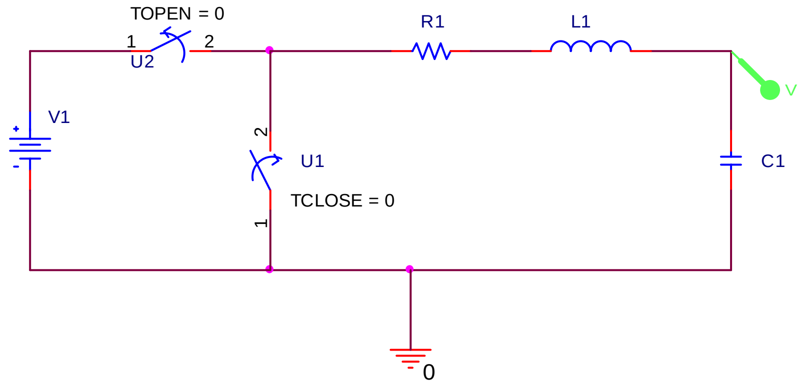
ENGR4300 Test 2A
Spring 2005

Name____solution________________

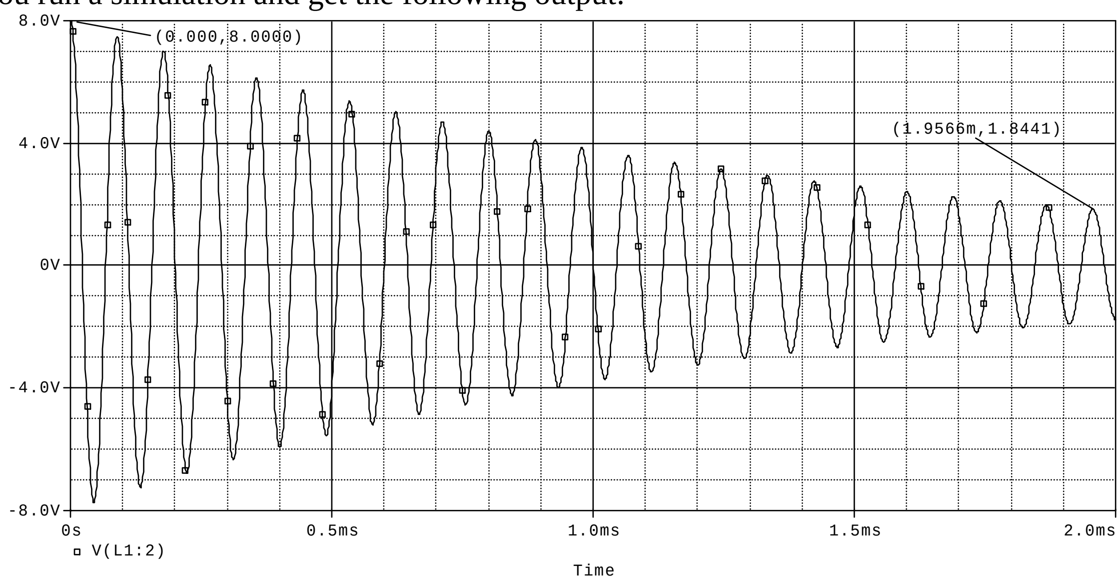
Section ________________________

1) Damped Sinusoids (25 points)

You wire the following circuit in PSpice.



You run a simulation and get the following output:



a) How would you set up the PSpice simulation screen pictured below, to get the output pictured above? (3 points)

A: Run to time = 2ms
Start time =0
Maximum step size varies should be from about 1us to about 20us.

B: Run to time =1ms
Start time = 0
Maximum step size size varies should be from about 1us to about 10us.
(See your test for the correct plot.)

b) Using the output pictured, determine the damping constant, α, of the circuit. (3 points)

A: *(t0,V0) = (0ms,8V) (t1,V1)=(1.96ms,1.84V)* $ln(1.84/8) = -\alpha\,(1.96m-0m)$

$\boldsymbol{\alpha = 749.8/s}$

B: *(t0,V0) = (0ms,8V) (t1,V1)=(0.99ms,1.1V)* $ln(1.1/8) = -\alpha\,(0.99m-0m)$
B: $\boldsymbol{\alpha = 2004/s}$

c) What is the resonant frequency of the circuit in Hertz? (3 points)

*A: f = 22 cycles/1.96ms = **11.22K Hz***

*A: f = 6 cycles/0.98ms = **6060 Hz***

d) Write an expression in the form $v(t) = Ae^{-\alpha t}\cos(\omega_0 t)$ for the output signal. (3 points)

A: $\omega = 2\pi f = 70.5K$ *rad/sec* $v(t) = 8Ve^{-749.8t}\cos(70.5K\,t)$

B: $\omega = 2\pi f = 38.08K$ *rad/sec* $v(t) = 8Ve^{-2004t}\cos(38.08K\,t)$

e) Use the general equations for capacitor and inductor behavior (located on the crib sheet for quiz 1), to describe what is happening in this circuit. What is causing the voltage to behave like a damped sinusoid? (5 points)

The equation for the capacitor is $I_C = C\frac{dV_C}{dt}$. *The equation for the inductor is* $V_L = L\frac{dI_L}{dt}$. *The 8 volt source places a charge on the capacitor at time t=0. Then the circuit is disconnected from the source and is allowed to oscillate on its own. The capacitor begins to discharge into the rest of the circuit and the voltage across it changes. The current, Ic, caused by the changing voltage of the capacitor (dVc/dt) starts to flow through the inductor. (This is also* I_L *because the circuit is in series.) The changing current in the inductor (*dI_I/dt*) induces a voltage,* V_L*. This changing voltage causes the capacitor to change up again. The process continues. During each cycle some of the energy in the circuit is dissipated by the resistance of the resistor and this causes the signal to decay and eventually disappear.*

f) The differential equation that governs the behavior of a damped sinusoid is given by $\frac{d^2V}{dt^2} + 2\alpha \frac{dV}{dt} + \omega_0^2 V = 0$. In a simple RLC circuit like the one in this question, the angular resonant frequency of the circuit, ω_0, is given by $\omega_0 = \frac{1}{\sqrt{LC}}$ and the decay constant, α, is given by $\alpha = \frac{R}{2L}$. In the circuit above, the value of the resistor, R1, is 30 ohms. What are the values of the capacitor, C1, and the inductor, L1? (6 points)

A: $\alpha = \frac{R}{2L}$ *749.8 = (30)/(2L) L=30/(2)(749.2) = 2****0mH***

$\omega_0 = \frac{1}{\sqrt{LC}}$ *70.5K=1/sqrt(20m×C) C=1/(Lω^2)=1/(20m×70.5K^2) =* ***0.01µF***

B: *R1=40 ohms*

$\alpha = \frac{R}{2L}$ *2004 = (40)/(2L) L=40/(2)(2004) = 1****0mH***

$\omega_0 = \frac{1}{\sqrt{LC}}$ 38.08*K=1/sqrt(10m×C) C=1/(Lω^2)=1/(10m×38.08K^2) =* ***0.083uF***

g) You want the damping constant of the circuit to be double what it is now. What new value of L1 would choose to make this occur? (2 points)

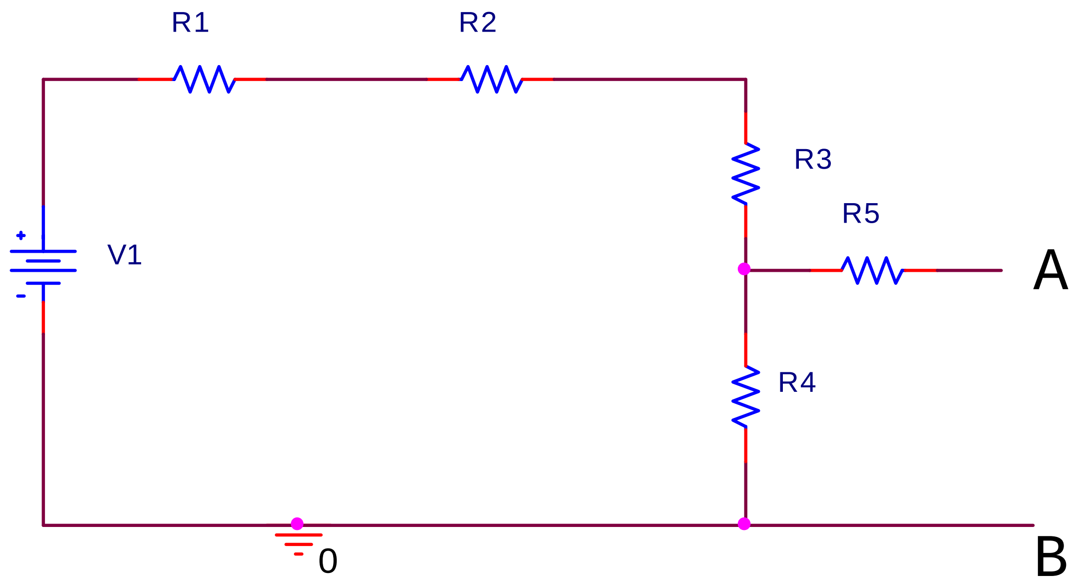
A: L=30/(2)(2)(749.2) = ***10mH***

B: You want damping constant to be ½ of what it is now.
L=40/(2)(1/2)(2004)= ***20mH***

ENGR4300 Test 2A **Name____solution_______________**
Spring 2005 **Section ________________________**

2) Thevenin Equivalent Sources (25 points)

Part A You build the circuit pictured below



Test A:
Given: R1=30 ohms, R2=2K ohms, R3 = 3K ohms, R4=2K ohms and R5=1K ohms.
Given: V1=6V

Test B:
Given: R1=40 ohms, R2=3K ohms, R3 = 4K ohms, R4=3K ohms and R5=1K ohms.
Given: V1=8V

a) Find the Thevenin Equivalent voltage, Vth, of this circuit between point A and point B. (6 points)

A: Vth = VR4 = V1(R4)/(R1+R2+R3+R4) = 1.707V

B: *Vth = VR4 = V1(R4)/(R1+R2+R3+R4)=2.390V*

b) Find the Thevenin Equivalent Resistance, Rth, of this circuit between point A and point B. (6 points)

A: Rth = R5 + (R1+R2+R3)//R4 R1+R2+R3=5030
R123//R4 = (5030)(2000)/(5030+2000)=1431

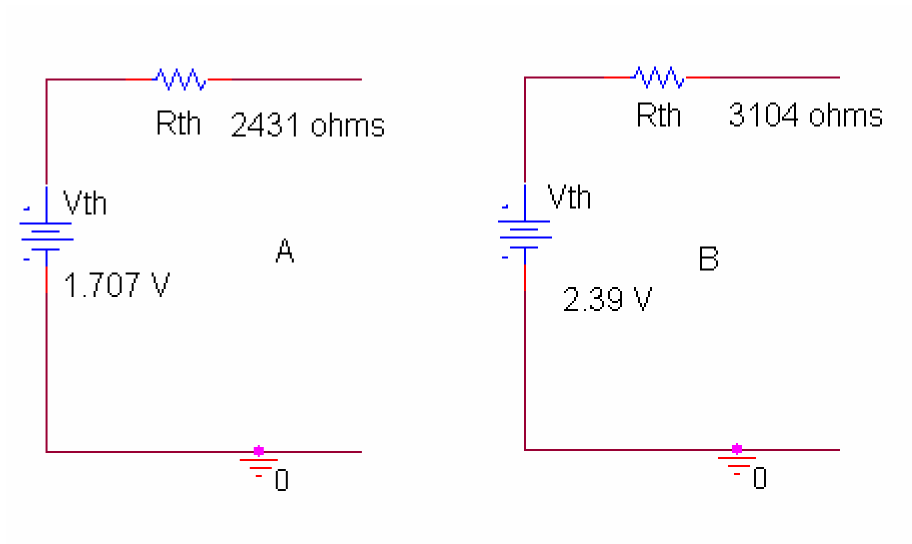
Rth = 1000+1431 = ***2431 ohms***

B: Rth = R5 + (R1+R2+R3)//R4 R1+R2+R3=7040
R123//R4 = (5030)(2000)/(5030+2000)=2103.6

Rth = 1000+1431 = ***3103.6 ohms***

Name____solution_______________
Section ________________________

c) Redraw the Thevenin equivalent model of the circuit (2 points).



d) If you place a 2K ohm load on the circuit, what will the output voltage be between A and B? (2 points)

A: *Vout =(2000/4431)1.707 =* ***0.770V***

B: *Vout = =(3000/3104)2.39* ***= 1.175V***

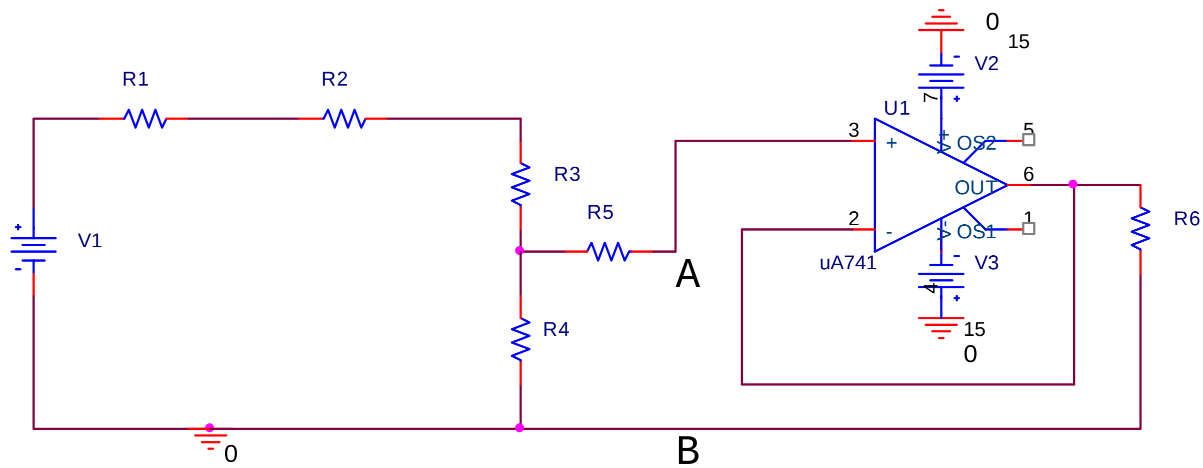
e) What is the current through the 2K ohm load resistor from d? (2 points)

A: I=V/R I=0.770/2K = ***0.385mA***

B: The load is 3k ohms.
I=V/R I=1.175/3K = ***0.392mA***

ENGR4300 Test 2A **Name____solution________________**
Spring 2005 **Section __________________________**

Part B You place a voltage follower into this circuit between A and B, as pictured below.



Test A:
Given: R1=30 ohms, R2=2K ohms, R3 = 3K ohms, R4=2K ohms and R5=1K ohms.
Given: V1=6V
Given: R6 is the load of 2K ohms

Test B:
Given: R1=40 ohms, R2=3K ohms, R3 = 4K ohms, R4=3K ohms and R5=1K ohms.
Given: V1=8V
Given: R6 is the load of 3K ohms

a) What does the voltage follower do in this circuit? (2 points)

The voltage follower isolates the circuit we modeled from the load R6. This means that whatever voltage the circuit puts out between A and B will be transferred to R6 without being influenced by the value of R6.

b) What is the voltage output between A and B for this circuit now? (3 points)

The voltage between A and B will always be the Thevenin voltage for the circuit because the buffer looks like an infinite impedance to it. Since an infinite impedance is much much bigger than Rth, it will not influence the circuit. Therefore,

A: *VAB = Vth =* ***1.707V***

B: *VAB = Vth =**2.39V***

c) What is the current though the load resistor, R6? (2 points)

A: I=V/R I=1.707/2K = ***0.854mA***

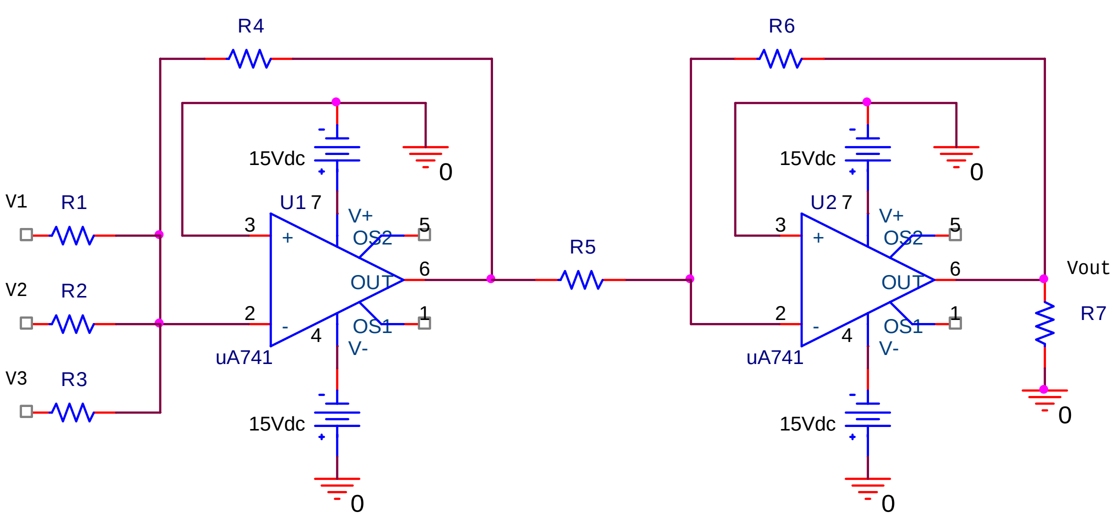
B: I=V/R I=2.39/3K = ***0.797mA***

3) Op Amp Applications – Digital to Analog Conversion (25 points)

One family of digital logic we'll learn about later in the course uses signals that switch discretely between zero and +5V, corresponding to logic levels of 0 (low) and 1 (high), respectively. Here is a chart of the binary numbers from 1 to 16, their decimal equivalents, and corresponding voltage inputs:

Decimal Value	Binary Value				Corresponding Voltage Inputs			
	x2^3	x2^2	x2^1	x2^0	Va	Vb	Vc	Vd
0	0	0	0	0	0	0	0	0
1	0	0	0	1	0	0	0	5
2	0	0	1	0	0	0	5	0
3	0	0	1	1	0	0	5	5
4	0	1	0	0	0	5	0	0
5	0	1	0	1	0	5	0	5
6	0	1	1	0	0	5	5	0
7	0	1	1	1	0	5	5	5
8	1	0	0	0	5	0	0	0
9	1	0	0	1	5	0	0	5
10	1	0	1	0	5	0	5	0
11	1	0	1	1	5	0	5	5
12	1	1	0	0	5	5	0	0
13	1	1	0	1	5	5	0	5
14	1	1	1	0	5	5	5	0
15	1	1	1	1	5	5	5	5

The following op amp circuit is configured as a digital to analog (D/A) converter. Our D/A converter shown here is a three bit converter: logic voltages at V1, V2, and V3 generate an output voltage at Vout. V1 is the lowest order bit corresponding to Vd (in the chart), V2 corresponds to Vc, and V3 corresponds to Vb.



Component values for the circuit shown are as follows: R4 = 5k, R5 = 20k, R6 = 10k, R7 = 10Meg ohms. Values R1, R2, and R3 will be determined by you in part b of this question.

ENGR4300 Test 2A **Name____solution________________**
Spring 2005 **Section __________________________**

Test B:
Component values for the circuit shown are as follows: R4 = 4k, R5 = 40k, R6 = 10k, R7 = 1Meg ohms. Values R1, R2, and R3 will be determined by you in part b of this question.

a. Given the D/A circuit on the previous page, about what is the maximum voltage rounded to the nearest 0.5 volts (e.g. 1.0V, 1.5V, 2.0V,…) that the circuit should generate at Vout when it is acting correctly as a D/A converter? Describe your reasoning for this answer and show all equations or calculations used to arrive at this answer. (3 points)

The maximum output for a three bit binary number is 111. This corresponds to the decimal number 7, so the circuit should output a maximum of 7 volts.

There is another answer to this question. If you assumed that the output of the first op-amp could put out -15 (or anything from -15 to -13), and used the gain of the second op-amp, (-10k/20k), to predict that the circuit could deliver a maximum of 7.5 volts at the output, then this is right too.

b. The D/A should generate output voltages between 0V for a binary input of 000 to the full scale value you determined in part A when a binary input of 111 is present with a proportional increase in voltage as the binary value increases from 000 to 111. Determine the resistor values R1, R2, and R3 that achieve this in the circuit. (12 points)

A:

Vx=(-R4)[(VD/R1)+(VC/R2)+(VB/R3)] *Vout = (-R6/R5)Vx*

*Vout=(R4*R6/R5)[(VD/R1)+(VC/R2)+(VB/R3)]*

*(R4*R6/R5)=(5K*10K)/20K=2.5K*

Vout	*VB (V3)*	*VC(V2)*	*VD(V1)*	*plug in*	*solve*
1	*0*	*0*	*5V*	*2.5K(5/R1)=1*	*R1=12.5K*
2	*0*	*5V*	*0*	*2.5K(5/R2)=2*	*R2=6.25K*
4	*5V*	*0*	*0*	*2.5K(5/R3)=4*	*R3=3.125K*

A: R1 = 12500 ohms
R2 = 6250 ohms
R3 = 3125 ohms

B: *(R4*R6/R5)=(4K*10K)/40K=1K*

Vout	*VB (V3)*	*VC(V2)*	*VD(V1)*	*plug in*	*solve*
1	*0*	*0*	*5V*	*1K(5/R1)=1*	*R1=5K*
2	*0*	*5V*	*0*	*1K(5/R2)=2*	*R2=2.5K*
4	*5V*	*0*	*0*	*1K(5/R3)=4*	*R3=1.25K*

B: R1 = 5000 ohms
R2 = 2500 ohms
R3 = 1250ohms

c. Show the output voltage at Vout will give the correct decimal value for the following two binary input combinations: (7 points).

A:
100: *Vout=(2.5K)[(0/12500)+(0/6250)+(5/3125)] = 4V ?*
4V = 4V

011: *Vout=(2.5K)[(5/12500)+(5/6250)+(0/3125)] = 3V ?*
3V = 3V

B:

010: *Vout=(1K)[(0/5000)+(5/2500)+(5/1250)] = 2V ?*
2V = 2V

101: *Vout=(1K)[(5/5000)+(0/2500)+(5/1250)] = 5V ?*
5V = 5V

d. The second op amp (rightmost in the schematic shown) performs two primary functions in this circuit. One of these functions could instead be integrated into the first op amp circuit by using a different selection of resistor values. What function does this op amp perform that *cannot* be integrated with the first op amp? (A one sentence answer is worth 3 points.)

The second op-amp circuit inverts the signal and changes the gain. We could alter the gain to whatever we want with the adder, but we cannot do the inversion.

Invert the signal.

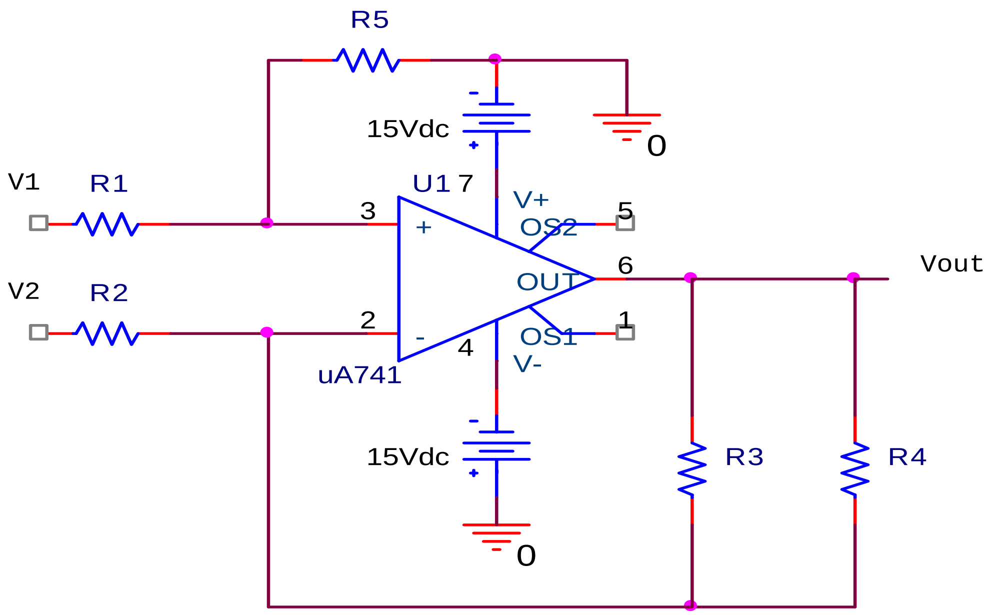
ENGR4300 Test 2A
Spring 2005

Name ___solution___________

Section ______________________

4) Op Amp Analysis (25 points)

Part A Given the op-amp circuit below:



Test A: R1 = R2 = 2k, R3 = R4 = 6k, R5 = 3k
Test B: R1 = R2 = 1k, R3 = R4 = 8k, R5 = 4k

a. What op-amp circuit given on your crib sheet does this circuit most closely represent? (Hint: disregard specific resistor values) (2 points)

Difference (differential) amplifier

b. What are the golden rules of op amp analysis? (2 points)

1) V+ = V-

2) I+ = I- = 0

c. Find an expression for Vout in terms of V1, V2 and resistor values R1, R2, R3, R4, and R5 (do not substitute actual resistor values) (12 points)

*Let R34 = R3*R4/(R3+R4)*

$$-: \quad i = \frac{V_2 - V_B}{R_2} = \frac{V_B - V_{out}}{R_{34}}$$

$$+: \quad V_A = \frac{R_5}{R_1 + R_5} V_1$$

$$solve \ \ for\, V_B: \quad V_B = \frac{\frac{V_2}{R_2} + \frac{V_{out}}{R_{34}}}{\frac{1}{R_2} + \frac{1}{R_{34}}} \quad V_B = \frac{\frac{R_{34}V_2 + R_2 V_{out}}{R_2 R_{34}}}{\frac{R_{34} + R_2}{R_2 R_{34}}} \quad V_B = \frac{R_{34}}{R_{34} + R_2} V_2 + \frac{R_2}{R_{34} + R_2} V_{out}$$

$$V_A = V_B: \quad \frac{R_5}{R_1 + R_5} V_1 = \frac{R_{34}}{R_{34} + R_2} V_2 + \frac{R_2}{R_{34} + R_2} V_{out}$$

$$\frac{R_2}{R_{34} + R_2} V_{out} = \frac{R_5}{R_1 + R_5} V_1 - \frac{R_{34}}{R_{34} + R_2} V_2$$

$$V_{out} = \frac{R_{34} + R_2}{R_2} \frac{R_5}{R_1 + R_5} V_1 - \frac{R_{34} + R_2}{R_2} \frac{R_{34}}{R_{34} + R_2} V_2$$

$$R_{34} = \frac{R_3 \times R_4}{R_3 + R_4} \Rightarrow \quad V_{out} = \frac{\frac{R_3 \times R_4}{R_3 + R_4} + R_2}{R_2} \frac{R_5}{R_1 + R_5} V_1 - \frac{\frac{R_3 \times R_4}{R_3 + R_4}}{R_2} V_2$$

That is as far as I'm going with this one. If they reduce further that is good, but this is ok.

d. Substitute resistor values in this equation and write the equation for Vout in terms of V1 and V2 input signals. (3 points)

A:

$$R_{34} = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{6k \times 6k}{6k + 6k} = 3k \Rightarrow \quad V_{out} = \frac{3k + 2k}{2k} \frac{3k}{2k + 3k} V_1 - \frac{3k}{2k} V_2 = \frac{3}{2}(V_1 - V_2)$$

$$V_{out} = \frac{3}{2}(V_1 - V_2)$$

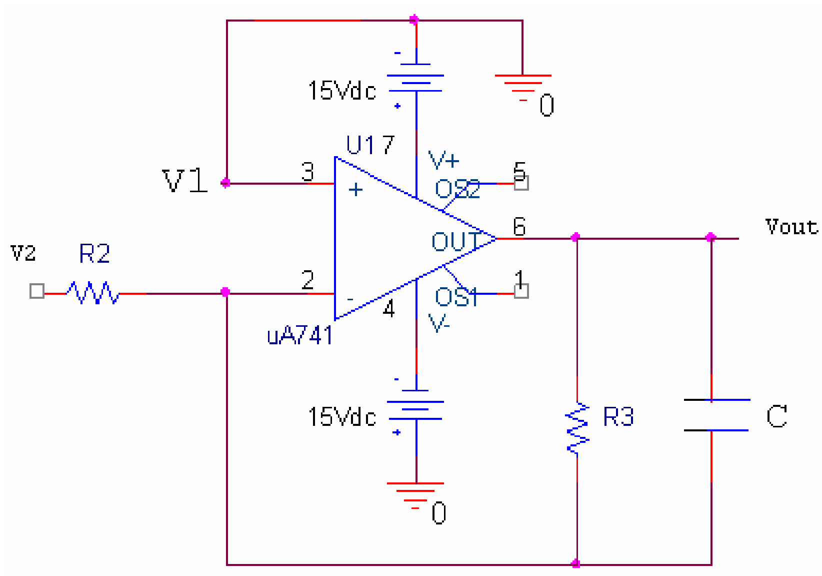
B:

$$R_{34} = \frac{R_3 \times R_4}{R_3 + R_4} = \frac{8k \times 8k}{8k + 8k} = 4k \Rightarrow \quad V_{out} = \frac{4k + 1k}{1k} \frac{4k}{1k + 4k} V_1 - \frac{4k}{1k} V_2 = 4(V_1 - V_2)$$

$$V_{out} = 4(V_1 - V_2)$$

ENGR4300 Test 2A | **Name ___solution___________**
Spring 2005 | **Section ________________________**

Part B What if R4 is replaced by a 10uF capacitor and V1 is grounded, as shown below?



a. What function is this circuit designed to perform? (2 points)

This circuit does integration. It is a Miller integrator.

b. Write the transfer function Vout/V2 for this circuit (2 points)

A: $$\frac{V_{out}}{V_2} = -\frac{Z_f}{Z_{in}} = -\frac{\frac{R_3}{j\omega R_3 C + 1}}{R_2} = -\frac{R_3}{j\omega R_2 R_3 C + R_2} = -\frac{6k}{j\omega 2k6k10\mu + 2k} = -\frac{6k}{j\omega 120 + 2k}$$

B: $$\frac{V_{out}}{V_2} = -\frac{Z_f}{Z_{in}} = -\frac{\frac{R_3}{j\omega R_3 C + 1}}{R_2} = -\frac{R_3}{j\omega R_2 R_3 C + R_2} = -\frac{8k}{j\omega 1k8k10\mu + 1k} = -\frac{8k}{j\omega 80 + 1k}$$

c. Over about what frequency range is the desired function of the circuit reliably performed? [You can assume that the operation is being performed even when the output amplitude is very small.] (2 points)

A: This circuit works at frequencies much greater than $fc = \frac{1}{2\pi R_3 C} = \frac{1}{2\pi 6k10\mu}$ *= 2.65 Hz. I am going to choose 5 times 2 Hz since that worked well on the quiz we went over in class. Therefore, this circuit works as an integrator* ***at frequencies above 13 Hz.*** *(Anything between 5 and 27 hertz is ok.)*

B: This circuit works at frequencies much greater than $fc = \frac{1}{2\pi R_3 C} = \frac{1}{2\pi 8k10\mu}$ *= 2 Hz.*

I am going to choose 5 times 2 Hz since that worked well on the quiz we went over in class. Therefore, this circuit works as an integrator ***at frequencies above 10 Hz****. (Anything between 4 and 20 hertz is ok.)*