Extra Credit – Spring 2006

Up to 20 extra credit points can be earned for Fields and Waves I. The first option is to do the extra credit project (information on the project page). The second option is answering some quiz questions listed on WebCT. This document has some figures, etc. that are used in the questions. The geometries for these problems are found in the appendix posted on WebCT that describes the basic configurations for capacitors, inductors and transmission lines.

1. A spherical capacitor whose inner radius is 10cm and outer radius is 10m is insulated with air. Assuming that a voltage of 1000V is placed across the capacitor, what is the charge density on the surface of the inner sphere?

$C = \frac{4\pi\varepsilon}{\left(\frac{1}{a} - \frac{1}{b}\right)} = \frac{4\pi\varepsilon_o}{\left(\frac{1}{0.1} - \frac{1}{10}\right)} = 11.2\, pF$ *. Some wrong answers 1.25pF, 1.11nF*

The charge on either of the plates is then

$Q = CV = 11.2nCoulombs = 11.2x10^{-9}\, Coulombs$ *. Some wrong answers 1.25nCoulombs and 1.11microCoulombs.*

*The surface areas of the two spheres is 0.1257*m^2 *and 1257*m^2

The correct answer is, thus, 88.2 nCoulombs per square meter. Some wrong answers are 8.82 mCoulombs per square meter, 8.82 nCoulombs per square meter, 88.2 microCoulombs per square meter, 88.2 mCoulombs per square meter.

2. A standard RG/58 cable is built with a new experimental insulator that has a dielectric constant 9 times that of solid polyethylene. What are the characteristic impedance and propagation velocity for this modified cable?

$\frac{C}{l} = \frac{2\pi\varepsilon}{\ln\frac{b}{a}}$ *so that increasing the dielectric constant by a factor of 9 will increase the capacitance per unit length by 9. This will reduce both the characteristic impedance and velocity by a factor of 3 since* $Z_o = \sqrt{\frac{\mu}{\varepsilon}}$ *and* $u = \sqrt{\frac{1}{\mu\varepsilon}}$ *. For the standard RG/58 cable, the impedance is 50 Ohms and the velocity is 2/3 the speed of light. Thus the answer is* $Z_o = \sqrt{\frac{\mu}{\varepsilon}} = \frac{50}{3} = 16.7\Omega$ *and* $u = \sqrt{\frac{1}{\mu\varepsilon}} = \frac{2x10^8}{3} = 6.7x10^7\, m/s$

3. A lossless transmission line is being used in a matched configuration (the source, line and load impedances are all 50 Ohms). The load impedance is replaced by a Tee with two 50 Ohm loads connected to it. How much power will be reflected from this load?

$\Gamma = \frac{25-50}{25+50} = -0.333$ *. The power reflection coefficient for lossless lines and resistive loads is the square of this value. Thus* $\Gamma_{power} = (-0.333)^2 = 0.1111$ *or about 11%.*

4. The inductance of a particular square cross-section torus is found to be 10H. If it is tightly wound with 5000 turns of wire and the dimensions of the core are width

and height equal to 2cm and inner radius equal to 5cm, what is the relative permeability μ_r of the core material?

$$L = w\frac{\mu N^2}{2\pi}\ln\frac{a+w}{a} = (0.02)\frac{\mu(5000)^2}{2\pi}\ln\frac{0.05+0.02}{0.05} = 10H$$

Solving for the permeability $\mu = \mu_r\mu_o = \dfrac{10}{(0.02)\dfrac{(5000)^2}{2\pi}\ln\dfrac{0.05+0.02}{0.05}} = 3.735x10^{-4}$

$\mu = \mu_r\mu_o \approx 300\mu_o$

5. A parallel plate inductor consists of two conducting plates (*length = 10m*, *width = 10cm, thickness = 2mm*) separated by a distance of *1cm*. If the plates carry a current of *1000A*, what is the total force they exert on one another?

The inductance per unit length of this structure is given by $L' = \dfrac{\mu_o d}{w} = \dfrac{\mu_o(0.01)}{0.1} = \dfrac{\mu_o}{10}$

Thus, the inductance of the entire line is $L = (10)L' = \dfrac{\mu_o}{10}10 = \mu_o$. *In terms of the separation, the inductance is* $L = \mu_o\dfrac{d}{0.01}$. *The energy carried by the structure is* $W_m = \dfrac{1}{2}LI^2 = \dfrac{1}{2}\mu_o\dfrac{d}{0.01}(1000)^2 = \dfrac{10^8}{2}\mu_o d$. *The derivative of this with respect to d is the force. Thus,* $Force = \dfrac{10^8}{2}\mu_o = \dfrac{10^8}{2}\left(4\pi 10^{-7}\right) = 20\pi$. *A second way of solving this problem is to find the field due to one of the currents and then the force experienced by the other. In this case, the field is* $H = \dfrac{1}{2}\dfrac{I}{Width} = \dfrac{1000}{0.2} = 5000$. *Then*

$B = \mu_o H = 5000\mu_o$. *The force on a current is*

$F = ILB = (1000)(10)5000(4\pi 10^{-7}) = 20\pi$

6. An air insulated parallel plate structure consists of a grounded conducting plate at *x=0* and a conducting plate at *x=d* where the second plate is connected to a voltage V_o. The region between the plates has a uniform charge density ρ_o. What is the voltage as a function of position *V(x)* in the region between the plates?

This requires Poisson's Equation rather than Laplace's equation. That is

$\nabla^2 V(x) = \dfrac{d^2V(x)}{dx^2} = -\dfrac{\rho_o}{\varepsilon_o}$ *Integrating this two times gives the voltage as a function of position as* $V(x) = -\dfrac{\rho_o}{\varepsilon_o}\dfrac{x^2}{2} + C_1 x + C_2$. *Since V(0)=0,* $C_2=0$. *Then*

$V(d) = -\dfrac{\rho_o}{\varepsilon_o}\dfrac{d^2}{2} + C_1 d = V_o$ *gives us* $C_1 = \dfrac{\rho_o}{\varepsilon_o}\dfrac{d}{2} + \dfrac{V_o}{d}$ *or*

$$V(x) = -\frac{\rho_o}{\varepsilon_o}\frac{x^2}{2} + \left(\frac{\rho_o}{\varepsilon_o}\frac{d}{2} + \frac{V_o}{d}\right)x$$

7. Sunlight is incident obliquely on a glass ($\varepsilon_r = 5$) surface at an angle of 45 degrees. Since sunlight is randomly polarized, it has an equal amount of power in each polarization. What is the ratio of the perpendicularly polarized electric field $E_\perp$ to the parallel polarized electric field $E_\parallel$?

It is only necessary to find the ratio of the two reflection coefficients, using

$\eta_1 = \eta_o = 120\pi$, $\eta_1 = \frac{\eta_o}{\sqrt{5}} = \frac{120\pi}{\sqrt{5}} = 53.6656\pi$, $\cos 45 = .707$, *Snell's Law*

$n_1 \sin\theta_1 = n_2 \sin\theta_2$ *gives us* $\sin 45 = \sqrt{5}\sin\theta_2$ *and* $\theta_2 = \sin^{-1}\frac{.707}{\sqrt{5}} = 18.4321$ *so that*

$\cos 18.4321 = 0.9487$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_1 - \eta_1\cos\theta_2}{\eta_2\cos\theta_1 + \eta_1\cos\theta_2} = -0.5$$

$$\Gamma_\parallel = \frac{\eta_2\cos\theta_2 - \eta_1\cos\theta_1}{\eta_2\cos\theta_2 + \eta_1\cos\theta_1} = -0.25$$

so the ratio equals 2. Check result with some online applet. For example, look at the Fresnel Laws applet at http://www.ub.es/javaoptics/index-en.html

8. A conducting ($\sigma = 1x10^6$) pipe (length *d=10m*), is rolling down an incline at a speed of *3m/s*. If the earth's magnetic field is approximately *1 Gauss = 10^{-4} Tesla*, what voltage is induced between the ends of the pipe?

The induced electric field is vB, which is integrated along the length to get the voltages or $V = vBd = (3)(10^{-4})(10) = 3mV$

9. A sinusoidal voltage source (V_s*=100V, f=1MHz,* R_s*=50 Ohms*) is connected to a *100 Ohm* load through a lossless *75 Ohm* transmission line. The transmission line is insulated with Polyethylene. If the power delivered to the load is *24.9 Watts*, what is the input impedance of the line if it is known to be real?

$$P_{in} = \frac{1}{2}\mathrm{Re}\left\{\frac{V_{in}^{\ 2}}{Z_{in}}\right\} \text{ and } V_{in} = \frac{Z_{in}V_o}{Z_{in}+R_s} = \frac{Z_{in}100}{Z_{in}+50} \text{ so } P_{in} = \frac{1}{2}\mathrm{Re}\left\{\frac{\left(\frac{Z_{in}100}{Z_{in}+50}\right)^2}{Z_{in}}\right\} = 24.9$$

There are only two possible real input impedances $Z_{in} = Z_L$ *for line lengths that are multiples of a half wavelength and* $Z_{in} = \frac{Z_o^{\ 2}}{Z_L}$ *for line lengths that are an odd multiple of a quarter wavelength. Thus, the answer must be either 100 Ohms or 56.25*

Ohms. For the former $P_{in} = \frac{1}{2}\mathrm{Re}\left\{\frac{\left(\frac{(100)(100)}{100+50}\right)^2}{100}\right\} = 22.22 Watts$ *while for the latter*

$$P_{in} = \frac{1}{2}\mathrm{Re}\left\{\frac{\left(\frac{(56.25)(100)}{56.25+50}\right)^2}{56.25}\right\} = 24.9 Watts \ \ Thus, \ Z_{in} = \frac{Z_o^{\ 2}}{Z_L} = 56.25\Omega$$

10. If the conducting pipe of question 8 is hollow (*inner radius = 3cm, outer radius equal 3.5cm*), what is its resistance?

The resistance is $R = \frac{d}{\sigma A} = \frac{d}{\sigma\pi\left(r_o^{\ 2} - r_i^{\ 2}\right)} = \frac{10}{(10^6)\pi\left(.035^2 - .03^2\right)} = 9.8 m\Omega$ *Some wrong answers 26 and 35mOhms.*