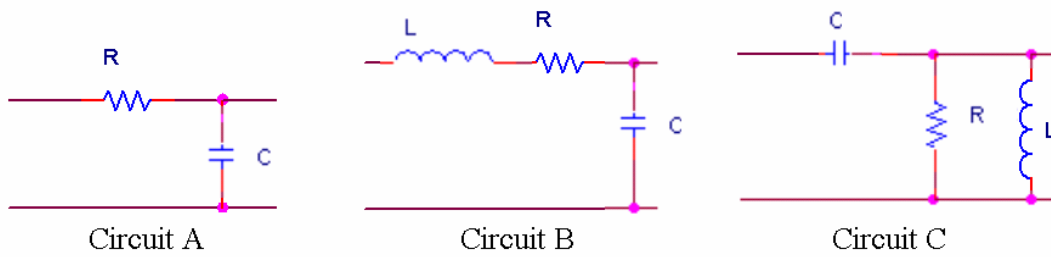
2) What type of filter could this be at point B (circle one)? (2 points)

Low Pass

High Pass

Neither

Question 3 – Transfer Functions (30 points)



A. Transfer Functions

1) What is the transfer function for circuit A? You must simplify. (3 points)

$$H_A(j\omega) = \frac{1/j\omega C}{R + 1/j\omega C} = \frac{1}{j\omega RC + 1}$$

2) What is the transfer function for circuit B? You must simplify. (4 points)

$$H_B(j\omega) = \frac{1/j\omega C}{j\omega L + R + 1/j\omega C} = \frac{1}{-\omega^2 LC + j\omega RC + 1}$$

3) What is the transfer function for circuit C? You must simplify. (5 points)

$$H_C(j\omega) = \frac{\frac{j\omega L \times R}{j\omega L + R}}{1/j\omega C + \frac{j\omega L \times R}{j\omega L + R}} = \frac{-\omega^2 RLC}{j\omega L + R - \omega^2 RLC}$$

B. We want to determine what type of filter circuit B is

1) What are the simplified transfer function, the magnitude, and the phase of circuit B at low frequencies? (3 points)

$$H_{\text{BLO}}(j\omega) = \frac{1}{1} = 1$$

$$|H_{\text{BLO}}| = 1$$

$$\angle H_{\text{BLO}} = 0$$

2) What are the simplified transfer function, the magnitude, and the phase of circuit B at high frequencies? (3 points)

$$H_{\text{BHI}}(j\omega) = \frac{1}{-\omega^2 LC}$$

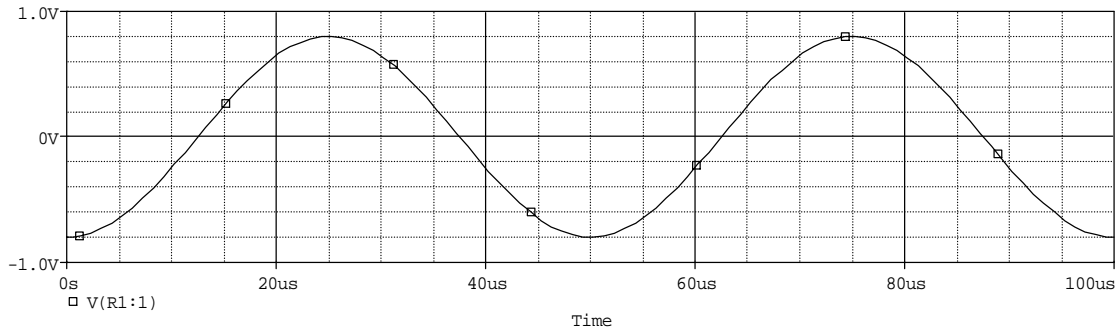
$$|H_{\text{BHI}}| = 0$$

$$\angle H_{\text{BHI}} = \pi \text{ or } -\pi$$

3) What type of filter is circuit B? (1 point)

Low Pass Filter

C. We want to know what the output of circuit A will look like for the input shown below



1) Write an equation in the form $V_{in}(t) = A_{in} \sin(\omega t + \phi_{in})$ which describes the input signal shown. (3 points)

$$V_{in}(t) = 800mV \sin(40K\pi t - \pi/2)$$

2) If $C=0.01\mu F$ and $R=10K$, what are the magnitude and phase of the transfer function of circuit A? (4 points)

$$|H_A| = 0.079$$

$$|H_A| = \left| \frac{1}{j\omega RC + 1} \right| = \left| \frac{1}{j(40k\pi)(10k)(0.01\mu) + 1} \right| = \left| \frac{1}{j(12.57) + 1} \right|$$

$$|H_A| = \frac{1}{\sqrt{12.57^2 + 1^2}} = 0.079$$

$$\angle H_A = -1.49 \text{ radians}$$

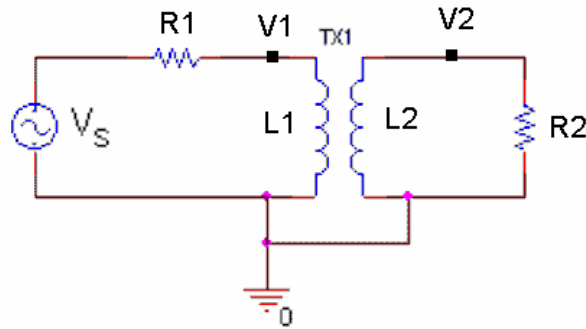
$$\angle(1) - \angle(1+j(12.57)) = 0 - \tan^{-1}(12.57) = 0 - 1.49 = -1.49$$

3) What are the amplitude and phase of the output of circuit A, when the input signal from part C-1 is applied to the circuit? (4 points)

$$A_{out} = (0.079)(800m) = 63.2mV$$

$$\phi_{out} = -1.571 - 1.49 = -3.06 \text{ radians}$$

Question 4 – Transformers and Inductors (25 points)



In the transformer circuit above, $R_1 = 400$ ohms, $R_2 = 20K$ ohms, and the turns ratio of the transformer is 1:5. Assume the transformer has perfect coupling. [Hint: You cannot assume that R_1 is much smaller than R_2 .]

1) What is the input impedance of the transformer? (2 points)

$$Z_{in} = (20k)/(25) = 800 \text{ ohms}$$

2) Find an expression for the voltage at point V_1 in terms of the input voltage, V_s . (3 points)

$$V_1 = V_s (800)/(400+800) = 0.667(V_s)$$

3) Find an expression for the voltage across the load resistor, R_2 , in terms of the input voltage, V_s . (2 points)

$$V_2 = V_1(5) \quad V_2 = (5)(0.667)(V_s) = 3.33(V_s)$$

4) What is the *current* through R_2 when the input voltage at V_s is 5V? (3 points)

$$I_2 = V_2/R_2 = 3.33(5)/20k = 0.83mA$$

- 5) Find the inductance of L1, given that it has the following properties: (3 points)
- property 1: Number of turns = 40
 - property 2: Length of coil = 4 cm = 0.04 m
 - property 3: Wire Gauge = 26 (wire diameter = 0.40 mm)
 - property 4: Core material = iron (permeability = 6.28×10^{-3} H/m)
 - property 5: Core diameter = 0.60 cm (radius = 3×10^{-3} m)

This is a long, thin coil. Therefore,

$$L1 = \frac{(6.28 \times 10^{-3})(40)^2 \pi (3 \times 10^{-3})^2}{(0.04)} = \frac{2.84 \times 10^{-4}}{0.04} = \mathbf{7.10 \text{ mH}}$$

- 6) In order for the transformer to function as predicted by the transformer ratio equations, you must make three basic assumptions. What are these three assumptions? (3 points)

Assumption 1 : perfect coupling ($k=1$)

Assumption 2: Very high frequency or very big inductance

Assumption 3: Both inductors have roughly the same physical properties.

- 7) Assuming L2 has properties similar to L1 as discussed in question 5, what must be the inductance of L2? How many turns must it have? (3 points)

$$5 = \sqrt{\frac{L2}{L1}} \quad 25 = \frac{L2}{L1} \quad L2 = 25(L1) = 25(7.10 \text{ mH}) = \mathbf{177.6 \text{ mH}}$$

The coil must have $40(5)=200$ turns.

8) Assuming the voltage at point Vs is shown on the plot below, sketch and label the voltage at V1 and at V2. (6 points)

