Corrections & Comments for Old Quizzes and Homework Posted on Class Reserves

Quiz 2 – Spring 2003

Question 1, iv, D: How does the charge density ρ_s affect the direction of the electric field?
This is one of the answers that is incorrect. The author of the problem intended to have the charge density $\rho_s = -3 \, C/m^2$ but used the wrong sign. Note that the E field on the

lower side of the boundary has a normal component of $E_n = E_y = \dfrac{3}{\varepsilon_o} = \dfrac{D_y}{\varepsilon_o}$. Since D_y is

directed into the boundary and $D_{n1} - D_{n2} = \rho_s$, D_y can be made equal to zero on the other side of the boundary if $D_y = \rho_s$. Where the author went wrong was forgetting that electric field lines must end on negative charges, not positive.

Question 3: Could you explain where E_r for a<r<b. why doesn't the answer have ε_r since all we do to figure out the E field is divide by the electric permittivity of the area we have the Gaussian surface in. is it just a mistake?

Actually, there is a mistake in this problem, but it is in the region r<a, since that is where the dielectric material is. For the electric field in this region, the answer should be

$E_r = \dfrac{\rho_o r^2}{4 \varepsilon_r \varepsilon_o}$. There should also be an ε_r in the denominator of the stored energy (the last

part of this question).

Question 4: How do you go from the second to the third line in this solution?

The Laplacian is set equal to zero in cylindrical coordinates. That is,
$\nabla^2 V = \dfrac{1}{r} \dfrac{\partial}{\partial r} \left(r \dfrac{\partial V}{\partial r} \right) = 0$. Multiply the left side by r and we are left with an expression in

the parentheses whose derivative is equal to zero. Thus $\left(r \dfrac{\partial V}{\partial r} \right) = const = C_1$ since an

expression whose derivative is zero must be equal to a constant. Then the final solution is obtained by integrating this equation. The simplest way to integrate this is to divide both

sides of this expression by r so that $\dfrac{\partial V}{\partial r} = \dfrac{C_1}{r}$ and then $V(r) = C_1 \ln r + C_2$ and the

constants are determined from the BC.

Test 1 – Fall 2001

Question 2: Shouldn't the answer have an ε_o in the denominator for C? Yes, this is an error in the solution.

Question 4: Shouldn't the primary equation be Poisson's instead of Laplace's Equation? Yes, this entire answer is wrong since the right hand side of the equation is missing. With the charge density included, the solution is easier to obtain using Gauss' Law. However, it is not that much more difficult using Poisson's equation. Using the same method as above:

$$\nabla^2 V = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = -\frac{\rho_o r}{\varepsilon_o} \qquad \Longrightarrow \qquad \frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = -\frac{\rho_o r^2}{\varepsilon_o}$$

$$r\frac{\partial V}{\partial r} = -\frac{\rho_o r^3}{\varepsilon_o 3} + C_1$$

$$\frac{\partial V}{\partial r} = -\frac{\rho_o r^2}{\varepsilon_o 3} + \frac{C_1}{r}$$

$$V(r) = -\frac{\rho_o r^3}{\varepsilon_o 9} + C_1 \ln r + C_2$$

Then the BC are used to get the constants. This is where things get a bit messy but not really hard.

$$V(a) = -\frac{\rho_o a^3}{\varepsilon_o 9} + C_1 \ln a + C_2 = 0 \text{ and } V(b) = -\frac{\rho_o b^3}{\varepsilon_o 9} + C_1 \ln b + C_2 = V_o$$

Subtracting the first equation from the second, we get

$$\frac{\rho_o a^3}{\varepsilon_o 9} - \frac{\rho_o b^3}{\varepsilon_o 9} + C_1 \ln b - C_1 \ln a = V_o \text{ or } C_1 = \frac{V_o + \frac{\rho_o}{9\varepsilon_o}\left(b^3 - a^3\right)}{\ln\left(\frac{b}{a}\right)} \text{ which can be plugged}$$

back into either equation to obtain $C_2 = \frac{\rho_o a^3}{9\varepsilon_o} - \left(\ln a\right)C_1$

Quiz 3 – Spring 2003

Question 2, Part v: The solution is presented as reluctances in series, but this problem clearly calls for reluctances in parallel. The circuit diagram should look like the one in problem 3 of homework 6. In that case the three reluctances in series with the source could be combined to produce the same simpler diagram shown below.
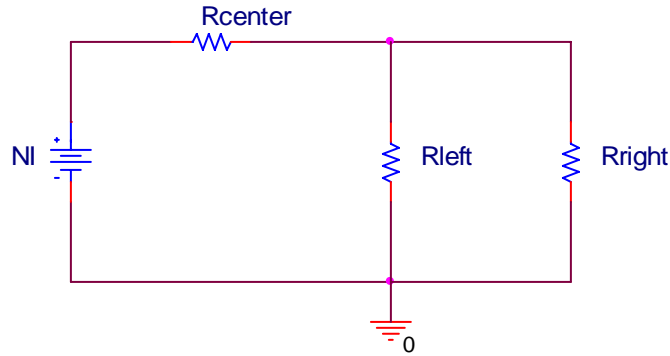
HW 8 – Fall 2002

Question 6 on Electrostatics: In part b, since there is no charge for $r<a$, the limits of integration should be a and r for $a<r<b$ and a and b for $b<r$. Unfortunately, when the solution was written up, the person who did it checked it only with the divergence equation, which works, since the error has no divergence. The charge enclosed in the two regions should be $Q_{encl} = 2\pi l \int_a^r \frac{a\rho_o}{r} r\, dr = 2\pi l a\rho_o(r-a)$ and

$$Q_{encl} = 2\pi l \int_a^b \frac{a\rho_o}{r} r\, dr = 2\pi l a\rho_o(b-a)$$ for the two regions.

HW 8 – Spring 2003

The Ampere's Law question is stated inconsistently. The currents in the two coils are equal, not equal and opposite for the solution provided. If they were equal and opposite, the field inside the inner coil would be zero.

The Gauss' Law problem has an error in the determination of the energy stored. The ε_r should be $\varepsilon = \varepsilon_r \varepsilon_o$. In many physics books the ε_o is suppressed since they like to use Gaussian units. However, it should always be included here. In addition, the solution has the $\varepsilon = \varepsilon_r \varepsilon_o$ in the numerator and it should be in the denominator.

Quiz 3 – Spring 2003

In Problem 3 on Ampere's Law, the last two expressions on page 12 are incorrect. The total current should be multiplied by 2 and the inductance per unit length should be divided by 2. They should read $I = 2\pi \frac{J_o a^8}{8}$ and $L/m = \frac{\psi_m/m}{I} = \frac{\mu}{2\pi} \ln\frac{b}{a}$, respectively.

This expression for the inductance is the same as found on page 41 of Ulaby.

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