ENGR-4300
Spring 2008
Test 2 CONFLICT

Name: _______SOLUTION__________

Section: 1(MR 8:00) 2(TF 2:00) 3(MR 6:00)
(circle one)

Question I (23 points): ___________

Question II (16 points): ___________

Question III (16 points): ___________

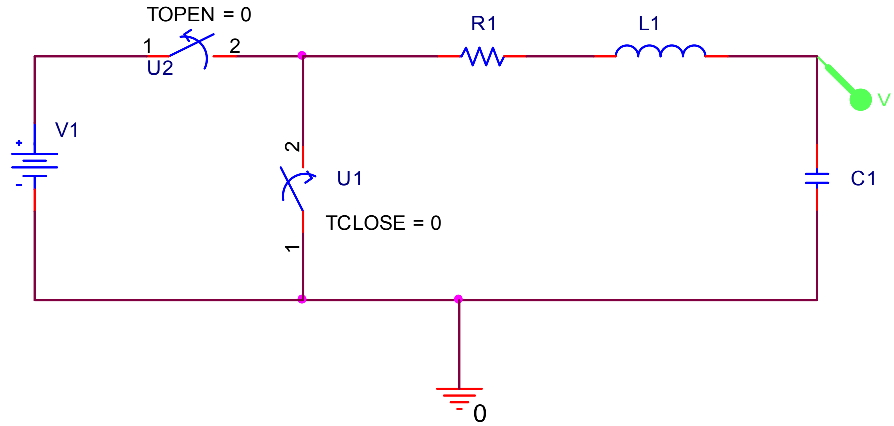
Question IV (20 points): ___________

Question V (25 points): ___________

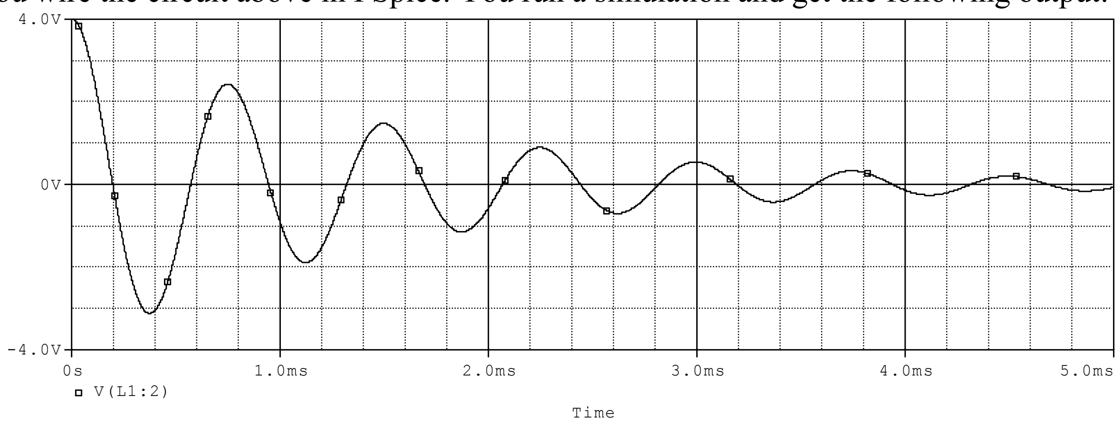
Total (100 points): ______________

On all questions: SHOW ALL WORK. BEGIN WITH FORMULAS, THEN SUBSTITUTE VALUES AND UNITS. No credit will be given for numbers that appear without justification.

Question I – Bridges and Damped Sinusoids (23 points)



You wire the circuit above in PSpice. You run a simulation and get the following output:



1) (3pt) Using the output pictured, determine the damping constant, α, of the circuit.

@ t = 3ms v(t) = 0.52 $4e^{-\alpha(0.003)} = 0.52 \Rightarrow \alpha = \dfrac{-\ln\frac{0.52}{4}}{0.003} = 680$

2) (3pt) What is the resonant frequency of the circuit in Hertz?

f = 1/T T = 4.5ms/6 ⇒ f = 6/0.0045 = 1.333kHz
ω = 2πf = 8378rad/sec

3) (3pt) Write an expression in the form $v(t) = Ae^{-\alpha t}\cos(\omega_0 t)$ for the output signal.

$v(t) = 4e^{-680t}\cos(8378t)$

Question I – Bridges and Damped Sinusoids (continued)

4) (6pt) The differential equation that governs the behavior of a damped sinusoid is given by $\frac{d^2V}{dt^2} + 2\alpha\frac{dV}{dt} + \omega_0^2 V = 0$. In a simple RLC circuit like the one in this question, the angular resonant frequency of the circuit, ω_0, is given by $\omega_0 = \frac{1}{\sqrt{LC}}$ and the decay constant, α, is given by $\alpha = \frac{R}{2L}$. In the circuit in this question, the value of the resistor, L1, is 40mH. What are the values of the capacitor, C1, and the inductor, R1?

$$\alpha = \frac{R}{2L} \Rightarrow R1 = 2\alpha L1 = 2 \cdot 680 \cdot 0.040 = 54.4\Omega$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \Rightarrow C1 = \frac{1}{L1\omega^2} = \frac{1}{0.040 \cdot 8378^2} = 0.356\mu F$$

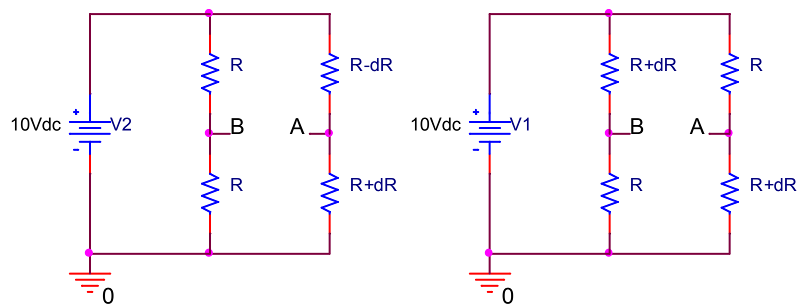
5) (2pt) What new value of capacitance for C1 would make the resonant frequency of the circuit be ten times higher than what it is now?

$$10\omega = \frac{10}{\sqrt{LC}} = \frac{1}{\frac{1}{10}\sqrt{LC}} = \frac{1}{\sqrt{\frac{1}{100}LC}}$$

$$C1' = \frac{C1}{100} = 3.56nF$$

Question I – Bridges and Damped Sinusoids (continued)

6) (6pt) Below are 2 variations of bridges circuits. The left configuration is the one used in Experiment 5. The right configuration has both strain gauges on the top of the beam. They are configured with a DC voltage source as shown below. Note that the nominal resistance of each strain gauge is represented as *R* and the change in the resistance of the strain gauge due to the deflection of the beam is *dR*. The voltage source is set to *10V*. The output voltage observed is A – B. Using this information, determine the sensitivity to deflection in terms of R and dR of each circuit and which is more sensitive.



Left bridge:

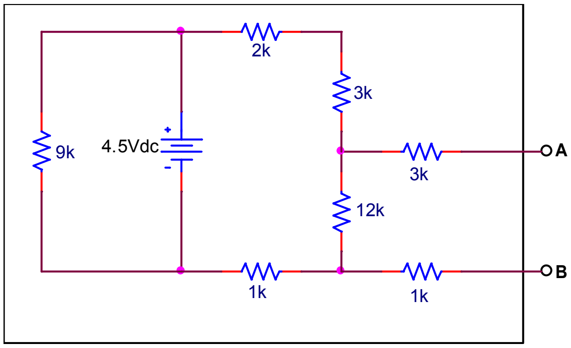
$$\frac{V_A - V_B}{10} = \frac{R+dR}{R+dR+R-dR} - \frac{R}{R+R} = \frac{R+dR}{2R} - \frac{R}{2R} = \frac{dR}{2R}$$

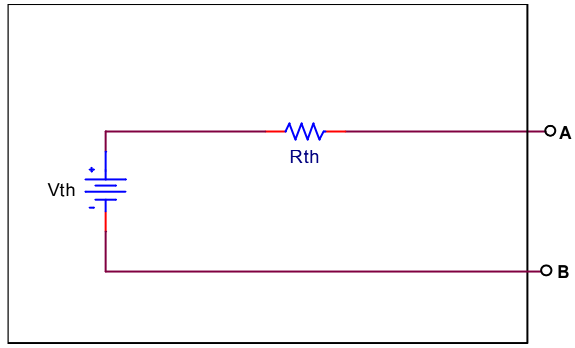
Right bridge:

$$\frac{V_A - V_B}{10} = \frac{R+dR}{R+dR+R} - \frac{R}{R+R+dR} = \frac{R+dR}{2R+dR} - \frac{R}{2R+dR} = \frac{dR}{2R+dR}$$

$$\frac{dR}{2R} \geq \frac{dR}{2R+dR} \quad \therefore \text{ left bridge is slightly more sensitive to deflection}$$

Question II – Thevenin Equivalents (16 points)





1) (5pt) Find the Thevenin equivalent voltage with respect to A and B for the circuit shown above left with the Thevenin equivalent circuit on the right.

$$V_{th} = 4.5\frac{12}{12+1+2+3} = 4.5\frac{12}{18} = 3V$$

2) (5pt) Find the Thevenin equivalent resistance with respect to A and B for the circuit shown above left.

$$R_{th} = 12k \| (1k + 2k + 3k) + 1k + 3k = 12k \| 6k + 4k = 4k + 4k = 8k\Omega$$

Note: 9k is shorted by supply in R_{th} calculation.

3) (4pt) If terminal A is shorted to terminal B in the left circuit, what is the maximum current that will flow through the connection? (Hint: use what can be done with Thevenin equivalent circuits to simplify this calculation.)

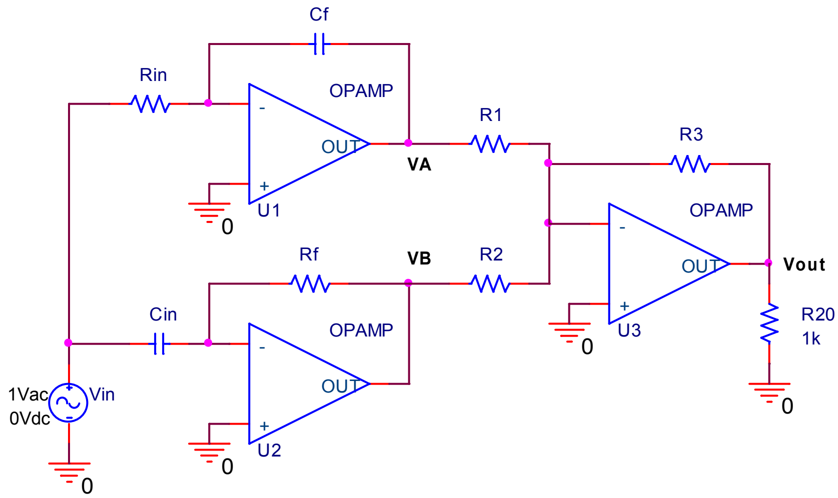
Short circuit current = V_{th}/R_{th} = 3V/8k = 0.375mA

4) (2pt) How would the V_{th} and R_{th} values change if the power supply voltage in the left circuit doubled?

R_{th} would be unchanged

V_{th} would double = 6V

Question III – Op-Amp Applications (16 points)



Assume that **±9 Volt power supplies** have been properly connected to all three op-amps in the circuit above.

1) (3pt) The circuit has 3 op-amps labeled as U1, U2, and U3. State what the op-amp circuit is for each. Choices are: 1. Follower/Buffer, 2. Inverting Amp, 3. Non-inverting Amp, 4. Differentiator, 5. Integrator, 6. Adding (Mixing) Amp, 7. Difference (Differential) Amp.

U1 Circuit: _INTEGRATOR_ U2 Circuit: _DIFFERENTIATOR_ U3 Circuit: _ADDING AMP_

2) (6pt) Determine the voltage (a function of t), relative to ground, at points VA and VB as functions of Vin(t) with Rin = 40k, Cf = 25µF, Cin = 33µF, Rf = 10k, R1 = 5k, R2 = 3k, and R3 = 15k.

a) Voltage at point VA(t):

$$VA(t) = -\frac{1}{40k \cdot 25\mu}\int Vin(t)dt = -\int Vin(t)dt$$

b) Voltage at point VB(t):

$$VB = (-10k \cdot 33\mu)\frac{dVin(t)}{dt} = (-0.33)\frac{dVin(t)}{dt}$$

Question III – Op-Amp Applications (continued)

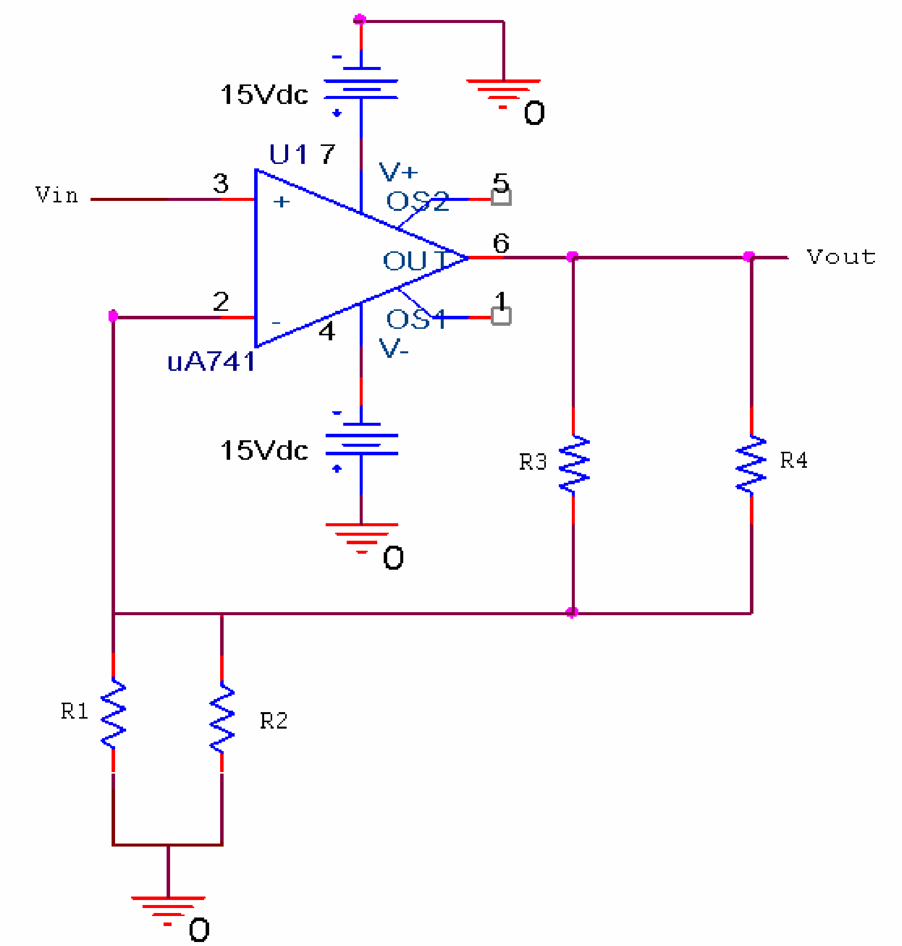
3) (3pt) Determine the output voltage, Vout(t), as a function of VA(t) and VB(t).

$$Vout(t) = -\left(\frac{R3}{R1}\right)VA(t) - \left(\frac{R3}{R2}\right)VB(t) = -\left(\frac{15k}{5k}\right)VA(t) - \left(\frac{15k}{3k}\right)VB(t) = -3VA(t) - 5VB(t)$$

4) (4pt) Now find Vout(t) as a function of Vin(t).

$$Vout = -3VA(t) - 5VB(t) = (-3)(-1)\int Vin(t)dt - 5(-.33)\frac{dVin(t)}{dt} = 3\int Vin(t)dt + 1.65\frac{dVin(t)}{dt}$$

Question IV – Op-Amp Analysis (20 points)



R1 = R2 = 6k, R3 = R4 = 8k

1) (2pt) What op-amp circuit given on your crib sheet does this circuit most closely represent? (Hint: disregard specific resistor values)

Non-inverting Amplifier

2) (2pt) What are the two golden rules of op-amp analysis?

1. The output attempts to do whatever is necessary to make the voltage difference between the inputs zero (the + and – terminals will have the same voltage).

2. The inputs (+ and – terminals) draw no current.

Question IV – Op-Amp Analysis (continued)

3) (12pt) **Derive** an expression for Vout using the rules for op-amp analysis. You must use Ohm's Law and current laws to describe how the op-amp functions by the golden rules. Your result should be in terms of Vin and resistor values R1, R2, R3, and R4. (Do not substitute actual resistor values. You cannot assume R1=R2 or R3=R4.)

R34 = R3||R4 = R3R4/(R3+R4)   R12 = R1||R2 = R1R2/(R1+R2)

$V^+ = V^- = Vin$

I_f = (Vout - Vin)/R34 I_g = (Vin - 0)/R12 = Vin/R12

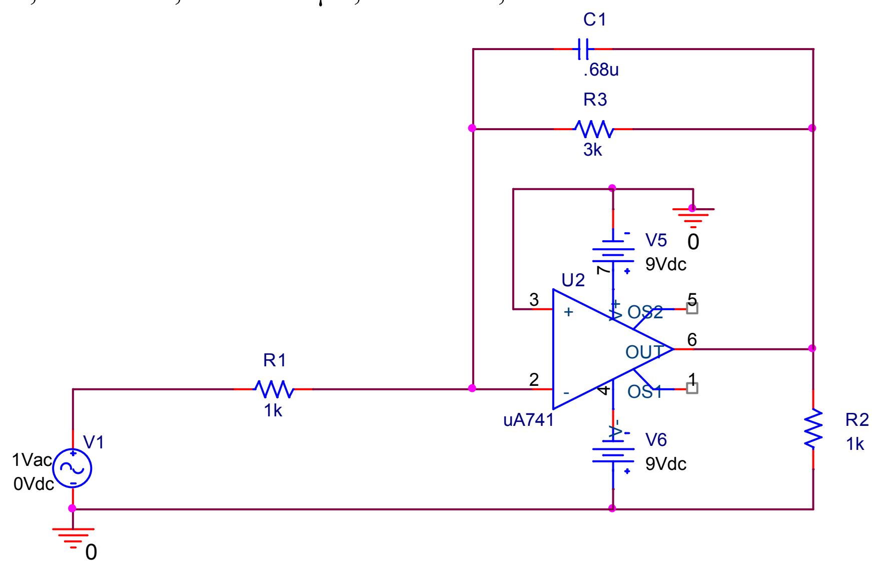
$I_f = I_g$ (Vout - Vin)/R34 = Vin/R12
Vout - Vin = Vin x R34/R12
Vout = Vin(1 + R34/R12)
Vout = Vin{1 + [R3R4/(R3+R4)]/[R1R2/(R1+R2)]}

4) (4pt) Substitute resistor values in this equation and write the equation for Vout in terms of Vin.

$$Vout = \left(1 + \frac{R_f}{R_g}\right)Vin = \left(1 + \frac{R3\|R4}{R1\|R2}\right)Vin = \left(1 + \frac{\frac{R3\cdot R4}{R3+R4}}{\frac{R1\cdot R2}{R1+R4}}\right)Vin = \left(1 + \frac{\frac{8k\cdot 8k}{8k+8k}}{\frac{6k\cdot 6k}{6k+6k}}\right)Vin = \frac{7}{3}Vin$$

Question V – Op-Amp Integrators and Differentiators (25 points)

In the circuit below, R1 = 1kΩ, C1 = 0.68μF, R3 = 3kΩ, and R2 = 1kΩ



1) (2pt) What function is this circuit designed to perform?

Practical Miller Integration

2) (4pt) Write the transfer function Vout/V1 for this circuit. (Substitute in the values provided for the components.)

$$\frac{V_{out}}{V_1} = -\frac{R_f}{R_{in}(1 + j\omega R_f C_f)} = -\frac{R_3}{R_1(1 + j\omega R_3 C_1)} = -\frac{3k}{1k(1 + j\omega 3K \cdot 0.68\mu)} = -\frac{3}{1 + j\omega(0.00204)}$$

3) (4pt) Over about what frequency range (in Hertz) is the desired function of the circuit reliably performed? [You can assume that the operation is being performed even when the output amplitude is very small.]

$f >> f_c = 1/(2\pi R_f C_f) = 1/(2\pi \text{ x } 1k \text{ x } 0.68\mu) = 78Hz$ $\quad$ $f \geq \sim 240Hz$

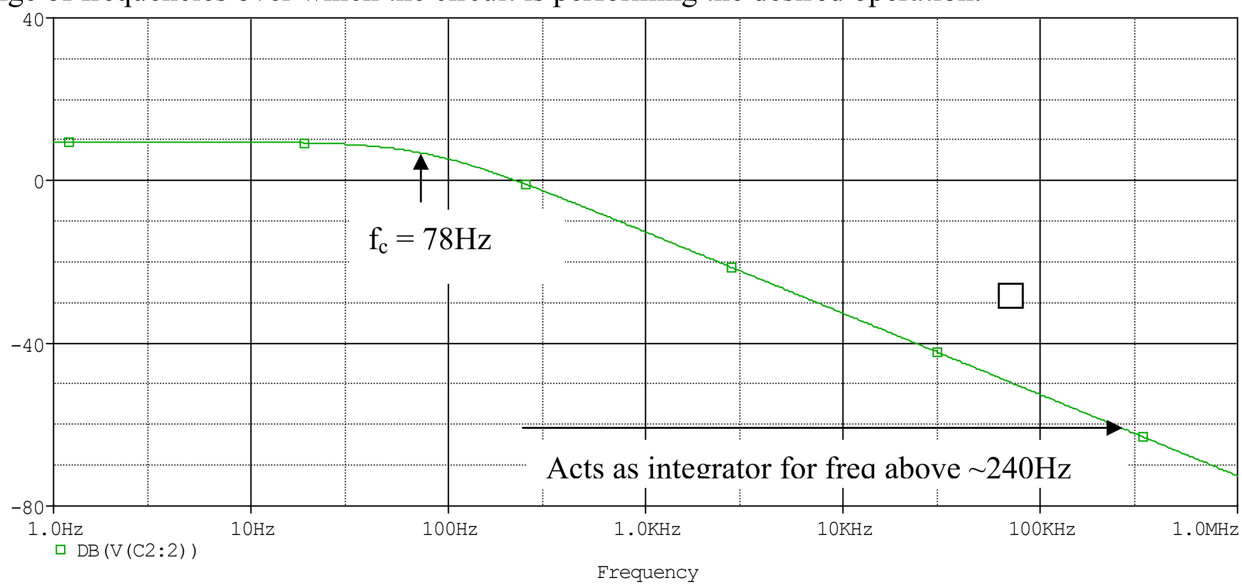
Question V – Op-Amp Integrators and Differentiators (continued)

4) (4pt) Assuming that you have created this circuit in PSpice, how would you set up a simulation to observe how it behaves over a range of frequencies?

Add dB voltage probe to Vout and set AC amplitude of V1 to 1V (Normal voltage probe is OK)
For Simulation-
 Analysis type: AC Sweep/Noise
 Start Frequency: 1Hz
 Stop Frequency: 1MegHz
 Points/Decade: 100
Run simulation

For plot in5 below, linear (non-log) plot is OK too.

5) (4pt) Make a sketch of the frequency response of this circuit. Mark the corner frequency and the range of frequencies over which the circuit is performing the desired operation.



6) (3pt) What type of filter does this frequency response most closely represent?
a) low pass filter b) high pass filter c) band pass filter d) band reject filter

a) Low Pass Filter

7) (4pt) for an input V1(t) = cos(6000t), what would be the amplitude of the output signal? [Assume the circuit is in its operating frequency and voltage range.]

$$Vout(t) \approx -1470 \int V_1(t)dt = -1470 \int \cos(3000\,t)dt = \frac{-1470}{-6000}\sin(6000\,t) = \boxed{0.245}\sin(6000\,t)$$

8) (EXTRA CREDIT 1pt) At low frequency this circuit in 5) acts like a ________________.
Fill in the blank using one of the following: a) Inverting Amplifier, b) Non-inverting Amplifier, c) Differential Amplifier, d) Adder, e) Integrator, f) Differentiator, g) none of a thru f.

a) Inverting Amplifier