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## Question 1 -- Astable Multivibrator (23 points)



The circuit above has been simulated using PSpice. The output below shows traces from probes placed at pins 2,3,6 and 7 .
a. Label which trace goes with which pin $(2,3,6,7)$ in each time period. Be sure that you label the traces in both the on and off parts of the pulse cycle. (8 points)

b. Find an equation which relates the duty cycle of the output to the values of R1 and R2. Do not substitute in values for R1 and R2. (3 points)
$D C=T 1 / T * 100=[0.693(R 1+R 2) C 1] /[0.693(R 1+2 R 2) C 1] * 100$
$D C=[R 1+R 2] /[R 1+2 R 2] * 100=[(R 1 / R 2)+1] /[(R 1 / R 2)+2] * 100$

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c. Use the equation you found in $b$ to determine the approximate duty cycle of this circuit when: (6 points)
$\mathrm{R} 1 \gg \mathrm{R} 2 \quad[(\infty / 0)+1] /[(\infty / 0)+2] * 100 \approx 100 \%$
$\mathrm{R} 1=\mathrm{R} 2 \quad[1+1] /[1+2] * 100=66.67 \%$
$\mathrm{R} 1 \ll \mathrm{R} 2 \quad[(0 / \infty)+1] /[(0 / \infty)+2] * 100 \approx 50 \%$
d. Using the output shown, on the previous page, determine the duty cycle of this circuit. (4 points)
$T 1=389 \mathrm{~m}-273 \mathrm{~m}=116 \mathrm{~ms} \quad T 2=450 \mathrm{~m}-389 \mathrm{~m}=61 \mathrm{~ms}$
$T=116 m+61 m=177 m s$
$D=(T 1 / T) * 100=116 \mathrm{~m} / 177 \mathrm{~m} * 100=65.5 \%$

A-e. What values do R1 and C1 have to have in order to create this output, if R2 is 47 K ? (4 points)
$T 2=0.693(R 2)(C 1) \quad 61 m=.693(47 \mathrm{~K}) C 1 \quad C 1=1.9 \mu F$
$\begin{array}{rl}T 1=0.693(R 1+R 2) C 1 & 116 m=0.693(R 1+47 K)(1.9 \mu) \quad \text { Note: } m / \mu=K \\ & 89.4 K=R 1+47 K \\ & R 1=42 K \text { ohms }\end{array}$

B-e. What values do R1 and C1 have to have in order to create this output, if R2 is 22 K ? (4 points)

$$
\begin{array}{ll}
T 2=0.693(R 2)(C 1) \quad 61 m=.693(22 K) C 1 \quad C 1=4.0 \mu F \\
T 1=0.693(R 1+R 2) C 1 \quad & 116 m=0.693(R 1+22 K)(4 \mu) \quad \text { Note: } m / \mu=K \\
41.8 K=R 1+22 K \\
R 1=19.8 K \text { ohms }
\end{array}
$$

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2) Inductance Measurement (19 points)

The RLC circuit below consists of an inductor, a capacitor and a resistor. The input is measured at point A and the output at point B .

a) Find the transfer function, $H(j \omega)$, at point $B$. Determine the value of the function, the magnitude and the phase at high and low frequencies.

$$
\begin{array}{rlrl}
(2 \mathrm{pts}) \mathrm{H}(\mathrm{j} \omega) & =[(1 / j \omega L C)+j \omega L] /[R+(1 / j \omega L C)+j \omega L] \\
& =\left[1-\omega^{2} L C\right] /\left[j \omega R C+1-\omega^{2} L C\right] & & \\
(1 \mathrm{pt}) \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=1 & & (1 \mathrm{pt}) \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=-\omega^{2} L C /-\omega^{2} L C=1 \\
(1 \mathrm{pt})\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)\right|=1 & & (1 \mathrm{pt})\left|\mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)\right|=1 \\
(1 \mathrm{pt}) \angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{lo}}\right)=0 \mathrm{rad} & & (1 \mathrm{pt}) \angle \mathrm{H}\left(\mathrm{j} \omega_{\mathrm{hi}}\right)=0 \mathrm{rad}
\end{array}
$$

b) Also calculate the value of $|\mathrm{H}(\mathrm{j} \omega)|$ at the resonant frequency, $\omega_{0}$. (2 points)

$$
\begin{aligned}
H\left(j \omega_{0}\right) & =\left[1-(1 / \sqrt{ }(L C))^{2} L C\right] /\left[j(1 / \sqrt{ }(L C)) R C+1-(1 / \sqrt{ }(L C))^{2} L C\right] \\
& =[0] /[j(R C /(\sqrt{ }(L C))]=0 \\
\left|H\left(j \omega_{0}\right)\right| & =0
\end{aligned}
$$

A-c) Calculate the theoretical inductance if your coil has a coin shape. Note this coil has an air core with a diameter of 3 cm . It has 50 turns and is built from 28 gauge wire. The diameter of 28 gauge wire is 0.32 mm and $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$. (3 points)
$L=\mu_{0} N^{2} r_{c}\left[\ln \left(8 r_{c} / r_{w}\right)-2\right] \quad r_{c}=1.5 \mathrm{~cm}=0.015$ meters $\quad r_{w}=.16 \mathrm{~mm}=0.00016$ meters
$L=(4 \pi)(50)^{2}(.015)[\ln (8(.015) /(.00016))-2] \times 10^{-7}=2.18 \times 10^{3} \times 10^{-7}$
$L=2.18 \times 10^{-4} H=0.218 \mathrm{mH}$

B-c) Calculate the theoretical inductance if your coil has a coin shape. Note this coil has an air core with a diameter of 4 cm . It has 80 turns and is built from 26 gauge wire. The diameter of 26 gauge wire is 0.40 mm and $\mu_{0}=4 \pi \times 10^{-7} \mathrm{H} / \mathrm{m}$. ( 3 points)
$L=\mu_{0} N^{2} r_{c}\left[\ln \left(8 r_{c} / r_{w}\right)-2\right] \quad r_{c}=2 c m=0.02$ meters $\quad r_{w}=.2 \mathrm{~mm}=0.0002$ meter $s$
$L=(4 \pi)(80)^{2}(.02)[\ln (8(.02) /(.0002))-2] \times 10^{-7}=7.35 \times 10^{3} \times 10^{-7}$
$L=7.35 \times 10^{-4} H=0.735 \mathrm{mH}$

A-d) If the measured value of your capacitor is $0.691 \mu \mathrm{~F}$ and the measured value of your resistor is 1004 ohms, estimate the value of the resonant frequency for this circuit in Hertz. (3 points)
$f_{0}=1 /(2 \pi \sqrt{ }(L C))=1 /\left(2 \pi \sqrt{ }\left(0.218 m^{*} .691 \mu\right)\right)=12.967 \mathrm{~K} \mathrm{~Hz} \approx 13 \mathrm{~K} \mathrm{~Hz}$

B-d) If the measured value of your capacitor is $0.677 \mu \mathrm{~F}$ and the measured value of your resistor is 987 ohms, estimate the value of the resonant frequency for this circuit in Hertz. (3 points)
$f_{0}=1 /(2 \pi \sqrt{ }(L C))=1 /\left(2 \pi \sqrt{ }\left(0.735 m^{*} .677 \mu\right)\right)=7.135 K \mathrm{~Hz} \approx 7 \mathrm{~K} \mathrm{~Hz}$
e) Based on your results from the previous parts of this question, sketch $|\mathrm{H}|$ of this circuit at point B from very low to very high frequencies. Clearly mark the resonant frequency in Hertz. (3 points)


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## Question 3 - Zener Diodes (18 points)

Below is a circuit built with a zener diode and a picture of the characteristic curve of the diode.


Here are the input and output, when a 10 Volt, 1 K Hertz signal is applied at the input.

a) Write the letter of the appropriate region on the zener diode characteristic curve next to each of the five descriptions below. (5 points)

| Forward Bias Region $B$ | Reverse Bias Region $A$ |
| :--- | :--- |
| Zener Region $E$ | Zener Voltage $\left(-\mathrm{V}_{\mathrm{Z}}\right) D$ |

Saturation Current $C$
b) Identify the following areas on the plot of the input and output. (5 points)

1. The input signal
2. The output signal
3. An area on the output signal where the diode is forward biased.
4. An area on the output signal where the saturation current runs through the diode.
5. An area on the output signal where the voltage across the diode is the zener voltage.
c) Using the output plot, estimate the zener voltage and the forward bias cutoff voltage of the diode. (4 points)

$$
\begin{aligned}
& \mathrm{V}_{\mathrm{Z}}=4.7 \mathrm{~V} \text { (Vout is } 4.7 \text { volts less than Vin) } \\
& \text { Von }=0.7 \text { (Vout is } 0.7 \text { volts more than Vin) }
\end{aligned}
$$

A-d) If $\mathrm{R}=5 \mathrm{~K}$, what is the current through the load resistor when the input voltage is 10V? (2 points)
$I=(-10+0.7) / 5 K=-1.86 m A$
$B-d)$ If $R=6 \mathrm{~K}$, what is the current through the load resistor when the input voltage is 10V? (2 points)
$I=(-10+0.7) / 6 K=-1.55 m A$

A-e) If $\mathrm{R}=5 \mathrm{~K}$, what is the current through the load resistor when the input voltage is +10 V ? (2 points)
$I=(10-4.7) / 5 K=1.06 m A$
$B-e)$ If $R=6 \mathrm{~K}$, what is the current through the load resistor when the input voltage is +10 V ? (2 points)
$I=(10-4.7) / 6 K=0.883 \mathrm{~mA}$
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## Question 4 -- (Op Amps) (25 points)

Below is a Capture schematic of two Op-amp circuits that you should recognize

## Circuit 1

Circuit 2


1. What is the function of circuit 1 ? (2 points)

Ideal Integrator (integrates the signal)
2. What is the function of the capacitor, C 3 , between the circuits? (2 points) DC bias blocker (removes the DC offset)
3. What is the function of circuit 2? (2 points) Inverting Op-Amp (inverts and amplifies the signal)
4. Find an expression for V3 in terms of Rf, Rin, and V2. (4 points)

$$
V 3=-(R f / R i n) V 2
$$

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The plot shown below is the input square wave applied to the schematic composed of circuits 1 and 2.

5. Mark on the plot which one is V2 and which one is V3. (4 points)

Note: When the input is -10 mV , the mathematical integration should slope down. However, the circuit integrates and inverts, so V2 is the smaller wave which slopes up. $V 3$ is the amplified version of V2. Circuit 2 is an inverting amplifier. The output should be larger and inverted.

A-6. Which value for Rf was used to produce this plot if Rin is 2 K ohms? (3 points)
$V 2$ has an amplitude of $-0.5 V$ and V3 has an amplitude of 2.3 volts
$2.3=(-R f / 2 K) /(-0.5) \quad R f=9.2 K$ ohms

B-6. Which value for Rf was used to produce this plot if Rin is 3 K ohms? (3 points)
$V 2$ has an amplitude of -0.5 V and $V 3$ has an amplitude of 2.3 volts
$2.3=(-R f / 3 K) /(-0.5) \quad R f=13.8 K$ ohms
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If the input to the complete system formed of the two circuit is a sine wave $v(t)=$ $A \sin ((\omega \mathrm{t})+\varphi)$, the simulation results for both the input and output will be as shown in the figure below.

7. What is the general expression for the input signal? Please give the numerical values for A and $\omega$ and $\varphi$ ? (3 points)

Input: $A=50 m V T=1 m s f=1 K \omega=2 \pi K$
$\operatorname{Vin}(t)=50 m V \sin (2 \pi K t)$
8. What is the general expression for the output signal? Please give the numerical values for $A$ and $\omega$ and $\varphi$ ( 3 points)

Output: $A=4 V \varphi=-2 \pi(1 / 4)=-\pi / 2 \quad T=1 m s f=1 K \quad \omega=2 \pi K$
$\operatorname{Vout}(t)=4 V \sin (2 \pi K t-\pi / 2)$
9. Explain the phase difference between Vin and Vout. (2 points)

The first circuit is an integrator and the input is a sine wave. If $\int \sin (t) d t=-\cos (t)$ then the output should have a phase shift of $-\pi / 2$. However, this circuit also inverts, so $-\pi / 2+\pi=+\pi / 2$. The second circuit inverts the signal again, so we get $+\pi / 2+\pi=-\pi / 2$. This is the $-\pi / 2$ phase shift we see in the output.
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## Question 5 -- Transformers (15 points)

Below is a circuit containing a transformer and an op-amp circuit you should recognize from the homework and experiment 8 .

A: Assume R1 $=50$ ohms, R2 $=5 \mathrm{~K}$ ohms, and $\mathrm{R} 3=200$ ohms.
B: Assume R1 $=50$ ohms, R2 $=3 \mathrm{~K}$ ohms, and R3 $=100$ ohms.

a) What is circuit 2 called and what does it do? (3 points)

Circuit 2 is a voltage follower (or buffer). It isolates the input from the output so the circuits on either side do not influence each other.

A-b) Given that the input inductance of the transformer, TX2, is 4 mH and the output inductance is 400 mH , determine the value of the constant "a" for the transformer. (2 points)
$a=\sqrt{ }(L 2 / L 1)=\sqrt{ }(400 m / 4 m)=10$

B-b) Given that the input inductance of the transformer, TX2, is 6 mH and the output inductance is 600 mH , determine the value of the constant "a" for the transformer. (2 points)
$a=\sqrt{ }(L 2 / L 1)=\sqrt{ }(600 m / 6 m)=10$

A-c) Given the amplitude of the voltage at point A is 50 mV and the transformer has perfect coupling, what is the amplitude of the voltage at point B ? (2 points)
$V 2 / V 1=V_{B} / 50 m=10 \quad V_{B}=500 \mathrm{mV}$

B-c) Given the amplitude of the voltage at point A is 40 mV and the transformer has perfect coupling, what is the amplitude of the voltage at point B ? (2 points)
$V 2 / V 1=V_{B} / 40 \mathrm{~m}=10 \quad V_{B}=400 \mathrm{mV}$

A-d) For the same signal as described in part c , what is the amplitude of the voltage at point C? (2 points)
$V_{C}=500 \mathrm{mV}$

B-d) For the same signal as described in part c , what is the amplitude of the voltage at point C? (2 points)
$V_{C}=400 \mathrm{mV}$

A-e) For the same signal in parts c and d, find the amplitude of the current through the load resistor, R3. (3 points)
$I=V / R=500 \mathrm{mV} / 200=2.5 \mathrm{~mA}$

B-e) For the same signal in parts c and d , find the amplitude of the current through the load resistor, R3. (3 points)
$I=V / R=400 \mathrm{mV} / 100=4 m A$

A-f) If we remove circuit 2 and connect the load resistor, R3, directly to point B (as shown below), would the new value of the voltage at point B be greater than, less than, or equal to the value you determined in part c? Why? (3 points)

The voltage at point $B$ would be less than the original circuit. The two resistors, $R 2$ and $R 3$ are now in parallel. The combined resistance is 192 ohms, $(200 * 5 \mathrm{~K}) /(200+5 \mathrm{~K})$. This is significantly smaller than the value of R2 alone, $5 K$. This would result in a much smaller voltage at point $B$.
$B-f$ ) If we remove circuit 2 and connect the load resistor, R 3 , directly to point B (as shown below), would the new value of the voltage at point $B$ be greater than, less than, or equal to the value you determined in part c? Why? (3 points)

The voltage at point $B$ would be less than the original circuit. The two resistors, $R 2$ and $R 3$ are now in parallel. The total resistance for test $B$ is 96.77 ohms, $(100 * 3 K) /(100+3 K)$. This is significantly smaller than the value of $R 2$ alone, 3 K . This would result in a much smaller voltage at point $B$.

Circuit 1


If we want to consider the transformer in the answer (this is not required), the argument is as follows:

Zin $=R 2 / a^{2}$. Therefore, if $R 2$ decreases, then Zin decreases. If Zin decreases, then $V_{A}$ will also decrease. ( $V_{A}$ is determined by a voltage divider between R1 and Zin. Less impedance at Zin, means less voltage at $A$.) You cannot argue that the voltage at point $B$ will not change because of the transformer. R2//R3 is much too close to the 50 ohm resistance of R1 to have no effect.

