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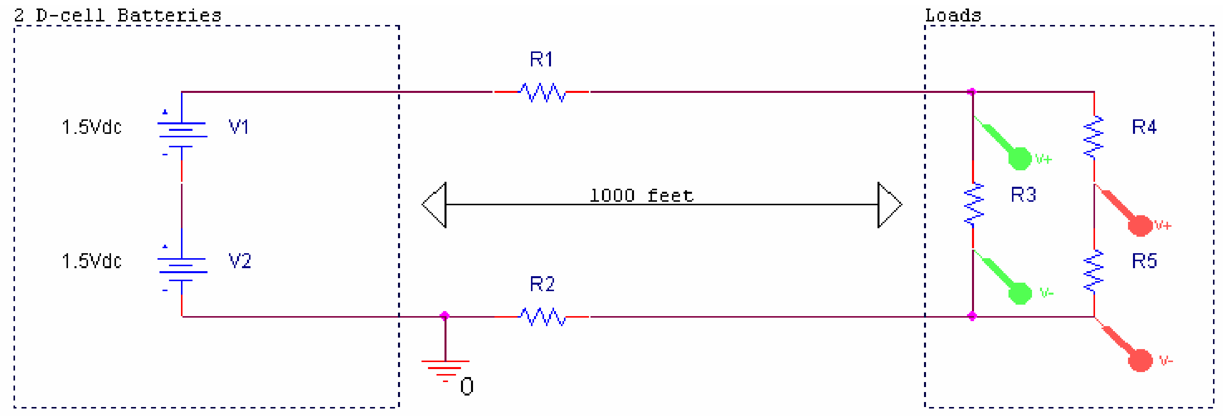
1) Resistive Circuits (25 points)

Part A Two series connected D-cell batteries power three loads over an extension cable 1000' long. The cable is made using 24AWG (American Wire Gauge) wire that has a resistance of 27.3Ω/1000'. The figure below shows this resistance as R1=27.3Ω & R2=27.3Ω. Loads R3=2kΩ, R4=800Ω, R5=1400Ω and are configured as shown in the circuit below. All calculations should be carried to three decimal places.



R1=27.3Ω R2=27.3Ω R3=2KΩ R4=800Ω R5=1400Ω

a) What is the total resistance seen by the batteries? (6 points):

R45 = 800+1400 = 2200 R345=(2000×2200)/(2000+2200) = 1047.619
RT = 27.3 + 1047.619 + 27.3= ***1102.219Ω***

b) Find the current that will be drawn from the batteries (2 points):

VT = 1.5+1.5=3   *I = VT/RT = 3/1102.219=* ***2.722 mA***

c) Assuming all resistors are exactly their stated value, find the voltage drop across R3 and R5. (6 points):

R3: V3 = V345 = VT×(R345)/RT = 3(1047.619)/1102.219 = ***2.851 V***

R5: V5 = V345×R5/(R45) = 2.851(1400)/(2200) = ***1.815 V***

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d) If the last color band on all resistors is red (+/- 2%), what are the minimum and maximum currents R3, R4 and R5 together should draw from the batteries? (3 points):

*Maximum = R345 + R345×0.02 = 1047.619+20.952=**1068.571Ω***
*Minimum = R345 - R345×0.02 = 1047.619-20.952=**1026.667Ω***

method 1 – Use voltage drop across the combination
$V_{R234} = V_{R3} = 2.851V$ $I_{R234} = V_{R3}/R345$
Minimum: I_{R234} = =2.851/1068.571 =**2.67 mA**
Maximum: : I_{R234} =2.851/1026.667 **= 2.78 mA**

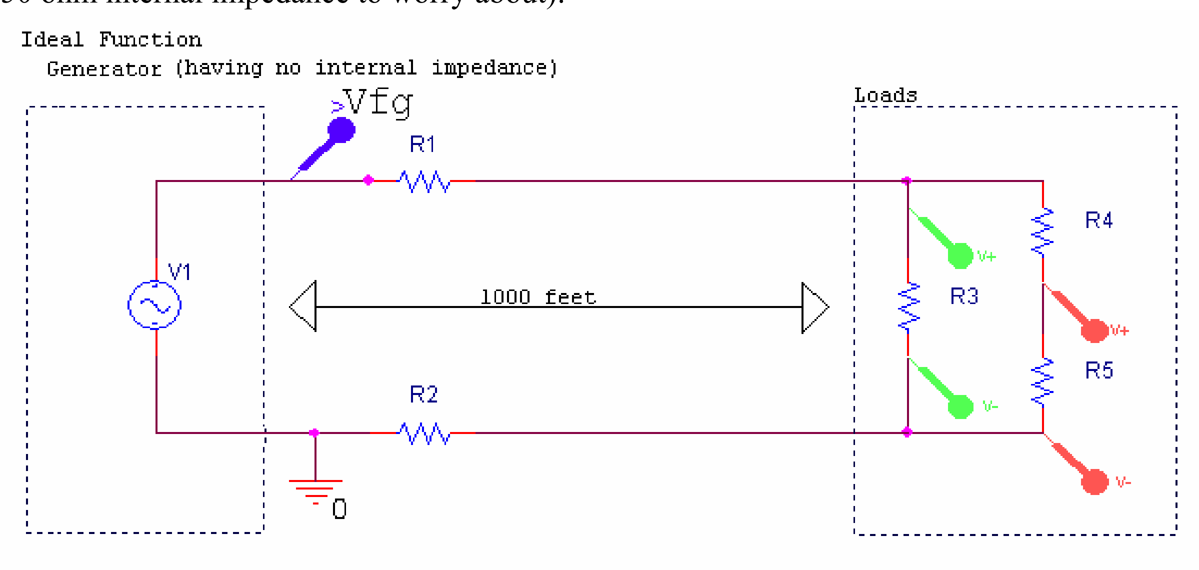
method 2 – Use input voltage and ignore wires
$V_{R234} = VT = 3$ $I_{R234} = V_{R3}/R345$
Minimum: I_{R234} = =3/1068.571 **=2.81mA**
Maximum: : I_{R234} =3/1026.667 **= 2.92mA**

method 3 – Recalculate voltages (This one is actually the correct method.)
$V_{R234} = 3(R234/RT)$ $I_{R234} = V_{R3}/R345$

Minimum: VT = 1068.571 + 2(27.3) =1123.171
VR234=(1068.571/1123.171)(3)= 2.854 V
I_{R234} = =2.854/1068.571 = **2.67mA**
Maximum: VT = 1026.667 + 2(27.3) = 1081.267
VR234=(1026.667/1081.267)(3)=2.849V
I_{R234} = =2.849/1026.667 =**2.8mA**

.

Part B Now suppose our D-cell batteries are replaced by an ideal function generator (no 50 ohm internal impedance to worry about):



V1: VOFF=4V VAMPL = 6V FREQ = 1KHz

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R1=27.3Ω R2=27.3Ω R3=2KΩ R4=800Ω R5=1400Ω

a) What is the maximum voltage at Vfg referenced to ground (1 point)?

Maximum Vfg = VOFF + VAMPL = 4+6 = ***10V***

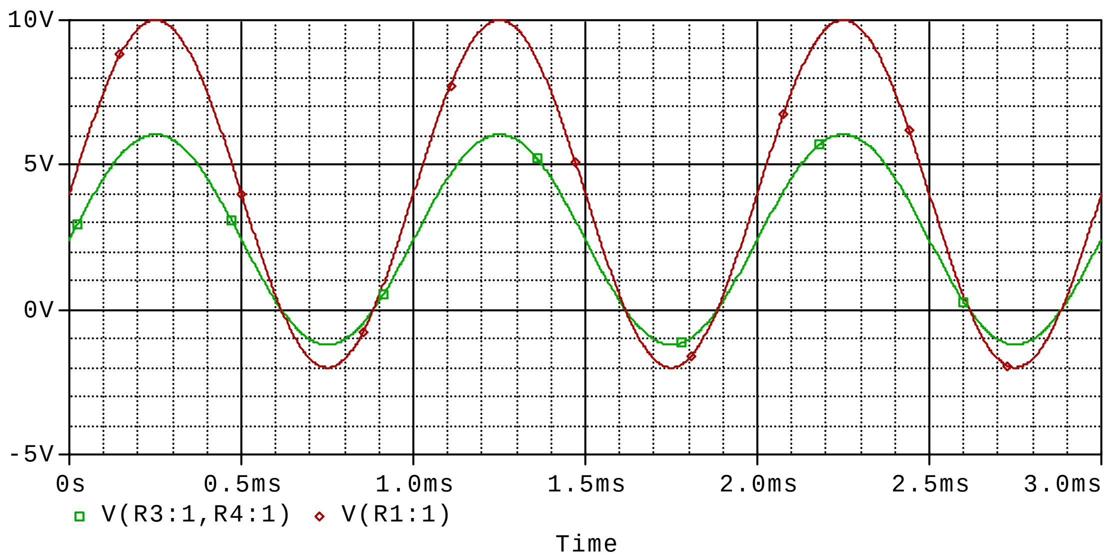
b) What is the minimum voltage at Vfg referenced to ground (1 point)?

Minimum Vfg = VOFF – VAMPL = 4-6 = ***-2V***

c) What is the average voltage at Vfg referenced to ground (1 point)?

Average Vfg = VOFF = ***4V***

d) On the graph below, using pencil if you have one, sketch and label the voltage at Vfg. Start the drawing with Vfg = Vavg at time zero (5 points). *(red trace is input)*



Extra Credit: On the same graph, sketch and label the voltage drop across R5 assuming the waveform drawn in (d) as the input signal. Show all calculations required below (1 extra point). *(green trace is output)*

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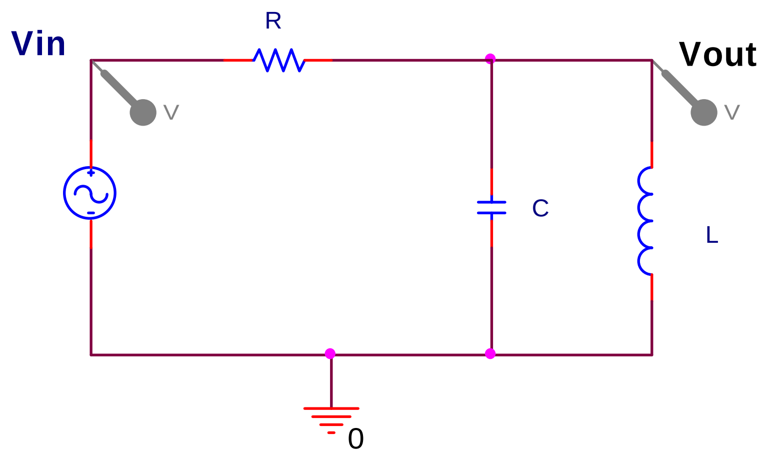
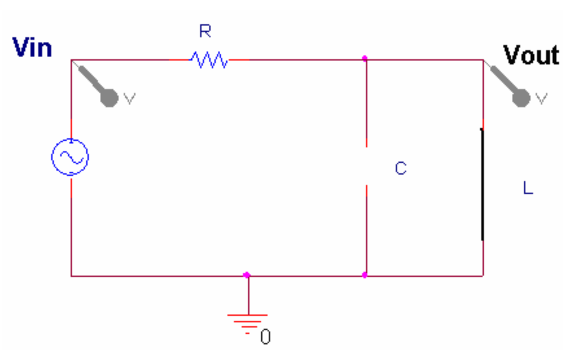
2) Filters (25 points)

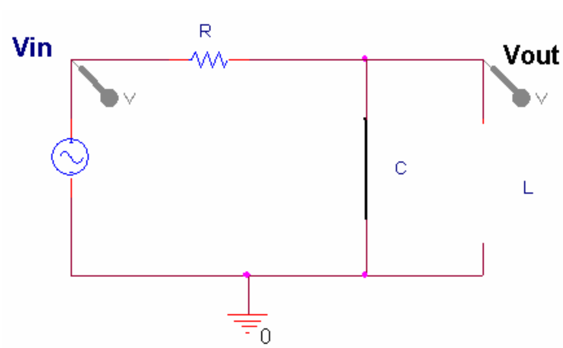
Part A: You want to determine what type of filter the following circuit is.



a) Redraw the circuit at very low frequencies. (2 points)



b) Redraw the circuit at very high frequencies. (2 points)



c) What is the value of Vout at very low frequencies? (1 point)

Vout = 0V

d) What is the value of Vout at very high frequencies? (1 point)

Vout = 0V

e) What type of filter is this? (1 point)

Band Pass Filter

Part B: You wire the circuit in Part A of this question on your protoboard with real components.

a) Your capacitor, C, has "474" written on it. What is its rated value in microfarads? (2 points)

$C = 47 \times 10^{4} \times 10^{-6} = \mathbf{0.47\mu F}$

b) Your resistor, R, has an red band, a black band, a orange band, and a gold band (5% tolerance) in that order. What is the rated value of the resistor? (2 points)

$R = 20\times10^{3} = \mathbf{20K\Omega\ +/-\ 5\%}$

c) Your inductor, L, is long and thin. It has 200 turns, a core diameter of 0.4 cm, a coil length of 6 cm, a wire gauge of 26 (diameter=0.4 mm). It is wound around an air core ($\mu=4\pi \times 10^{-7}$ H/m). Calculate an estimate for the inductance. (4 points)

$L = \mu N^{2}\pi r^{2}/d=4\pi(10^{-7})(200\text{^}2)\pi(.002\text{^}2)/.06=\mathbf{10.5\mu H}$

d) Based on the values of the components, give an estimate for the resonant frequency of the circuit in Hertz. (3 points)

$f = 1/(2\pi \mathrm{sqrt}(LC)) = 1/(2\pi\times \mathrm{sqrt}(10.5\mu\times 0.47\mu))=$ ***71.6K Hertz***

e) If input a signal, v(t)=3V sin (3πKt), is applied at the input to your circuit, will your output amplitude be less than, greater than, or about equal to the input amplitude? Explain why. (3 points)

$\omega=3\pi K$ $2\pi f=3K\pi$ $f = 1500$ *Hertz*

The resonance is at around 72000 Hertz. The band will pass only frequencies around the resonance. The input frequency of about 1500 Hertz is not near the resonance, it will not be passed, and ***the output will be less than the input.***

f) You connect your circuit to a source and find that the actual resonance occurs at f_0 hertz. Assuming you now have measured values for C, R, and the resistance of the inductor, describe how you could use PSpice to get a closer estimate (than that found in part c) for the inductance of your inductor. Be specific. (4 points)

1) Wire the circuit in PSpice. Model the inductor as an inductance and a resistance.
2) Set the values of the components -- R, C, and the resistance of the inductor -- to their measured values. Set the initial value of the inductor to the value calculated in part c.
3) Place a voltage marker at the output, Vout.
4) Set up an AC sweep from 1000 to 1Meg Hertz. (The exact minimum and maximum are not critical, but they must include the decade between 10,000 and 100,000.)
5) Run the AC sweep. The resonant frequency is the location where the output peaks. (We know it will be the highest point because it is a band pass filter.)
6) If the resonance in the plot is above f_0, then increase the value of L. If the resonance in the plot is below f_0, then decrease the value of L. Rerun the simulation and repeat the process until the resonance is exactly at the experimental resonant frequency, f_0.
7) The value of L obtained in this manner will be a better estimate that the calculated value in part c.

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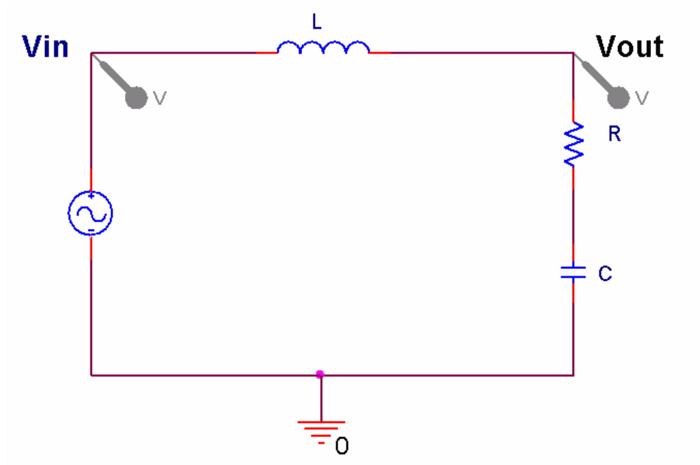
3) Transfer Functions and Phasors (25 points)



a) Find the transfer function for the above circuit. (2 points)

$H(j\omega) = [R+(1/j\omega C)] / [R+j\omega L+(1/j\omega C)] = [j\omega RC+1] / [1+j\omega RC-\omega^2 LC]$

b) Find the function to describe the behavior of the circuit at very low frequencies. Also determine the magnitude and phase of this circuit at very low frequencies. (3 points)

$H_{LO}(j\omega) = [1] / [1] = 1$

$|H_{LO}| = 1$ $\angle H_{LO} = 0$

c) Find the function to describe the behavior of the circuit at very high frequencies. Also determine the magnitude and phase of this circuit at very high frequencies. (3 points)

$H_{HI}(j\omega) = [j\omega RC] / [-\omega^2 LC] = -[jR] / [\omega L]$

$| H_{HI}| = 0$ $\angle H_{HI} = -\pi/2$

d) Find the function to describe the behavior of the circuit at the resonant frequency in terms of L, R and C. Also determine the magnitude and phase of the circuit at the resonant frequency. (6 points)

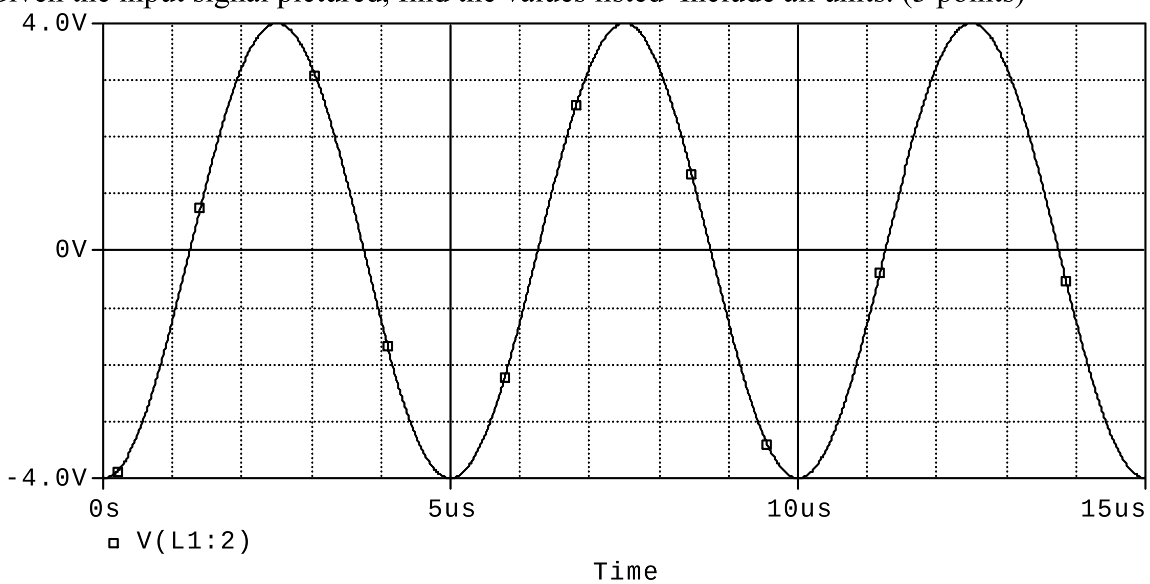
$$H_0 = \frac{j\frac{1}{\sqrt{LC}}RC+1}{1+j\frac{1}{\sqrt{LC}}RC-\frac{1}{LC}LC} = \left(j\frac{1}{\sqrt{LC}}RC+1\right)\times\frac{1}{j}\frac{\sqrt{LC}}{RC}$$

$$H_0 = 1 - j\frac{\sqrt{LC}}{RC}$$

$$|H_0| = \sqrt{1+\left(\frac{\sqrt{LC}}{RC}\right)^2} = \sqrt{1+\frac{LC}{(RC)^2}}$$

$$\angle H_0 = \tan^{-1}\left(-\frac{\sqrt{LC}}{RC}\right)$$

e) Given the input signal pictured, find the values listed Include all units. (3 points)



amplitude voltage: **4V** peak-to-peak voltage: **8V**

frequency (f): 1/5μs = **200K Hertz** angular frequency (ω): **400πK or 1257K rad/sec**

RMS voltage: 4/√2 = **2.83V** phase shift (ϕ): 2π(1/4) = **-π/2 or -1.57 rad**

f) Assuming the input signal pictured and given values C=0.01µF, L=1mH and R=10K, find a complex expression for the transfer function of the circuit at the input frequency. Also, find the magnitude and phase of the function. (6 points).

$$H = \frac{j\omega RC + 1}{j\omega RC + 1 - \omega^2 LC} = \frac{j(125.7) + 1}{j(125.7) + 1 - 15.79} = \frac{j(125.7) + 1}{j(125.7) - 14.79}$$

$$|H| = \frac{\sqrt{1^2 + 125.7^2}}{\sqrt{14.79^2 + 125.7^2}}$$ ***= 0.993***

$$\angle H = \tan^{-1}\left(\frac{125.7}{1}\right) - \tan^{-1}\left(\frac{125.7}{-14.79}\right)$$ *= 1.56-(π-1.45)= -0.13 rad (+6.15 rad)*

(Also acceptable: 1.56-(-1.45) ***= 3.01 rad or -3.27 rad)***

(Technically, the first is correct. The phase shift must be negative and between 0 and -π/2. The best way to get it right is to use reference angles (like in trig) to get the correct magnitudes. Subtract the magnitudes and then, consider the sign based on what you know about the transfer function.)

g) For the input signal given in e), what will be the amplitude and phase of the output? (2 points)

A_{out} *= Ain * |H| = 4 (0.079) =* ***3.972V***

$\phi_{out} = \phi in + \angle H = -1.57 - 0.31 =$ **-1.88 rad or +4.40 rad**
(also acceptable ϕ_{out} = -1.57 + 3.01 = 1.44 rad or 4.84 rad

(Technically, the first is correct.)

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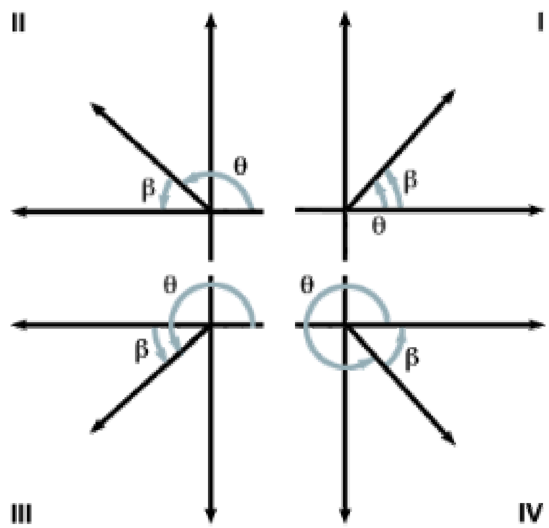
ASIDE -- *This is going into the Steady State handout for next semester. Since you did not get this information, we did not deduct for the phase calculation as long as it was substituted correctly into the equation.*

<u>Calculating phases using the inverse tangent function</u>
If the transfer function is given as a ratio of two complex numbers, then the phase is given by the difference between the phases of the numerator and denominator:

$$\text{if} \quad H(j\omega) = \frac{x_1 + j\,y_1}{x_2 + j\,y_2} \quad \text{then} \quad \angle H = \tan^{-1}\left(\frac{y_1}{x_1}\right) - \tan^{-1}\left(\frac{y_2}{x_2}\right)$$

If x_1, y_1, x_2 and y_2 are all positive, then the phase changes are all in the first quadrant, and the equation can be applied directly with a calculator. If one or more of them is negative, then one must worry about which quadrant the phase angle is in. The most reliable way to determine a phase change is to take the absolute value of the x and y coordinates of a complex number, calculate $\tan^{-1}(|y/x|)$ to find the reference angle, use the signs of x and y to determine the quadrant, and find the phase based on the reference angle and the quadrant.



In the figure above, β is the reference angle for θ. We want to find θ -- the actual phase. tan-1(|y/x|) will always give us the reference angle β. We can find θ based on the sign and the quadrant:

x	*y*	*quadrant*	*θ (radians)*	*θ (degrees)*
x>0	*y>0*	*I*	β	β
x<0	*y>0*	*II*	$\pi - \beta$	$180 - \beta$
x<0	*y<0*	*III*	$\beta - \pi$	$\beta - 180$
x>0	*y<0*	*IV*	$-\beta$	$-\beta$

Note that all angles in the above chart represent a phase shift between $-\pi$ and $+\pi$ radians (between -180 and +180 degrees).

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some examples: $H(j\omega) = \dfrac{3 + j4}{4 + j3}$

numerator: $\beta = \tan^{-1}(|4/3|) = 0.93$ *(x>0, y>0, Q1)*
$\angle num = 0.93$
denominator: $\beta = \tan^{-1}(|3/4|) = 0.54$ *(x>0, y>0, Q1)*
$\angle den = 0.54$
$\angle H = \angle num - \angle den = 0.93-0.54 = 0.39$ *rad*

$$H(j\omega) = \frac{-3 + j4}{4 - j3}$$

numerator: $\beta = \tan^{-1}(|4/3|) = 0.93$ *(x<0, y>0, Q2)*
$\angle num = 3.14-0.93 = 2.21$
denominator: $\beta = \tan^{-1}(|3/4|) = 0.54$ *(x>0, y<0, Q3)*
$\angle den = 0.54-3.14 = -2.60$
$\angle H = \angle num - \angle den = 2.21-(-2.60) = 4.81$ *rad* $= -1.47$ *rad*

$$H(j\omega) = \frac{-3 - j4}{4 + j3}$$

numerator: $\beta = \tan^{-1}(|4/3|) = 0.93$ *(x<0, y<0, Q4)*
$\angle num = -0.93$
denominator: $\beta = \tan^{-1}(|3/4|) = 0.54$ *(x>0, y>0,1)*
$\angle den = 0.54$
$\angle H = \angle num - \angle den = -0.93-0.54 = -1.47$ *rad*

And now back to the test…..

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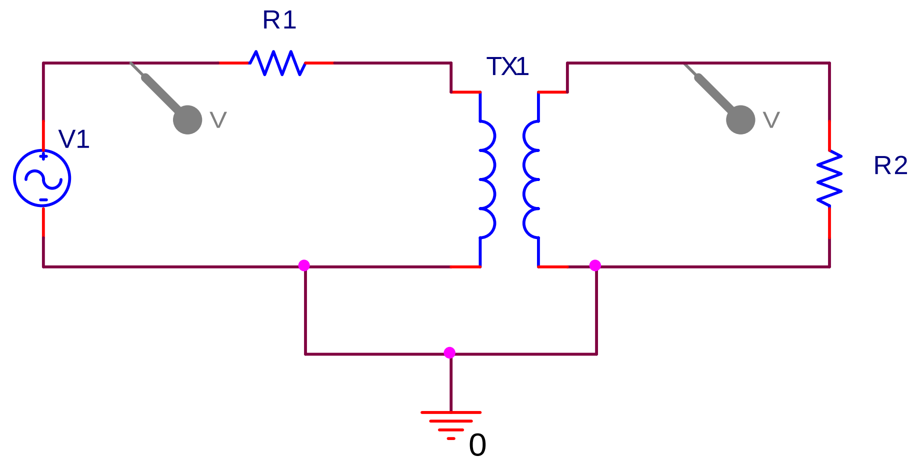
4) Transformers (25 points)

Part A In the transformer below, the value of the input inductance (the inductance of the primary winding of the transformer) is 1mH. You can assume that the coupling coefficient is 1 and that the resistance of R1 is negligible. Use the equations for an ideal transformer to determine the following:



L1 = 1mH, k=1, R1 = 3Ω, R2 = 3KΩ, V1 = 300mV

a) You would like the output voltage of the transformer to be 3 times that of the input voltage. What should the value of the constant, a, be? (1 point)

***a* = 3**

b) Given the value for a you found in part a, what should you set the value of the output inductance (the inductance of the secondary winding of the transformer) to in order to obtain the desired voltage ratio? (3 points)

a= √L2 / √L1   L2 = a^2×L1 = 9×1m = ***9m***

c) For the case in part b, what will be the amplitude of the output *current*? (3 points)

a=V2/V1 3=V2/300m V2=900mV V=IR 900mV=I (3K) ***I = 0.3 mA***

d) What is the input impedance, Z_{in}, of this transformer under the conditions given? (3 points)

Z=R2/a^2 = 3K/9 = ***333.3Ω***

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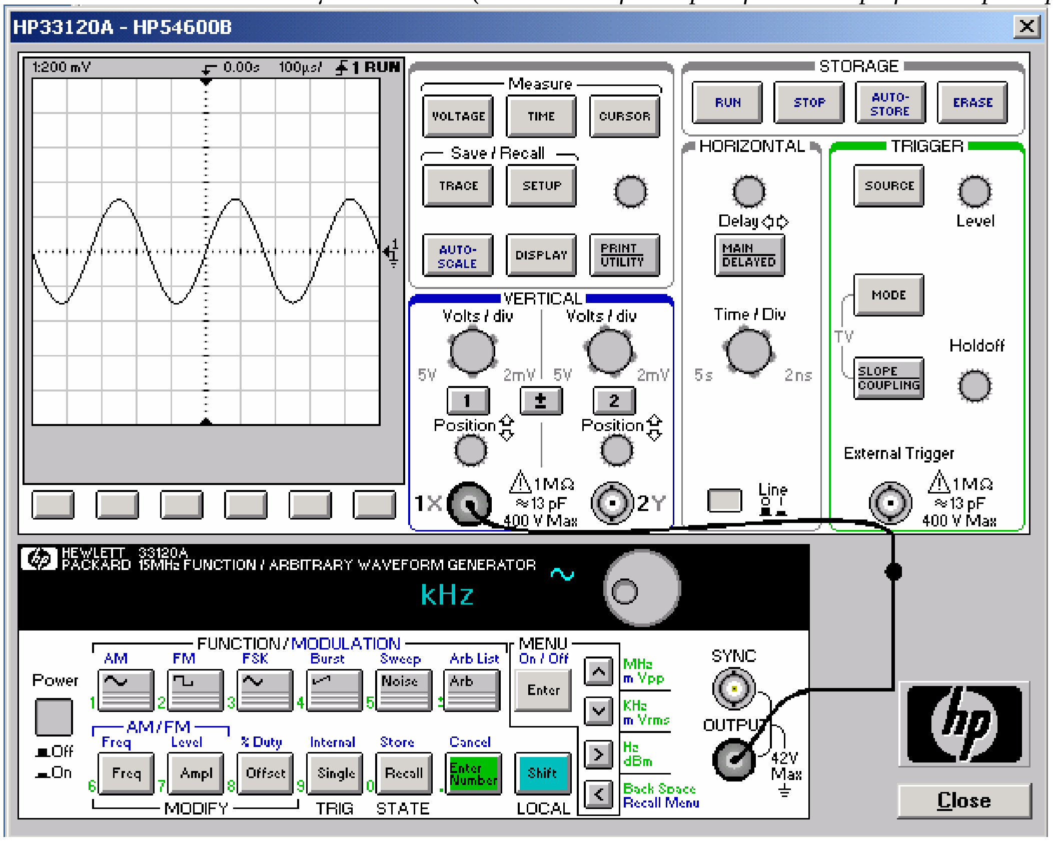
Part B Applying an input signal to your transformer.

1:200mV 0.00s 100μs/ 1 RUN (The text at left is copied from the top of the scope display)



a) Describe how you would set the function generator and manually adjust the scope to get the signal shown above. Assume that the scope is connected directly to the function generator as shown. Give specific details. (4 points)

1) Push the Freq button on the function generator. Set it to (1/333μ)=3K Hertz.
2) Push the Ampl button on the function generator. Set it so that the mV pp is equal to 300mV (This is twice the desired peak-to-peak voltage.)
3) Set the Volts/div knob on the voltage scale is 200 mV.
4) Set the Time/Div knob so that the time scale is 100μs/.

b) Describe two ways that you could verify that the output signal has the correct peak-to-peak amplitude voltage using the scope. (2 points)

1) You can use the vertical scale on the scope display. The peak-to-peak voltage of the trace goes from -300mV to 300mV. So the peak-to-peak voltage is 600mV.
2) You can push the VOLTAGE button and the Vp-p soft key. The peak-to-peak voltage will be displayed underneath the scope display.

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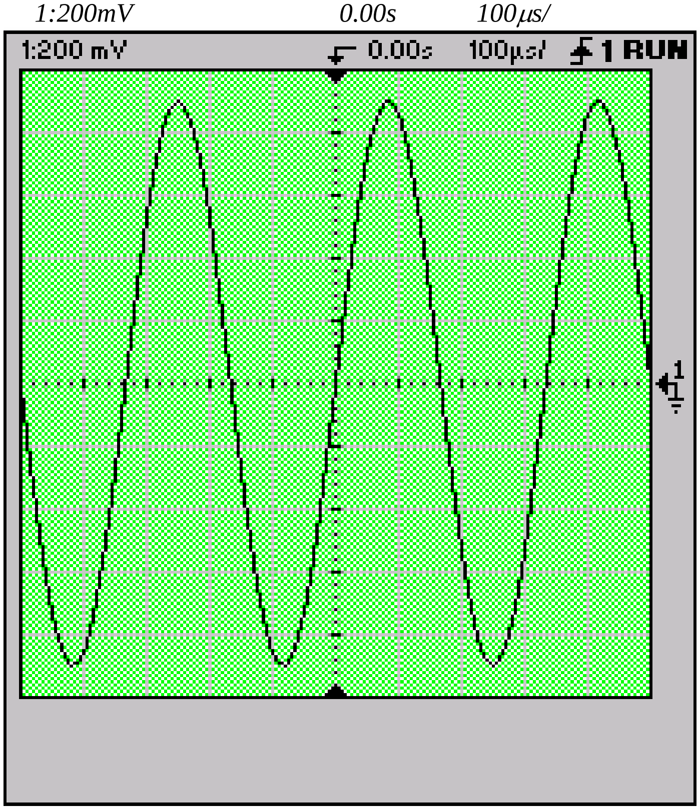
c) As you are setting up the input signal, you notice that the function generator and the scope display a different value for peak-to-peak voltage. Which is correct, the function generator or the scope? Why? (3 points)

The scope is correct. The function generator assumes that there is a 50 ohm impedance attached to it. Therefore, it displays ½ of what it is actually putting out. The scope has an impedance much larger than 50 ohms (1Meg). This discrepancy causes the display on the function generator to be incorrect.

d) Once you have set up the correct input to the circuit, you connect your transformer circuit to the input and you observe the output. Based on the ideal behavior of a transformer, sketch the expected output voltage on the scope screen below. (3 points)

1:200mV *0.00s* *100μs/*



e) When you observe the output on the scope, the amplitude of the output relative to the input is less than expected. How could you change the input signal to force the transformer to act more like an ideal transformer? (3 points)

Increase the frequency of the input signal.