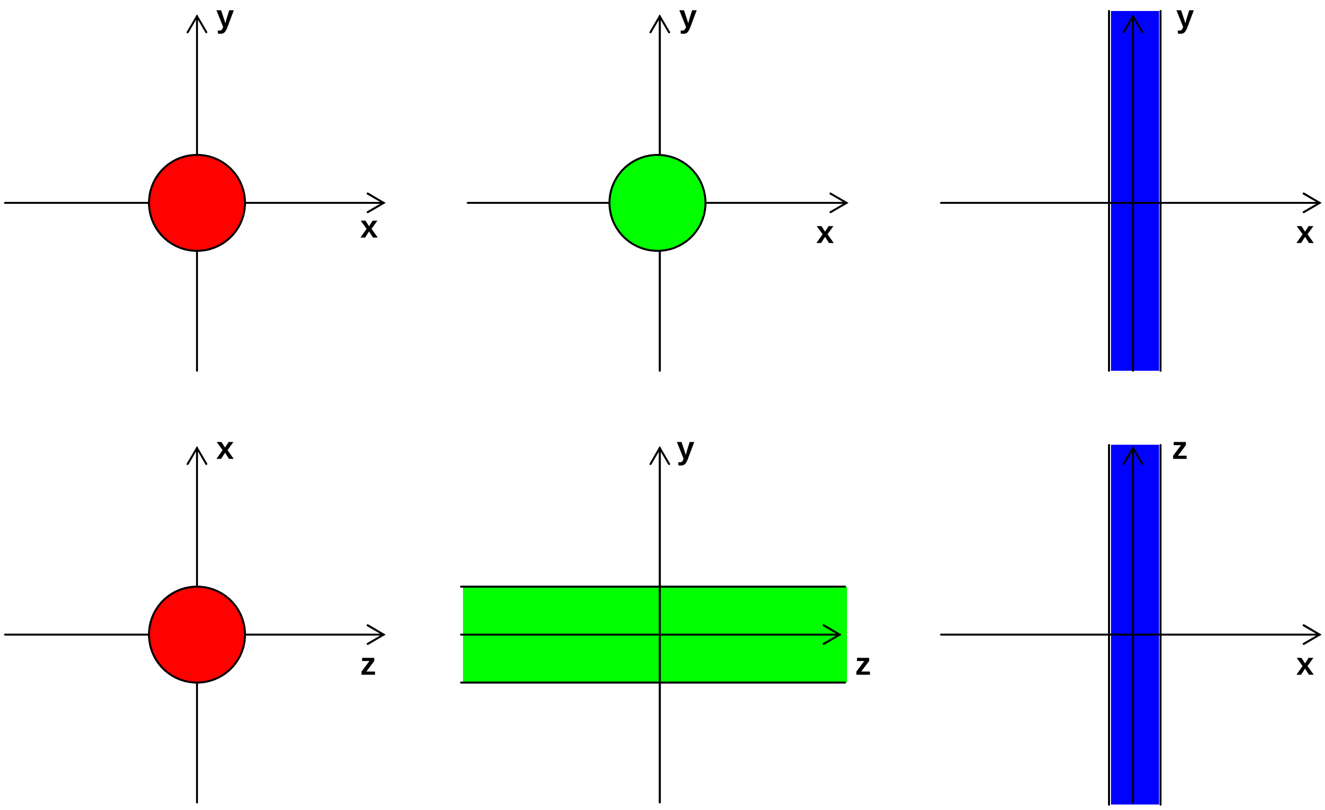
A Difficult Concept – The variation of electric and magnetic fields at large distances from sources.

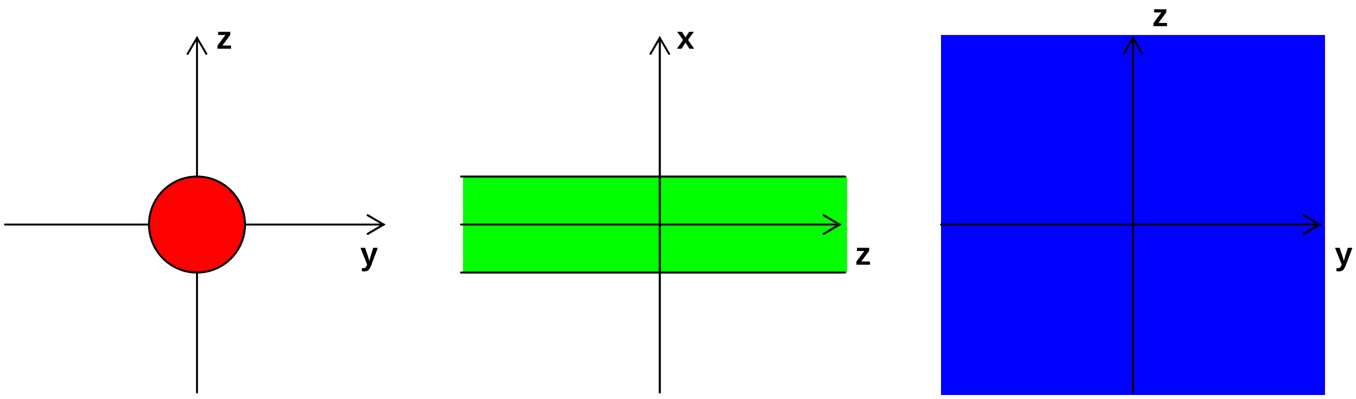
I think we are getting to a good part in the course since several students have begun to raise some excellent and subtle questions. One of these questions has to do with how fields vary well away from sources:

On the Fall 2002 exam, the Ampere's Law rectangular coordinates problem. Why is H not a function of distance when $|x| > a$*? The solution makes sense analytically but not intuitively to me.*

To answer this question in a relatively general context, let us consider what the three canonical geometries we address in this course look like.

1. Uniform spherical sources (generally used for electric fields but not very useful for magnetic fields, e.g. point charges, spherical volume charges, surface charges on a sphere).
2. Uniform cylindrical sources (e.g. line charges, long-straight current-carrying wires, coaxial cables).
3. Uniform planar sources (e.g. surface charges, parallel plate capacitors and inductors).

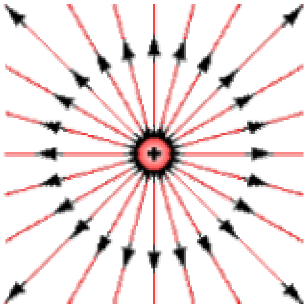


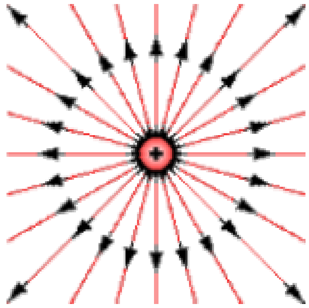


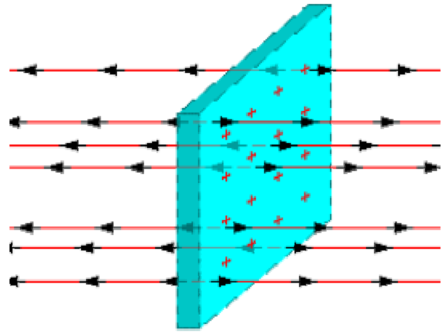
Shown above are the three 2D views of these sources. Now consider what each of these diagrams looks like as we move far away from the sources ($\rightarrow \infty$).

1. As we move far from the sphere, it gets smaller and smaller until it appears as a point and then finally becomes too small to see. Thus, as we move away from this source, we expect that its contribution to any field will also become smaller and smaller and eventually disappear. For electric fields, this results in field expressions that decay as $\frac{1}{r^2}$, eventually going to zero.
2. As we move far from the cylinder, it gets smaller and smaller until it appears as a line and then finally becomes too small to see. Thus, as we move away from this source, we expect that its contribution to any field will also become smaller and smaller and eventually disappear. For electric and magnetic fields, this results in field expressions that decay as $\frac{1}{r}$, eventually going to zero.
3. As we move far from the planar charge, it gets smaller and smaller in two views until it appears as a line and then finally becomes too small to see. However, in the third view, the charge always looks the same. If we could actually make an infinite featureless plane, we would never know how far above it we are, since it would always appear the same (as an infinite plane). Since the source always appears the same in this view, the field must always be the same (e.g. a constant). Thus, the solution is not a function of position.

There is another way to look at these three cases using the concept of flux to understand this. There is a somewhat outdated webpage (not modified in recent years) that has some good pictures involving electric fields.
http://www.slcc.edu/schools/hum_sci/physics/tutor/2220/e_fields/
I will extract a couple of these figures since they help to understand the dependencies we find mathematically. The electric fields due to the three types of charges look like the following.







Spherical Source　　Cylindrical Source　　Planar Source

Since electric fields must begin and end on charges and, since from Gauss' Law, $\oint \varepsilon \vec{E} \cdot d\vec{S} = Q_{encl}$, E field lines must diverge as they get further from a finite source (as they do in two dimensions for the spherical source and in one dimension for the cylindrical source). However since the source is infinite in two dimensions for the planar source, the field lines must be parallel to one another. Parallel field lines (as we have in a parallel plate capacitor) mean that the field magnitude is constant.

Finally, note that the area through which the flux must pass changes for the first two cases, but remains a constant for the planar case. The surface of a sphere is $4\pi r^2$ and the surface of a cylinder is $2\pi rl$ (where l is the length of the cylinder). Since the same flux must pass through these surfaces, no matter how large they are, spherical fields must decay as $\frac{1}{r^2}$ while cylindrical fields must decay as $\frac{1}{r}$ and planar fields must be constant.