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## 1) Damped Sinusoids ( 25 points) from Spring 2005

You wire the following circuit in PSpice.


You run a simulation and get the following output:

a) How would you set up the PSpice simulation screen pictured below, to get the output pictured above? (3 points)

A: Run to time $=2 \mathrm{~ms}$
Start time $=0$
Maximum step size varies should be from about 1us to about 20us.
b) Using the output pictured, determine the damping constant, $\alpha$, of the circuit. (3 points)

A: $(t 0, V 0)=(0 m s, 8 V)(t 1, V 1)=(1.96 m s, 1.84 V) \ln (1.84 / 8)=-\alpha(1.96 m-0 m)$
$\alpha=749.8 / s$
c) What is the resonant frequency of the circuit in Hertz? (3 points)

$$
\text { A: } f=22 \text { cycles } / 1.96 \mathrm{~ms}=11.22 \mathrm{~K} \mathrm{~Hz}
$$

d) Write an expression in the form $v(t)=A e^{-\alpha t} \cos \left(\omega_{0} t\right)$ for the output signal. (3 points)

$$
A: \omega=2 \pi f=70.5 \mathrm{~K} \mathrm{rad} / \mathrm{sec} \quad v(t)=8 V e^{-749.8 t} \cos (70.5 K t)
$$

e) Use the general equations for capacitor and inductor behavior (located on the crib sheet for quiz 1), to describe what is happening in this circuit. What is causing the voltage to behave like a damped sinusoid? (5 points)

The equation for the capacitor is $I_{C}=C \frac{d V_{C}}{d t}$. The equation for the inductor is $V_{L}=L \frac{d I_{L}}{d t}$. The 8 volt source places a charge on the capacitor at time $t=0$. Then the circuit is disconnected from the source and is allowed to oscillate on its own. The capacitor begins to discharge into the rest of the circuit and the voltage across it changes. The current, Ic, caused by the changing voltage of the capacitor ( $d V c / d t$ ) starts to flow through the inductor. (This is also $I_{L}$ because the circuit is in series.) The changing current in the inductor ( $d I_{L} / d t$ ) induces a voltage, $V_{L}$. This changing voltage causes the capacitor to change up again. The process continues. During each cycle some of the energy in the circuit is dissipated by the resistance of the resistor and this causes the signal to decay and eventually disappear.

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f) The differential equation that governs the behavior of a damped sinusoid is given by $\frac{d^{2} V}{d t^{2}}+2 \alpha \frac{d V}{d t}+\omega_{0}^{2} V=0$. In a simple RLC circuit like the one in this question, the angular resonant frequency of the circuit, $\omega_{0}$, is given by $\omega_{0}=\frac{1}{\sqrt{L C}}$ and the decay constant, $\alpha$, is given by $\alpha=\frac{R}{2 L}$. In the circuit above, the value of the resistor, R1, is 30 ohms. What are the values of the capacitor, C 1 , and the inductor, L 1 ? (6 points)

$$
\begin{aligned}
& \text { A: } \alpha=\frac{R}{2 L} \quad 749.8=(30) /(2 L) \quad L=30 /(2)(749.2)=20 \mathrm{mH} \\
& \omega_{0}=\frac{1}{\sqrt{L C}} \quad 70.5 K=1 / \operatorname{sqrt}(20 \mathrm{~m} \times C) \quad C=1 /\left(L \omega^{2}\right)=1 /\left(20 \mathrm{~m} \times 70.5 \mathrm{~K}^{2}\right)=0.01 \mu \mathbf{F}
\end{aligned}
$$

g) You want the damping constant of the circuit to be double what it is now. What new value of L1 would choose to make this occur? (2 points)

A: $L=30 /(2)(2)(749.2)=10 \boldsymbol{m H}$

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## 2) Thevenin Equivalent Sources ( 25 points) from Spring 2005

Part A You build the circuit pictured below


Test A:
Given: R1=30 ohms, R2=2K ohms, R3 = 3K ohms, R4=2K ohms and R5=1K ohms. Given: V1=6V
a) Find the Thevenin Equivalent voltage, Vth, of this circuit between point A and point B. (6 points)

A: $V t h=V R 4=V 1(R 4) /(R 1+R 2+R 3+R 4)=1.707 V$
b) Find the Thevenin Equivalent Resistance, Rth, of this circuit between point A and point B. (6 points)

$$
\begin{aligned}
& \text { A: Rth }=R 5+(R 1+R 2+R 3) / / R 4 \quad R 1+R 2+R 3=5030 \\
& R 123 / / R 4=(5030)(2000) /(5030+2000)=1431 \\
& \text { Rth }=1000+1431=\mathbf{2 4 3 1} \text { ohms }
\end{aligned}
$$

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c) Redraw the Thevenin equivalent model of the circuit (2 points).

d) If you place a 2 K ohm load on the circuit, what will the output voltage be between A and B ? (2 points)

$$
\text { A: Vout }=(2000 / 4431) 1.707=0.770 \mathrm{~V}
$$

e) What is the current through the 2 K ohm load resistor from d? (2 points)

A: $I=V / R \quad I=0.770 / 2 K=0.385 m A$

Part B You place a voltage follower into this circuit between A and B, as pictured below.


Test A:
Given: R1=30 ohms, R2=2K ohms, R3 = 3K ohms, R4=2K ohms and R5=1K ohms. Given: V1=6V
Given: R6 is the load of 2 K ohms
a) What does the voltage follower do in this circuit? (2 points)

The voltage follower isolates the circuit we modeled from the load R6. This means that whatever voltage the circuit puts out between $A$ and $B$ will be transferred to $R 6$ without being influenced by the value of $R 6$.
b) What is the voltage output between $A$ and $B$ for this circuit now? (3 points)

The voltage between $A$ and $B$ will always be the Thevenin voltage for the circuit because the buffer looks like an infinite impedance to it. Since an infinite impedance is much much bigger than Rth, it will not influence the circuit. Therefore,

$$
A: V A B=V t h=1.707 V
$$

c) What is the current though the load resistor, R6? (2 points)

A: $I=V / R \quad I=1.707 / 2 K=0.854 m A$

Question 3 - Op Amp Applications (25 points) from Fall 2005


The following questions refer to the circuit on the previous page.

1) What type of op-amp circuit is between points $A$ and $B$ ? (1 point)

## Inverting Amplifier

2) What type of op-amp circuit is between points $C$ and $D$ ? (1 point)

## Non-inverting amplifier

3) What type of op-amp circuit is between (B and D) and E? (1 point)

## (Weighted) Adder

4) Write an expression for the voltage at point $B, V_{B}$, in terms of the voltage at $A, V_{A}$. Please substitute values. (3 points)

$$
V_{B}=-(R f / R i n) V_{A}=-(3 k / 1 k) V_{A} \quad V_{B}=-3 V_{A}
$$

5) Write an expression for the voltage at point $D, V_{D}$, in terms of the voltage at $C, V_{C}$. Please substitute values. (3 points)

$$
V_{D}=(1+R f / R i n) V_{C}=(1+2 k / 1 k) V_{C} \quad V_{D}=3 V_{C}
$$

6) Write an expression for the voltage at point $\mathrm{E}, \mathrm{V}_{\mathrm{E}}$, in terms of the voltage at $\mathrm{B}, \mathrm{V}_{\mathrm{B}}$, and the voltage at $\mathrm{D}, \mathrm{V}_{\mathrm{D}}$. Please substitute values. (3 points)

$$
V_{E}=-R f(V 1 / R 1+V 2 / R 2) \quad V_{E}=-2 k\left(V_{B} / 2 k+V_{D} / 1 k\right) \quad V_{E}=-V_{B}-2 V_{D}
$$

7) Write an expression for the output voltage at $\mathrm{E}, \mathrm{V}_{\mathrm{E}}$, in terms of the two input voltages in the circuit, $\mathrm{V}_{\mathrm{A}}$ and $\mathrm{V}_{\mathrm{C}}$. (3 points)

$$
V_{E}=-V_{B}+2 V_{D} \quad V_{E}=-\left(-3 V_{A}\right)-2\left(3 V_{C}\right) \quad \boldsymbol{V}_{\boldsymbol{E}}=3 \boldsymbol{V}_{A}-\mathbf{6} \boldsymbol{V}_{C}
$$

8) Identify the signals at points $\mathrm{A}, \mathrm{B}, \mathrm{C}, \mathrm{D}$ and E in the PSpice plot below. (10 points)

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## Question 4 - Op Amp Analysis (30 points) From Fall 2005

You are given the circuit below:


1) What are the golden rules of op amp analysis (2 points):
1. $\boldsymbol{I}+=\boldsymbol{I}-=\mathbf{O}$
2. $V+=V-$
2) Without a feedback connection, can either of these rules be violated for an ideal op amp? If so, which one(s) and why (2 points)?

The terminals of the op amp are very high impedance - ideally an open circuit - so it is certainly possible to set the voltages at the two terminals to be different, consequently, $V+=V$ - can be violated and the device can still be useful as a comparator. The high intrinsic gain of the device will drive it to saturation at $+V d c$ if $V+>V$ - or at $-V d c$ if $V+$ $<V-. I+=+-=0$ cannot be violated if the input terminals are open circuit internal to the device.

Grading:1 pt for correct rule by itself (not if listed both rules), 1 pt for any sensible explanation.
$\qquad$
3) Use the golden rules of op amp analysis to derive an expression for Vout in terms of Vin and the resistors in the circuit as labeled. Do NOT substitute resistor values until instructed to do so.
a) Draw the circuit corresponding to the inverting terminal of the op-amp (3 points)

b) Based on the circuit in a) write the equation at the inverting terminal of the op amp (Do not substitute values) (3 points)
$\frac{V_{\text {in }}-V^{-}}{R 1}=\frac{V^{-}-V_{\text {out }}}{R 5+R 2}$
c) Draw the circuit corresponding to the non-inverting terminal of the op-amp (3 points)

d) Based on the circuit in c) write the equation at the non-inverting terminal of the op amp (Do not substitute values) (3 points)

$$
V^{+}=\frac{R 4}{R 3+R 4} 12 V
$$

e) Use these equations to solve for Vout in terms of Vin and resistor values R1-R6. (Do not substitute values.) (7 points)

$$
\begin{aligned}
& \frac{V_{\text {in }}-V^{-}}{R 1}=\frac{V^{-}-V_{\text {out }}}{R 5+R 2} \quad V^{+}=\frac{R 4}{R 3+R 4} 12 V \quad V^{+}=V^{-} \\
& \frac{V_{\text {in }}-\frac{R 4}{R 3+R 4} 12 V}{R 1}=\frac{\frac{R 4}{R 3+R 4} 12 V-V_{\text {out }}}{R 5+R 2} \\
& V_{\text {in }}(R 5+R 2)-12 R 4 \frac{R 5+R 2}{R 3+R 4}=12 R 1 \frac{R 4}{R 3+R 4}-V_{\text {out }}(R 1) \\
& -(R 1) V_{\text {out }}=V_{\text {in }}(R 5+R 2)-12 R 4 \frac{R 5+R 2}{R 3+R 4}-12 R 1 \frac{R 4}{R 3+R 4}
\end{aligned}
$$

$$
V_{\text {out }}=-V_{\text {in }} \frac{R 5+R 2}{R 1}+\frac{12 R 4}{R 1(R 3+R 4)}(R 5+R 2+R 1)
$$

f) Substitute resistor values into your equation from part e to get Vout in terms of Vin (3 points)
$V_{\text {out }}=-V_{\text {in }} \frac{5 k+3 k}{2 k}+\frac{12(1 k)}{2 k(11 k+1 k)}(5 k+3 k+2 k)$
$V_{\text {out }}=-4 V_{\text {in }}+5$
g) For the input signal shown in the circuit schematic, what is the minimum and maximum output voltages generated at Vout? Show your work. (4 points)
if $\operatorname{Vin}=1 V \sin (\omega t)+1 V$ then Vout $=-4(1 V \sin (\omega t)+1 V)+5 V$
Vout $=-4 V \sin (\omega t)+1 V$
Minimum: Negative Amplitude is $-4 V$, Offset is $1 V-4+1$ is $-3 V$
Maximum: Amplitude is 4 V , Offset is $1 \mathrm{~V} 4+1$ is 5 V
$\qquad$
Question 5 -- Op Amp Integrators and Differentiators (25 points) from Spring 2004
Here is a combined differentiator/integrator similar to the one you implemented in experiment 8 . Let $\mathrm{C} 1=0.01 \mu \mathrm{~F}, \mathrm{R} 2=100 \mathrm{~K}$ ohms, $\mathrm{C} 2=0.01 \mathrm{nF}$, and $\mathrm{R} 3=300 \mathrm{~K}$ ohms.

a. Below is an AC sweep for the above circuit

i) Identify the input and the output traces.(2 points) (Test A)
ii) If you built this circuit in the studio, in which of the circled regions would the output look like the following? (2 points each) [Total=8 points]
a reasonable integration of the input? D
a reasonable differentiation of the input? B
an amplified inversion of the input? C
disappear into the noise? $\boldsymbol{A}, \boldsymbol{E}$
b. What are the general equations for the following: (Give specific values based on the components in the circuit.)
i. The circuit when it is acting as an ideal integrator (3 points)

$$
v_{\text {out }}=\frac{-1}{R_{\text {in }} C_{f}} \int v_{\text {in }} d t=\frac{-1}{(100 K)(0.01 n)} \int v_{\text {in }} d t=-1 \mathrm{Meg} \int v_{\text {in }} d t
$$

(Note how the large gain compensates for $1 / \omega$ in the integration. $\omega$ is large, its inverse is small, so a large gain is needed to make the signal recognizable. )
ii. The circuit when it is acting as an ideal differentiator (3 points)

$$
v_{\text {out }}=-R_{f} C_{\text {in }} \frac{d v_{\text {in }}}{d t}=-(300 K)(0.01 \mu) \frac{d v_{\text {in }}}{d t}=-3 m \frac{d v_{\text {in }}}{d t}
$$

(Note how the small gain compensates for $\omega$ in the differentiation. $\omega$ is large, soothe gain must be small or the op-amp will saturate.)
iii. The circuit when it is acting as an inverting amplifier (3 points)

$$
v_{\text {out }}=\frac{-R_{f}}{R_{\text {in }}} v_{\text {in }}=\frac{-300 K}{100 K} v_{\text {in }}=-3 v_{\text {in }}
$$

(Note that the input amplitude is $1 V$ and at $C$, where the circuit is an inverting amplifier, the output amplitude is 3 )
cA. Sketch the AC sweep of an integrator that integrates between 1 K and 4 K hertz. Give a ballpark estimate of the corner frequency. Mark $1 \mathrm{~K}, 4 \mathrm{~K}$ and the corner frequency on the sketch. Justify your decisions. (6 points)


The corner frequency in this case is about 200 Hertz. Note that answers may vary. Corner frequencies should be at or below 500 Hz .
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cB. Sketch the AC sweep of an differentiator that differentiates between 1 K and 4 K hertz. Give a ballpark estimate of the corner frequency. Mark $1 \mathrm{~K}, 4 \mathrm{~K}$ and the corner frequency on the sketch. Justify your decisions. (6 points)


The corner frequency in this case is about 20K Hertz. Note that answers may vary. Corner frequencies should be at or above 8 K Hz

