Fields and Waves I

Lecture 1

Introduction to Fields and Waves

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- Jeffrey Braunstein – Chung-Ang University, Seoul, Korea

Materials from other sources are referenced where they are used.
Those listed as Ulaby are figures from Ulaby's textbook.

Overview

- Why study E&M?
- Introduction to transmission lines

Why Study E&M?

- Some good sources
  - http://www.ece.northwestern.edu/ecefaculty/taflove/WhyStudy.pdf Some info from this document follows. (Taflove)
  - Others?

Why take Fields and Waves?

- E and B are fundamental to Electrical Engineering
- if you have "**V**", there is an "**E**"
- if you have "**I**", there is a "**B**"

V is Voltage → E is Electric Field Intensity
I is Current → B is Magnetic Flux Density

Relationship with Circuit Theory

- Circuit Theory uses simplified (lumped) Models of components

The model, however, does not include:

- Details on how the components work
  - Components are not made up of C,L,R
- Distributed Properties for example Transmission Lines
- Electromagnetic Waves - like μWaves, Radio Waves, Optics
- Applications such as Capacitive Sensors
- Noise

Relationship with Circuit Theory

Many of these effects are more important at High Frequency

Need to be considered when designing for High Speed Applications

HOWEVER

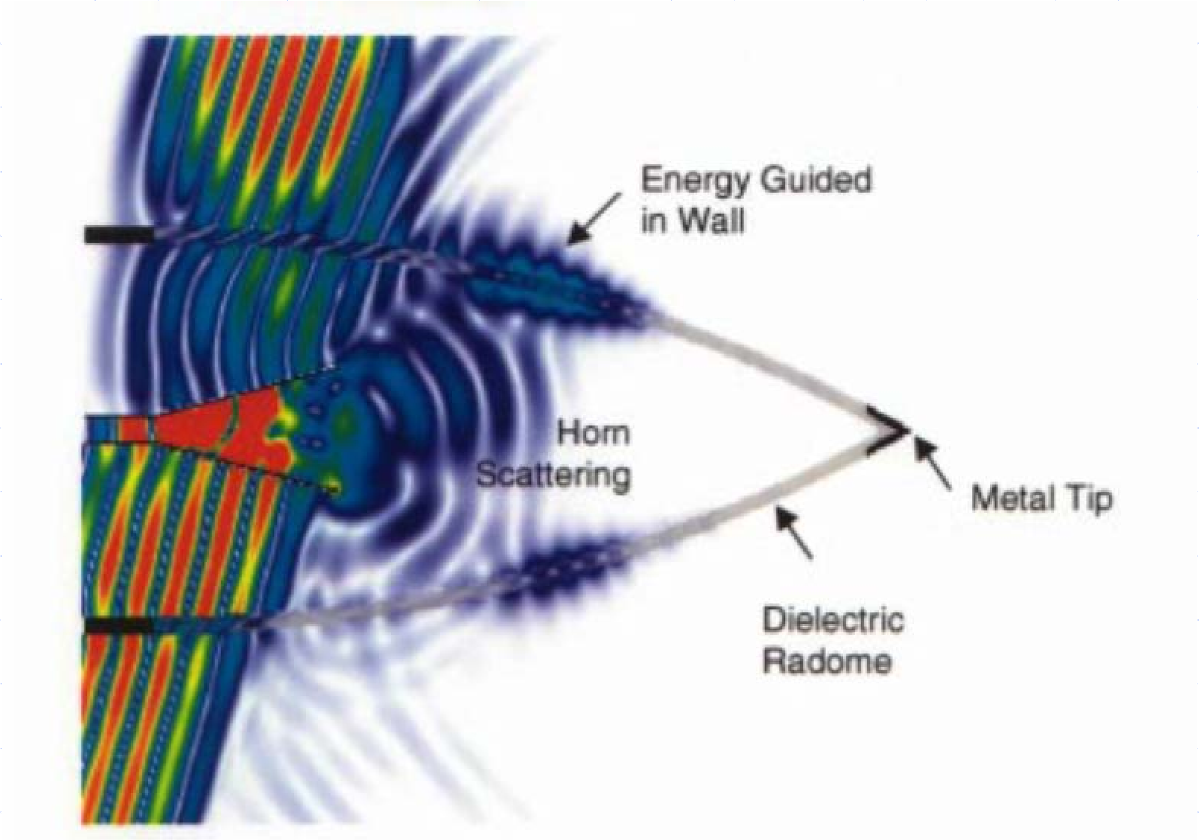
Current technology (with High Speed Computers) enables accurate circuit simulation

Simulation Packages include:
a) SPICE (from UC Berkeley)
b) SABER (systems approach)

- Accurate simulation requires understanding of " components" and interactions
- Interactions also need to be described by Models
- Models are obtained by an understanding of EM Fields

Question: Why do companies spend resources on developing models?

Why Study E&M?



(Taflove)

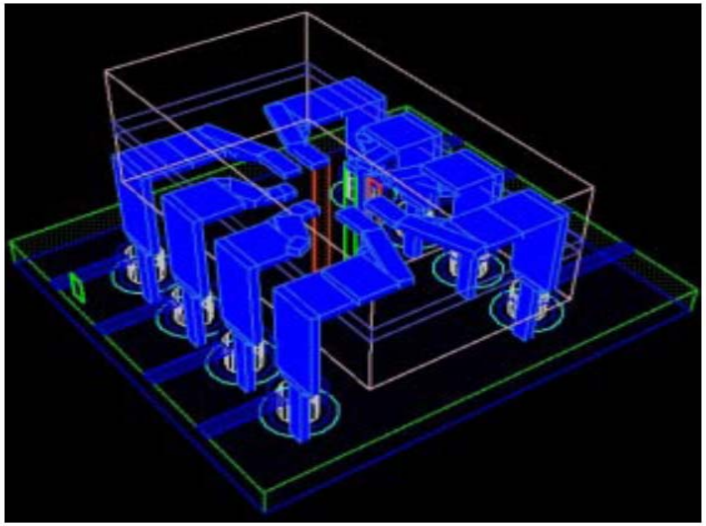
Microwave energy scattering from missile antenna radome.

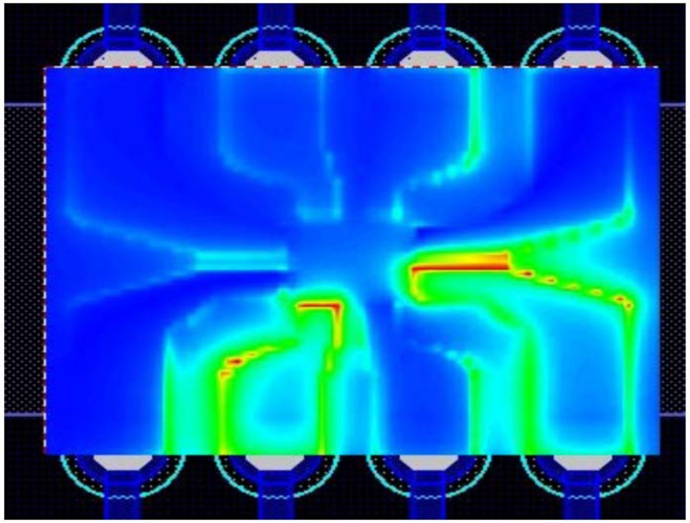
Why Study E&M?

- The bedrock of introductory circuit analysis, Kirchoff's current and voltage laws, fail in most high-speed circuits. These must be analyzed using E&M theory. Signal power flows are not confined to the intended metal wires or circuit paths.
  - Microwave circuits typically process bandpass signals at frequencies above 3 GHz. Common circuit features include microstrip transmission lines, directional couplers, circulators, filters, matching networks, and individual transistors. Circuit operation is fundamentally based upon electromagnetic wave phenomena.
  - Digital circuits typically process low-pass pulses having clock rates below 2 GHz. Typical circuits include densely packed, multiple planes of metal traces providing flow paths for the signals, dc power feeds, and ground returns. Via pins provide electrical connections between the planes. Circuit operation is nominally not based upon electromagnetic wave effects.
- The distinction between the design of these two classes is blurring.

(Taflove)

Why Study E&M?

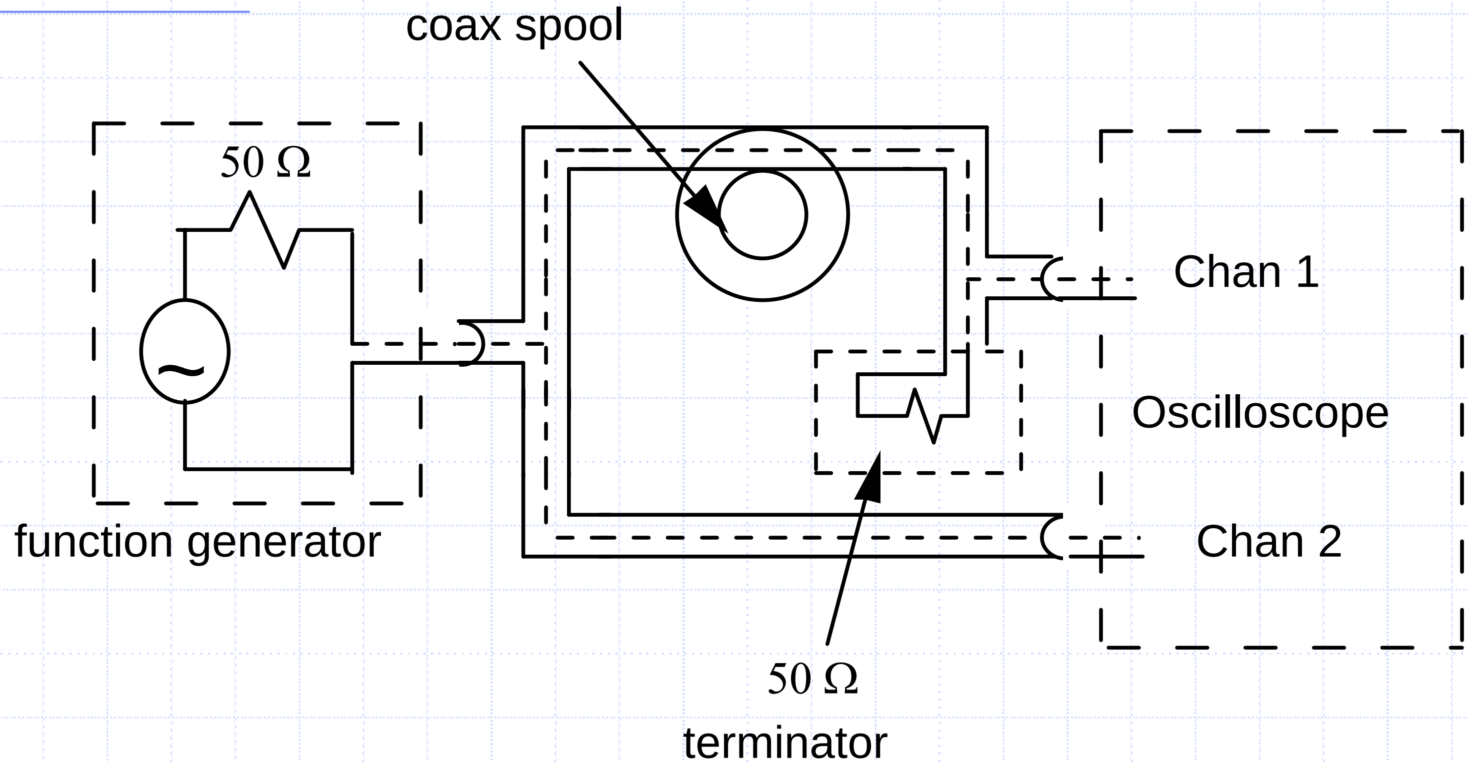




- False-color visualization (right) illustrating the coupling and crosstalk of a high-speed logic pulse entering and leaving a microchip embedded within a conventional dual in-line integrated-circuit package (left). The fields associated with the logic pulse are not confined to the metal circuit paths and, in fact, smear out and couple to all adjacent circuit paths.

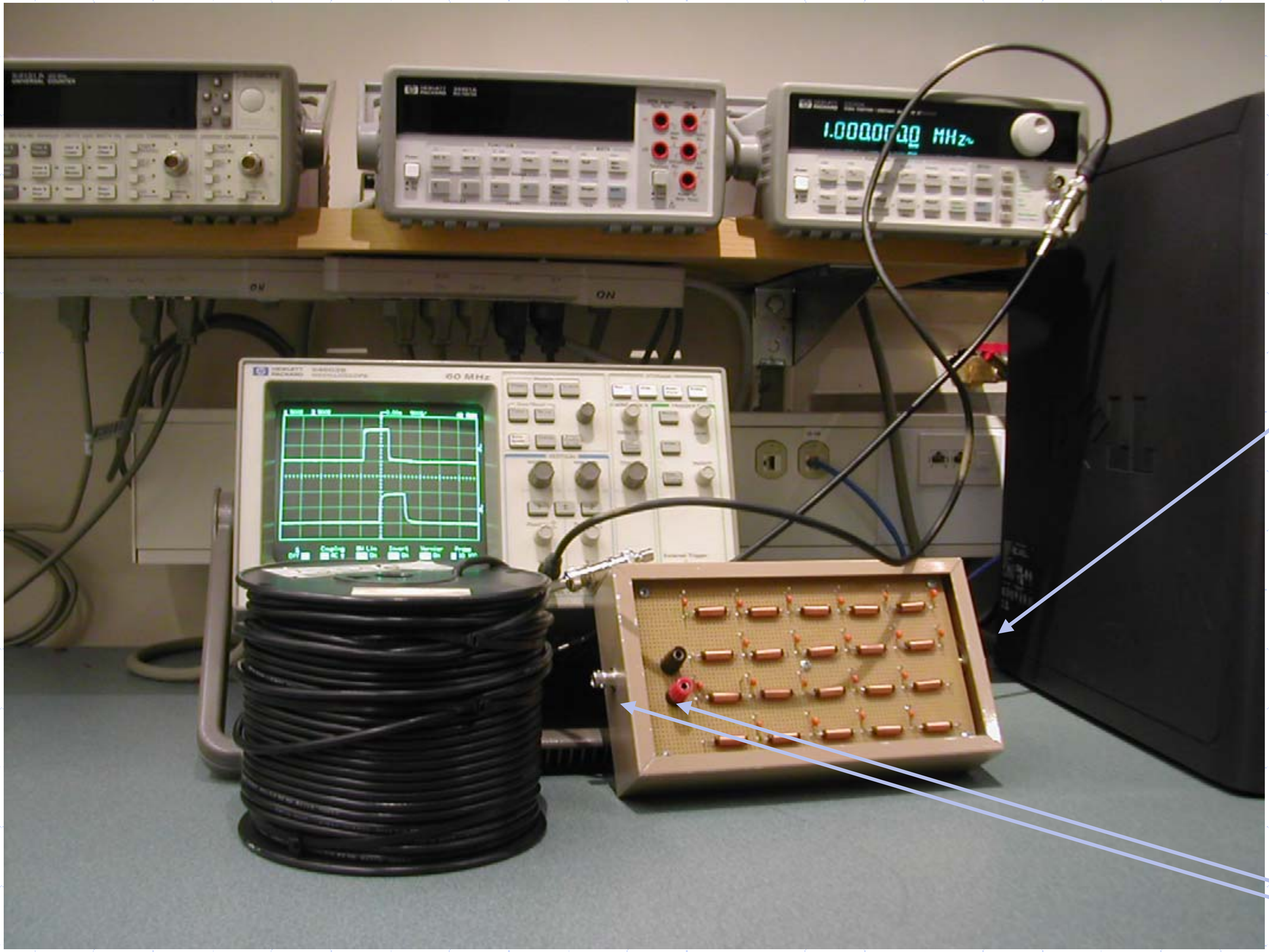
(Taflove)

Exp: Transmission Lines (You will work on this in the future)



The same signal passes through the short cable to channel 2 and the long cable (60-100 meters) to channel 1.

Lumped Transmission Line



Input Not Shown

Both Outputs Shown

Exp 5: Transmission Lines

- What is observed?
  - Input and output look largely the same
  - Phase shift between input and output
  - Output signal is somewhat smaller than input on the long cable
  - When the terminating resistor is removed, the signal changes
  - The wires have finite resistance ($\sim 50\, {}^{m\Omega}\!/_{meter}$)
- What can we conclude from this?
  - Wire resistance is low so, for shorter cables, we can consider transmission lines to be lossless
  - What else?

Workspace: General Mathematical Form of Voltage & Current Waves

Transmission Lines

- Connect to circuit theory
- Demonstrate the need to understand R, L, C, & G per unit length parameters
- Very useful devices (all EE, CSE, EPE students will use them someday)
- Can be easily analyzed to find electric and magnetic fields

Transmission Line

Fundamental Purpose of TL ⇒ Transfer signal/power from A to B

EXAMPLES:

- Power Lines (60Hz)
- Coaxial Cables
- Twisted Pairs
- Interconnects (approximates a parallel plate capacitor)

- All have two conductors

Transmission Line Effects

RELEVANT EFFECTS:

- Time Delays
- Reflections/Impedance Matching

TL effects more important at high ***f*** (or short ***t***) and long lengths

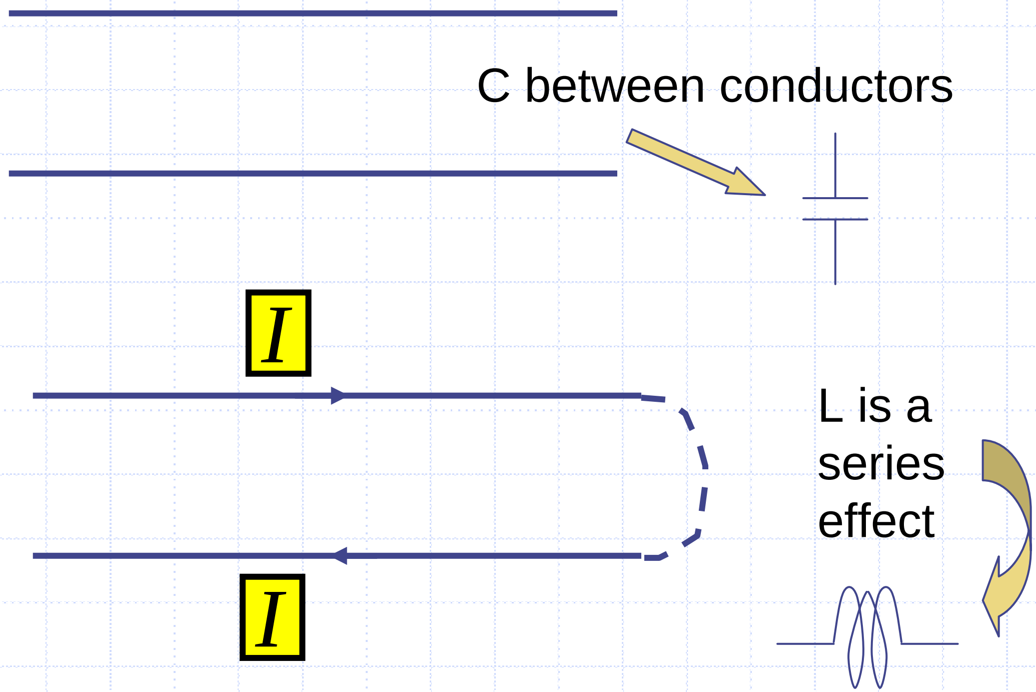
$\vec{E}$ and $\vec{H}$ effects are important for understanding

But, calculations use ***V*** and ***I*** for predicting effects

Transmission Line Model

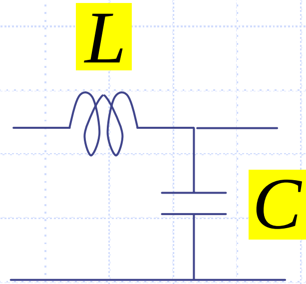
Cables have both ***L*** and ***C***:

2 wire example:



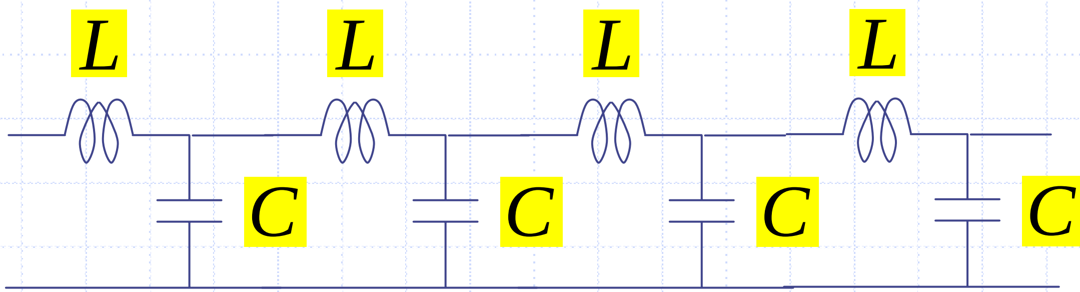
Transmission Line Model

Model of SHORT SECTION:



L and ***C*** are distributed through the length of the cable

Model the full length as:



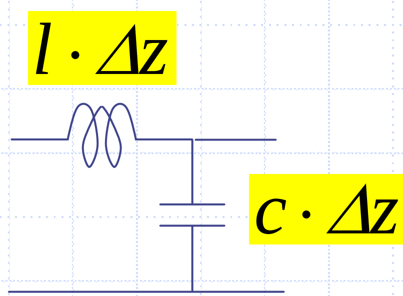
….etc.

Transmission Line Model

Does ***L*** -***C*** combination behave like a cable?
How would you know?

Time Delay ~ same as cable delay

Each,



, represents a length of cable

l = inductance/length

c = capacitance/length

Transmission Line Model

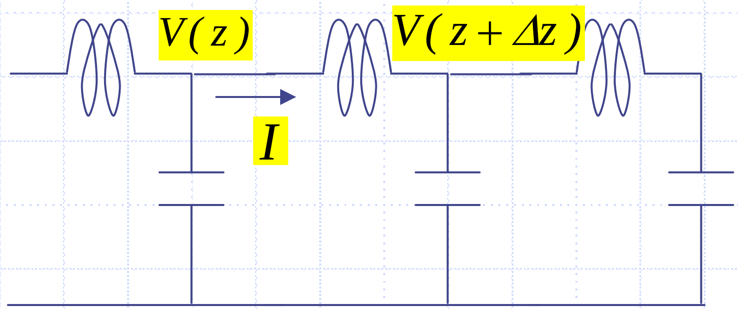
When is model of ***L*** - ***C*** combination valid?

- need Δz small ⇨ $\Delta z << \lambda$

What is λ ?

Transmission Line Representation

As $\Delta z \Rightarrow 0$ limit



$$V(z+\Delta z) - V(z) = -L\frac{\partial I}{\partial t} \quad \Rightarrow \quad -l \cdot \frac{\partial I}{\partial t} = \frac{\Delta V}{\Delta z} = \frac{\partial V}{\partial z}$$

$L \rightarrow l \cdot \Delta z$

Transmission Line Representation

Similarly, $$\frac{\partial I}{\partial z} = -c \cdot \frac{\partial V}{\partial t}$$

$$\frac{\partial^2 V}{\partial z^2} = \frac{\partial}{\partial z}(-l \cdot \frac{\partial I}{\partial t}) = -l \cdot \frac{\partial}{\partial t}(\frac{\partial I}{\partial z}) = lc\frac{\partial^2 V}{\partial t^2}$$

Obtain the following PDE: $$\frac{\partial^2 V}{\partial z^2} = lc\frac{\partial^2 V}{\partial t^2}$$

Solutions are: $f(t \pm \frac{z}{u})$

These are functions that move with velocity ***u***

Workspace – look at the general form of the solution

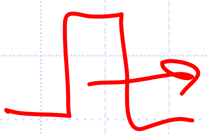
$$\frac{\partial^2 f}{\partial z^2} = lc \frac{\partial^2 f}{\partial t^2}$$

$$f\left(t \pm \frac{z}{u}\right)$$

$$f\left(t - \frac{z}{u}\right)$$

$$\frac{\partial f}{\partial z} = f'\left(-\frac{1}{u}\right)$$

$$\frac{\partial f}{\partial t} = f'$$



$$\frac{\partial^2 f}{\partial z^2} = f''\left(\frac{1}{u^2}\right)$$

$$\frac{\partial^2 f}{\partial t^2} = f''$$

$$\frac{1}{u^2} = lc \quad \text{or} \quad u = \frac{1}{\sqrt{lc}}$$

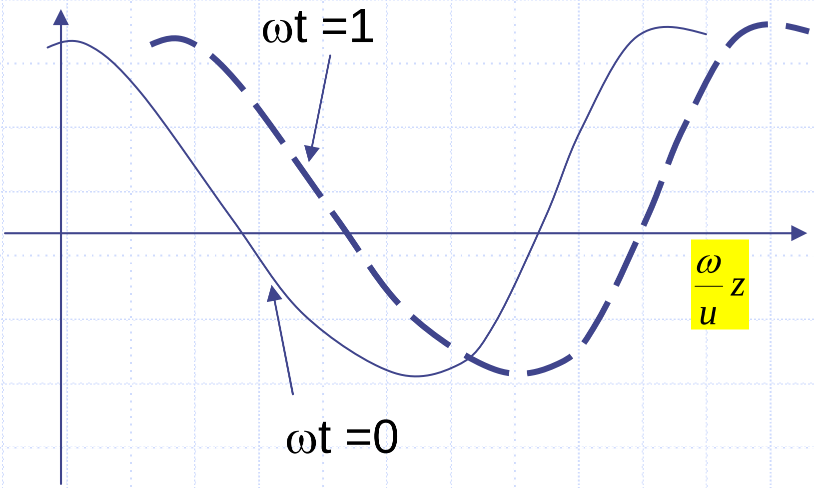
Transmission Line Representation

Functions that move with velocity ***u***

Example: $\cos(\omega t \pm \frac{\omega}{u} z)$

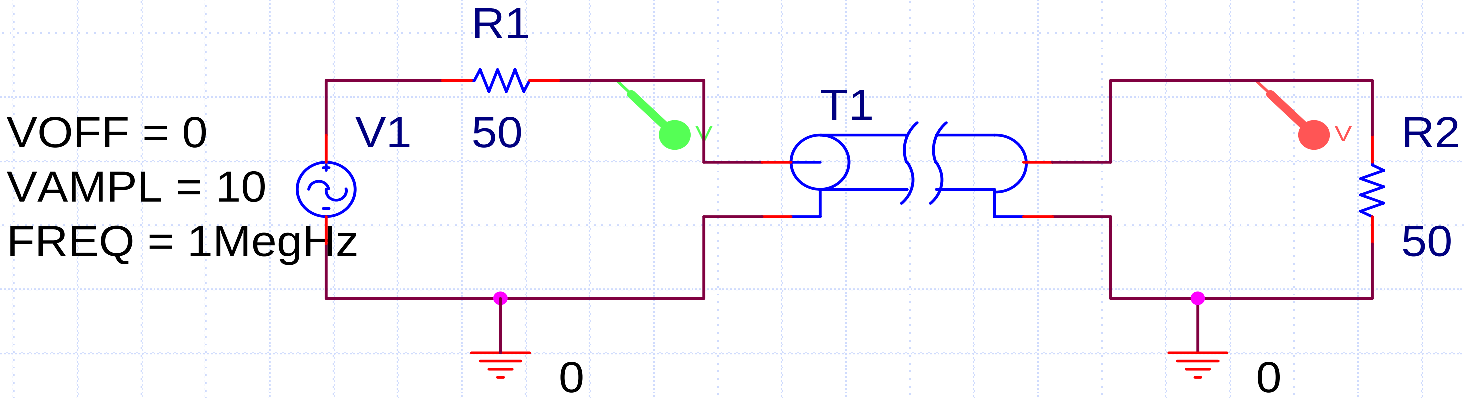
At t=0, $\cos(-\frac{\omega}{u} z)$ Wave moving to the right

At ωt =1 $\cos(1 - \frac{\omega}{u} z)$

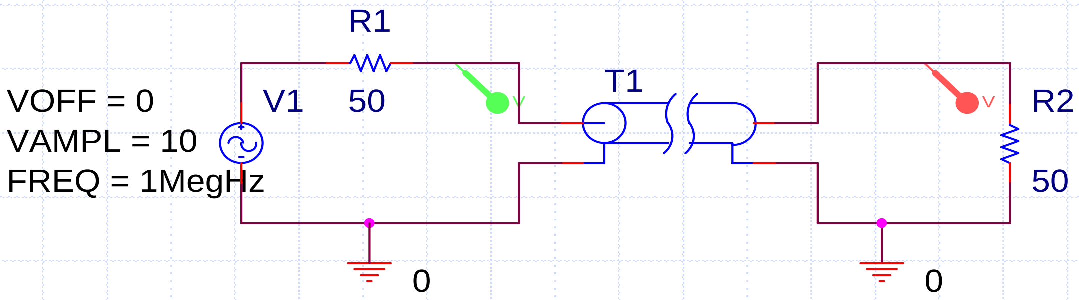


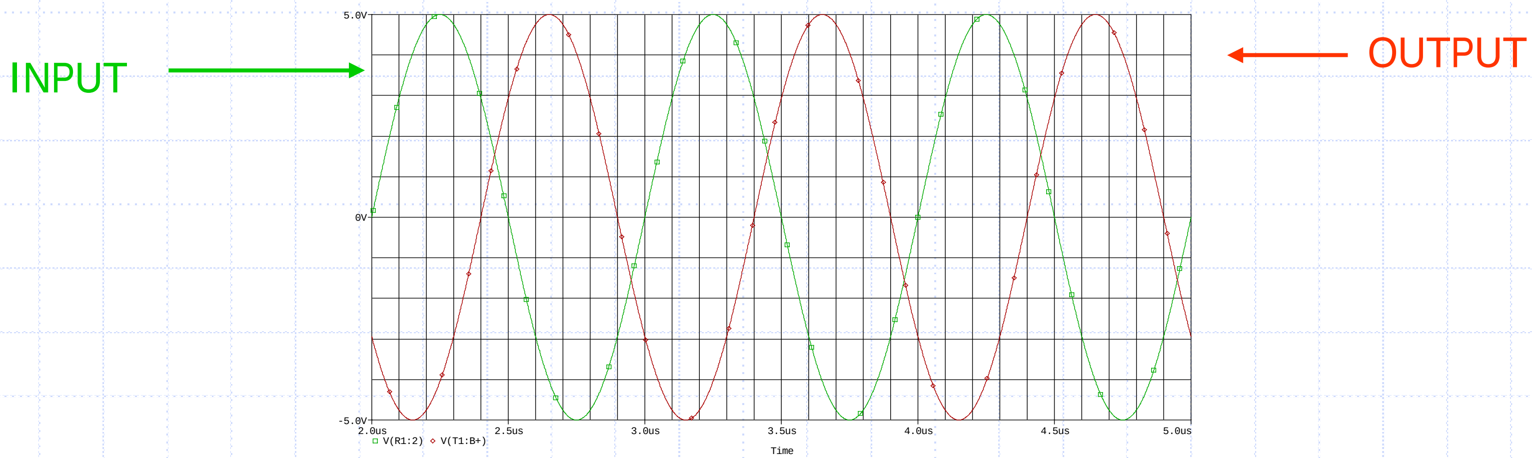
Using PSpice

- We can use PSpice to do numerical experiments that demonstrate how transmission lines work



PSpice


R1
VOFF = 0
VAMPL = 10
FREQ = 1MegHz
V1
50
T1
R2
50
0
0


INPUT
OUTPUT
5.0V
0V
-5.0V
2.0us
2.5us
3.0us
3.5us
4.0us
4.5us
5.0us
V(R1:2)
V(T1:B+)
Time

Sine Waves

- The form of the wave solution

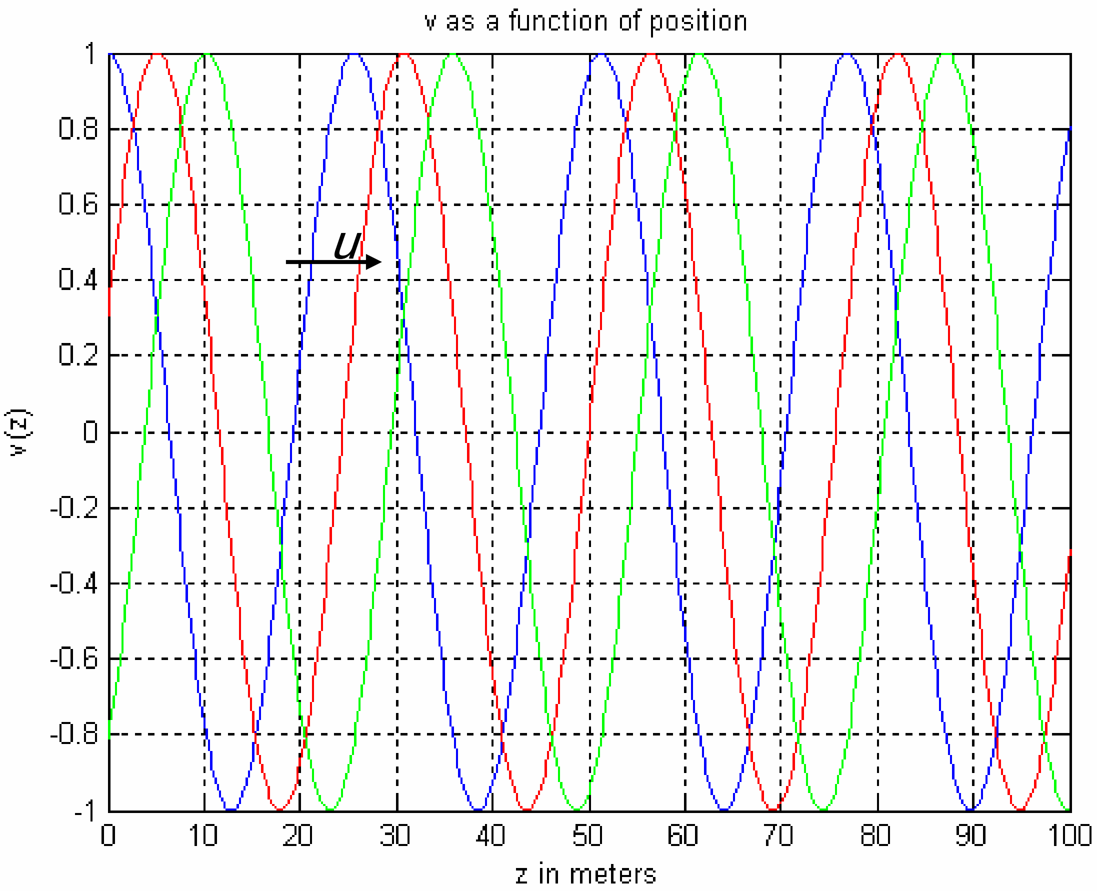
$$A\cos\left(\omega t \mp \frac{\omega}{u} z\right) = A\cos(\omega t \mp \beta z)$$

- First check to see that these solutions have the properties we expect by plotting them using a tool like Matlab

Sine Waves

$$\cos\left(\omega t - \frac{\omega}{u}z\right) = \cos\left(2\pi f t - \frac{2\pi f}{u}z\right)$$

- The positive wave



Workspace

$$\cos\left(\omega t - \omega\frac{z}{u}\right) = \cos\left(2\pi f t - 2\pi f\frac{z}{u}\right)$$

$$\cos(\omega t)$$

$$2\pi f = \omega$$

$$f = \frac{1}{T}$$

$$\cos\left(\underbrace{\frac{2\pi}{T}}_{\omega} t\right)$$

$$\cos\left(2\pi f\frac{z}{u}\right)$$

$$\frac{u}{f} = \lambda$$

$$\cos\left(\underbrace{\frac{2\pi}{\lambda}}_{\beta} z\right)$$

Solutions to the Wave Equation

- Now we check to see that the sine waves are indeed solutions to the wave equation

$$\frac{\partial}{\partial t}\cos(\omega t-\beta z)=-\omega\sin(\omega t-\beta z) \qquad \frac{\partial}{\partial z}\cos(\omega t-\beta z)=-(-\beta)\sin(\omega t-\beta z)$$

$$\frac{\partial^2}{\partial t^2}\cos(\omega t-\beta z)=\omega^2\cos(\omega t-\beta z) \qquad \frac{\partial^2}{\partial z^2}\cos(\omega t-\beta z)=\beta^2\cos(\omega t-\beta z)$$

$$\frac{\partial^2}{\partial z^2}A\cos(\omega t-\beta z)=\frac{\beta^2}{\omega^2}\frac{\partial^2}{\partial t^2}A\cos(\omega t-\beta z)=\frac{1}{u^2}\frac{\partial^2}{\partial t^2}A\cos\left(\omega t-\frac{\omega}{u}z\right)$$

Solutions to the Wave Equation

- Thus, our sine wave is a solution to the voltage or current equation $$\frac{\partial^2 V}{\partial z^2} = lc \frac{\partial^2 V}{\partial t^2}$$
- if $\beta = \dfrac{\omega}{u} = \omega\sqrt{lc}$ or $u = \dfrac{1}{\sqrt{lc}}$
- *u = the speed of wave propagation*
  = the speed of light

Velocity of Propagation

Hosfelt

- Check for RG58/U Cable
  - Inductance per unit length is 0.25 micro Henries per meter
  - Capacitance per unit length is 100 pico Farads per meter

$$u = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{(0.25x10^{-6})(100x10^{-12})}} = \frac{1}{\sqrt{25x10^{-18}}} = \frac{10^9}{5} = 2x10^8 \, {}^{m}\!/_{s}$$

or 2/3 the speed of light

From Digi-Key (Carol Cable)

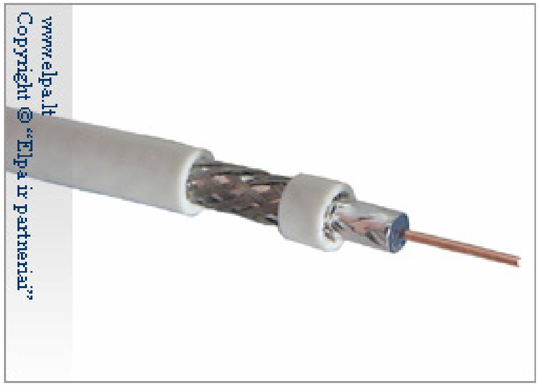
	Imped- ance (Ω)	Capacitance pF/FT (pF/M)	Digi-Key Part No
	50	30.00 (98.43)	W300-X-NI
	50	31.80 (104.34)	C1178-X-N

Fig.	RG Type	Conductor Size (AWG)	Jacket Type	Core O.D. Inch (mm)	Shield Type	Nominal O.D. Inch (mm)	Imped- ance (Ω)	Capacitance pF/FT (pF/M)	Digi-Key Part No	(X = No. of Ft. Per Roll) Price Per Roll 100 ft. (†, ◊)	500 ft. (#)	1000 ft. (#)	General Cable Part No.
RG-58													
1	58/U	20 (Solid BC)	Black PVC	0.116 (2.95)	95% TC Braid	0.195 (4.95)	50	30.00 (98.43)	W300-X-ND	34.01	92.58	185.16	C1166
1	58/U	20 (19X.0071)(TC)	Black PVC	0.116 (2.95)	95% TC Braid	0.195 (4.95)	50	31.80 (104.34)	C1178-X-ND ***NEW!***	29.88	—	182.79	C1178.21.01
2	58/U Thinnet	20 (19X32)(TC)	Gray PVC	0.100 (2.54)	100% Flexfoil + 81% TC Braid	0.186 (4.72)	50	25.40 (83.34)	C5779-X-ND ***NEW!***	31.03	—	193.02	C5779.41.10
1	58/U Plenum	20 (19X32)(TC)	Flexguard/White	0.100 (2.54)	100% Flexfoil + 95% TC Braid	0.165 (4.19)	50	26.00 (85.31)	C3579-1000-ND ***NEW!***	—	—	525.25	C3579.41.02
1	58/U Plenum	19 (Solid BC)	Flexguard/White	0.102 (2.59)	95% TC Braid	0.161 (4.09)	50	25.00 (82.00)	C3519-X-ND ***NEW!***	56.49	—	419.34	C3519.41.02

From Elpa (Lithuania)

RG58U

Purpose: Computer Networks, radiocommunication systems and other.



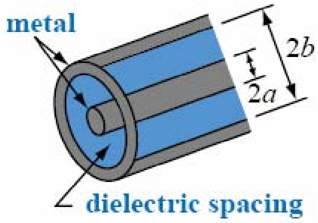
Capacitance

Velocity ratio = .68

Name	RG58U
Code	490503
Packing	**100m**
In box	6x100m=600m
Center conductor	0,81mm BC
Dielectric	2,9mm PE
Shielding foil	AL/PE
Outer conductor	96x0,12mm TC
Jacket	5mm PVC
Weight	20kg/600m
Inner Conductor DC resistance	34Ω/km
Outer Conductor DC resistance	17Ω/km
Capacitance	95pF/m
Impedance	50±3Ω
Screening efficiency	>95dB
Min. bending radius	5 x diameter of cable
Velocity ratio	0,68

Coaxial Cable Parameters

- Capacitance $$c = \frac{2\pi\varepsilon}{\ln\frac{b}{a}} \, F/m$$



(a) Coaxial line

Ulaby

- Inductance

$$l = \frac{\mu}{2\pi}\ln\frac{b}{a} \, H/m$$

$$\varepsilon_o = \frac{1}{36\pi} x10^{-9} \, F/m$$

$$\varepsilon = \varepsilon_r \varepsilon_o$$

$$\mu_o = 4\pi x10^{-7} \, H/m$$

$$\mu = \mu_r \mu_o$$

Coaxial Cable Parameters

$$a = 0.4mm$$

- For RG58/U $\varepsilon = \varepsilon_r \varepsilon_o = 2.3\varepsilon_o$ and

$$b = 1.4mm$$

- One can easily find the capacitance and inductance per unit length. Note that when a parameter is unspecified, you should assume that is has the default value. $\mu = \mu_o$

Workspace

- Why don't the numbers vary by much?

Sine Waves

$$\cos\left(\omega t - \frac{\omega}{u}z\right) = \cos\left(2\pi f t - \frac{2\pi f}{u}z\right)$$

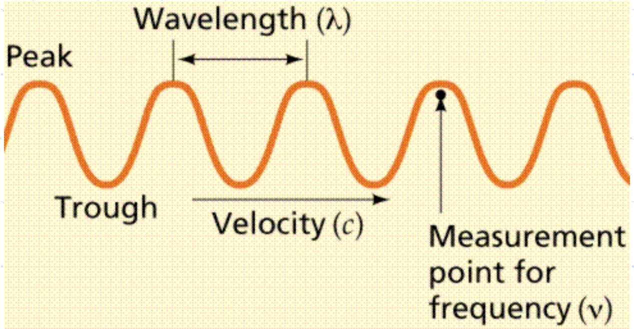
- Consider one other property. What is the distance required to change the phase of this expression by 2π ? We just did this qualitatively.

$$\beta z = \frac{\omega}{u}z = \frac{2\pi f}{u}z = 2\pi$$

- This distance is called the wavelength or

$$\beta\lambda = \frac{\omega}{u}\lambda = \frac{2\pi f}{u}\lambda = 2\pi \qquad \lambda = \frac{2\pi}{\beta} = \frac{u}{f}$$

Sine Waves



- Solutions look like $A\cos(\omega t \mp \beta z)$

$$\beta = \frac{\omega}{u} = \omega\sqrt{lc} = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$$

$$\omega = 2\pi f = \frac{2\pi}{T}$$

$$u = \frac{1}{\sqrt{lc}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{u}{f}$$

$$\varepsilon = \varepsilon_r \varepsilon_o \qquad \mu = \mu_r \mu_o$$

Figure from http://www.emc.maricopa.edu/

Phasor Notation

$$e^{j\omega t} = \cos \omega t + j \sin \omega t$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

- For ease of analysis (changes second order partial differential equation into a second order ordinary differential equation), we use phasor notation.

$$f(z,t) = A\cos(\omega t \mp \beta z) = \mathrm{Re}\left(\left\{Ae^{\mp j\beta z}\right\}e^{j\omega t}\right)$$

$$f(z) = Ae^{\mp j\beta z}$$

- The term in the brackets is the phasor.

Phasor Notation

- To convert to space-time form from the phasor form, multiply by $e^{j\omega t}$ and take the real part. $f(z,t) = \mathrm{Re}(Ae^{\mp j\beta z}e^{j\omega t}) = A\cos(\omega t \mp \beta z)$

- If A is complex $A = |A|e^{j\theta_A}$

$$f(z,t) = \mathrm{Re}(|A|e^{j\theta_A}e^{\mp j\beta z}e^{j\omega t}) = |A|\cos(\omega t \mp \beta z + \theta_A)$$

$$\mathrm{Re}\, e^{j\theta} = \cos\theta$$

Example

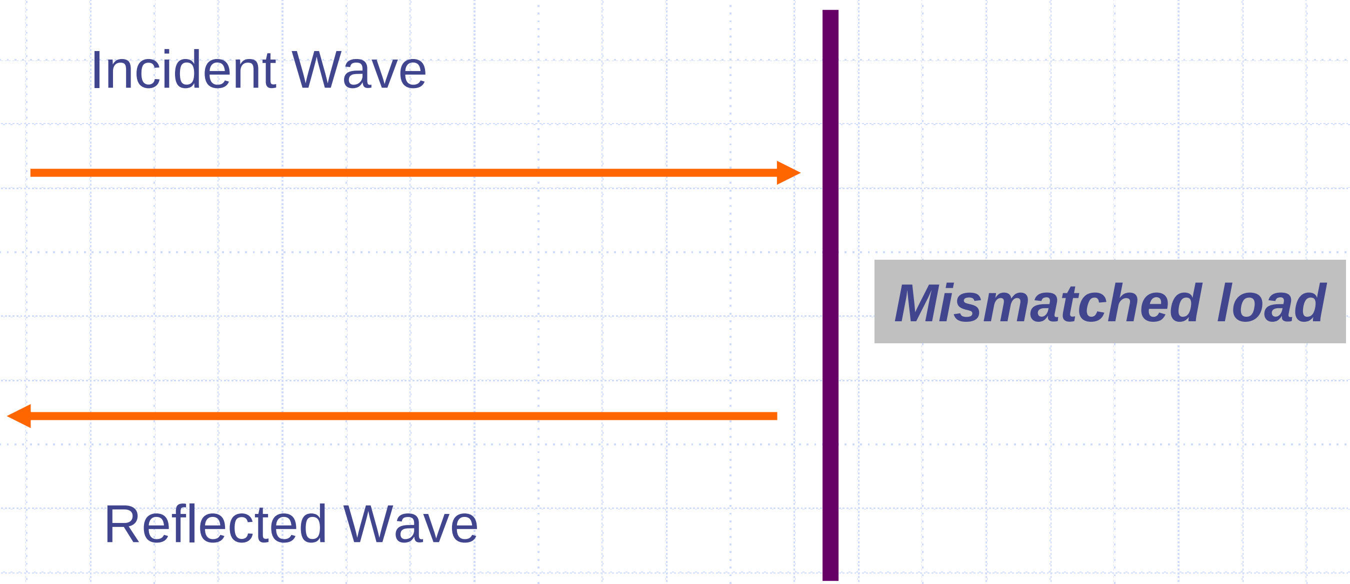
Example

$$\frac{\partial}{\partial t}(\ )e^{j\omega t}$$

$$(j\omega)(\text{const})\,e^{j\omega t}$$

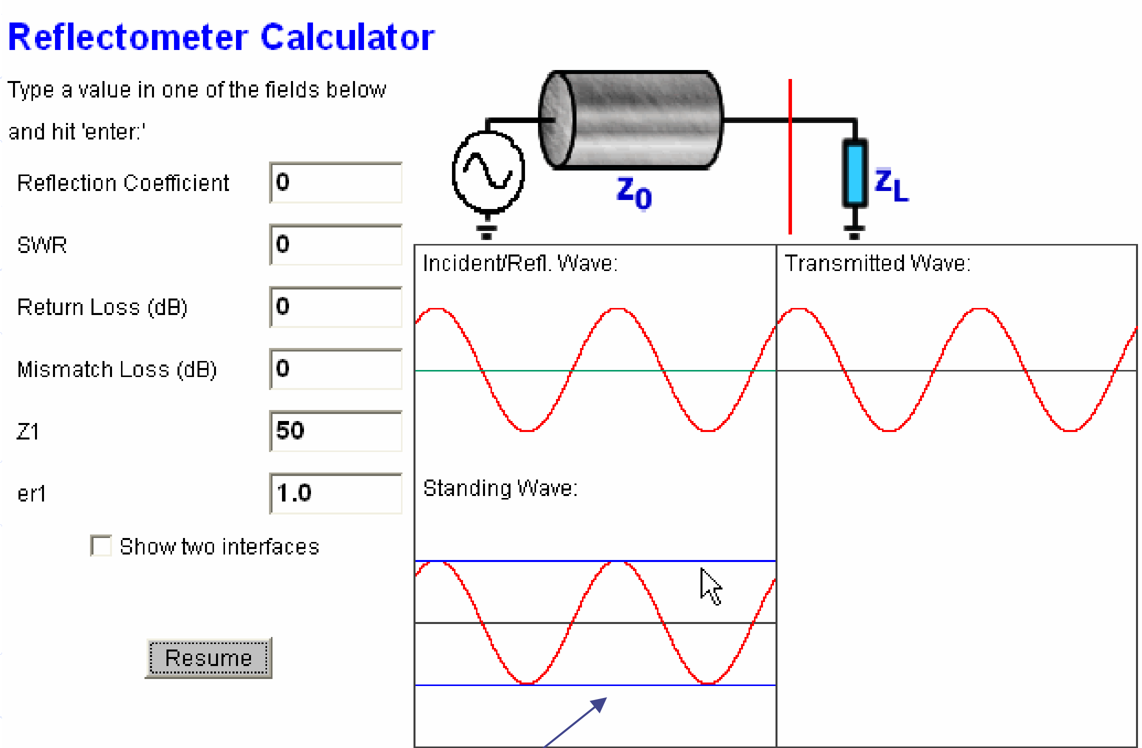
$$\frac{\partial^2}{\partial t^2} \Rightarrow (j\omega)^2 = -\omega^2$$

Transmission Lines



Standing wave due to interference

Standing Waves



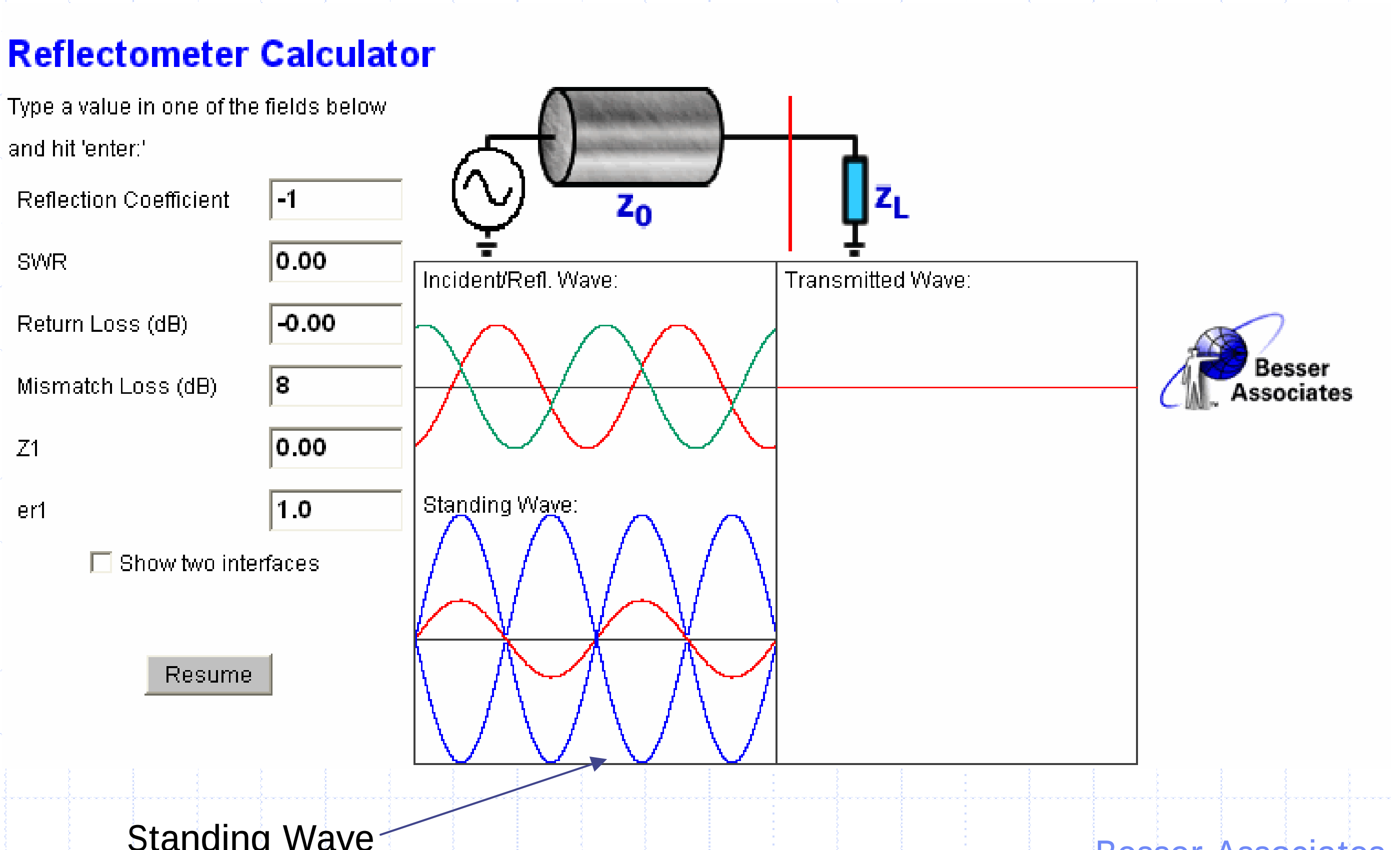
Besser Associates

http://www.bessernet.com

No Standing Wave

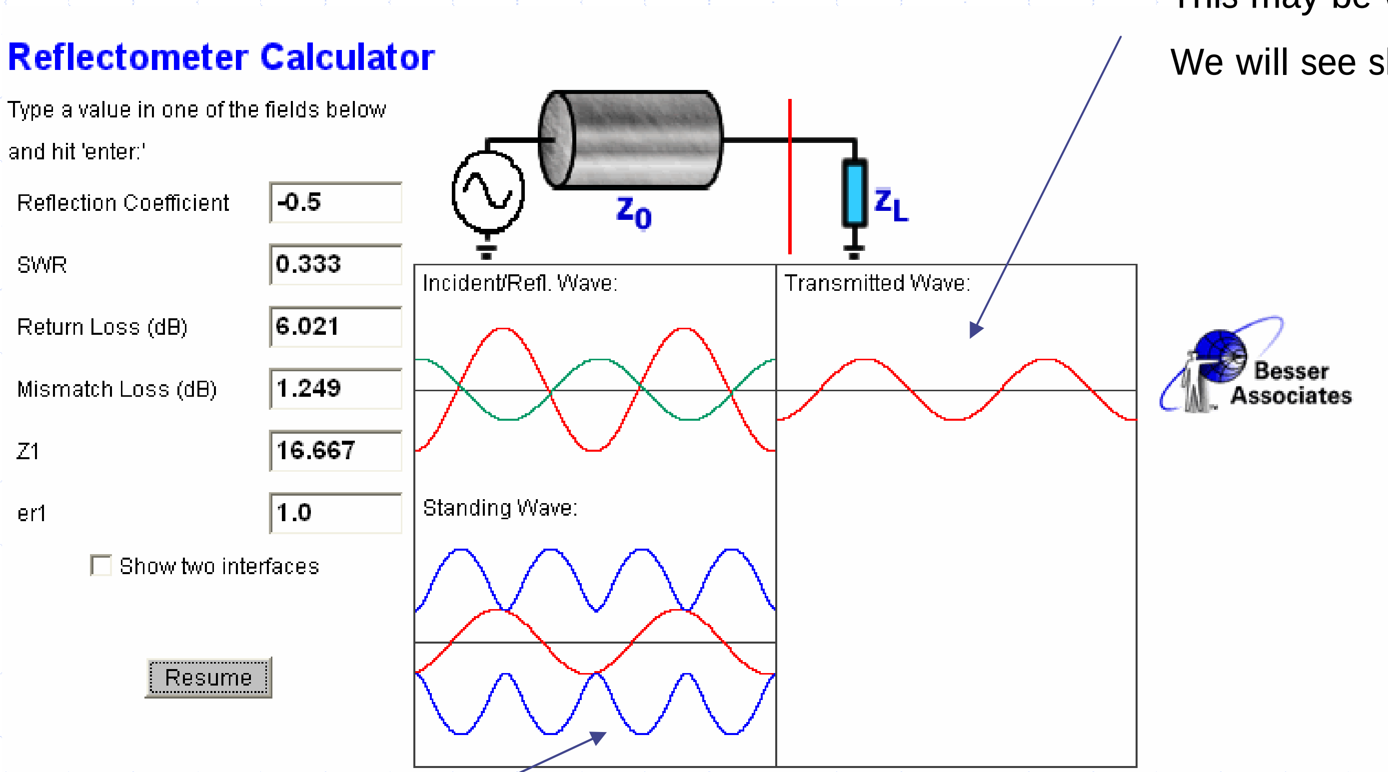
Besser Associates

Standing Waves



Besser Associates

Standing Waves



This may be wrong

We will see shortly

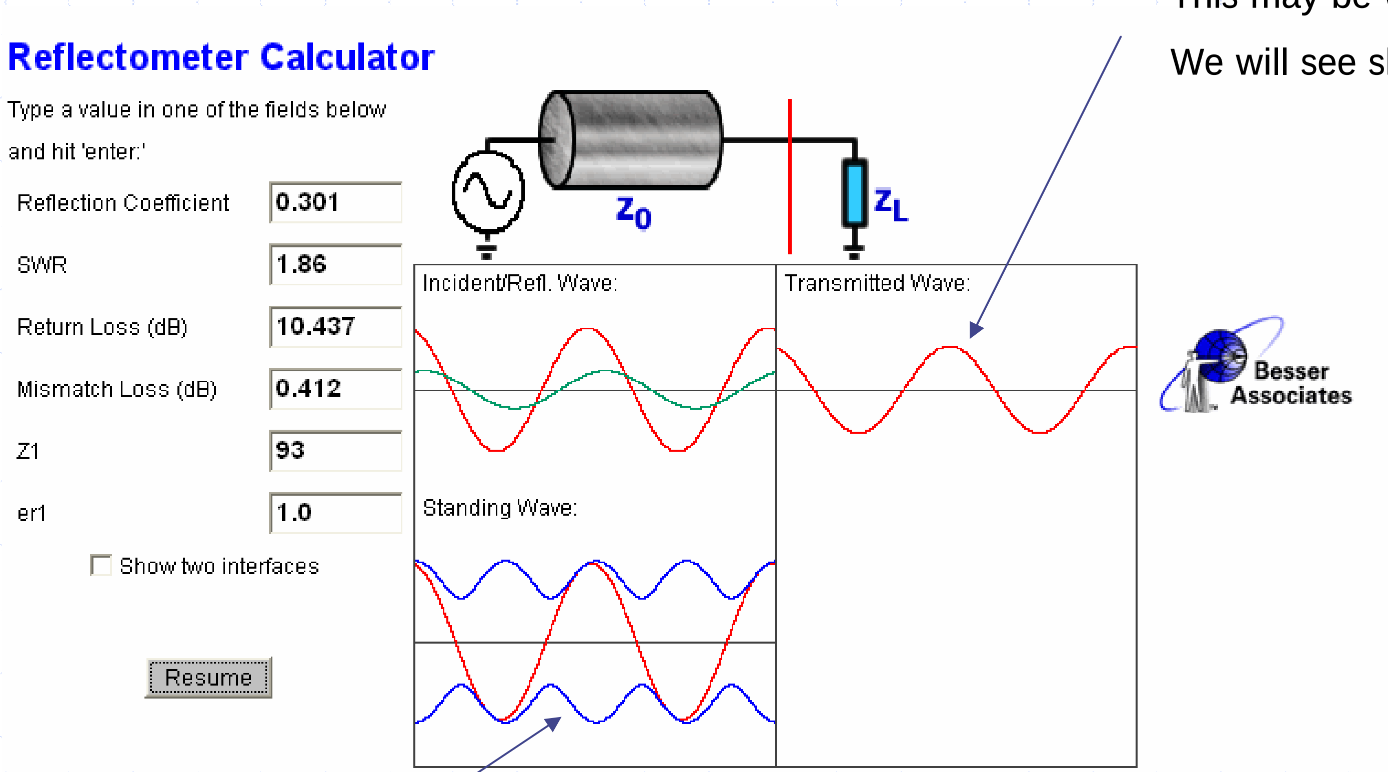
Standing Wave

[Besser Associates]

Standing Waves

This may be wrong

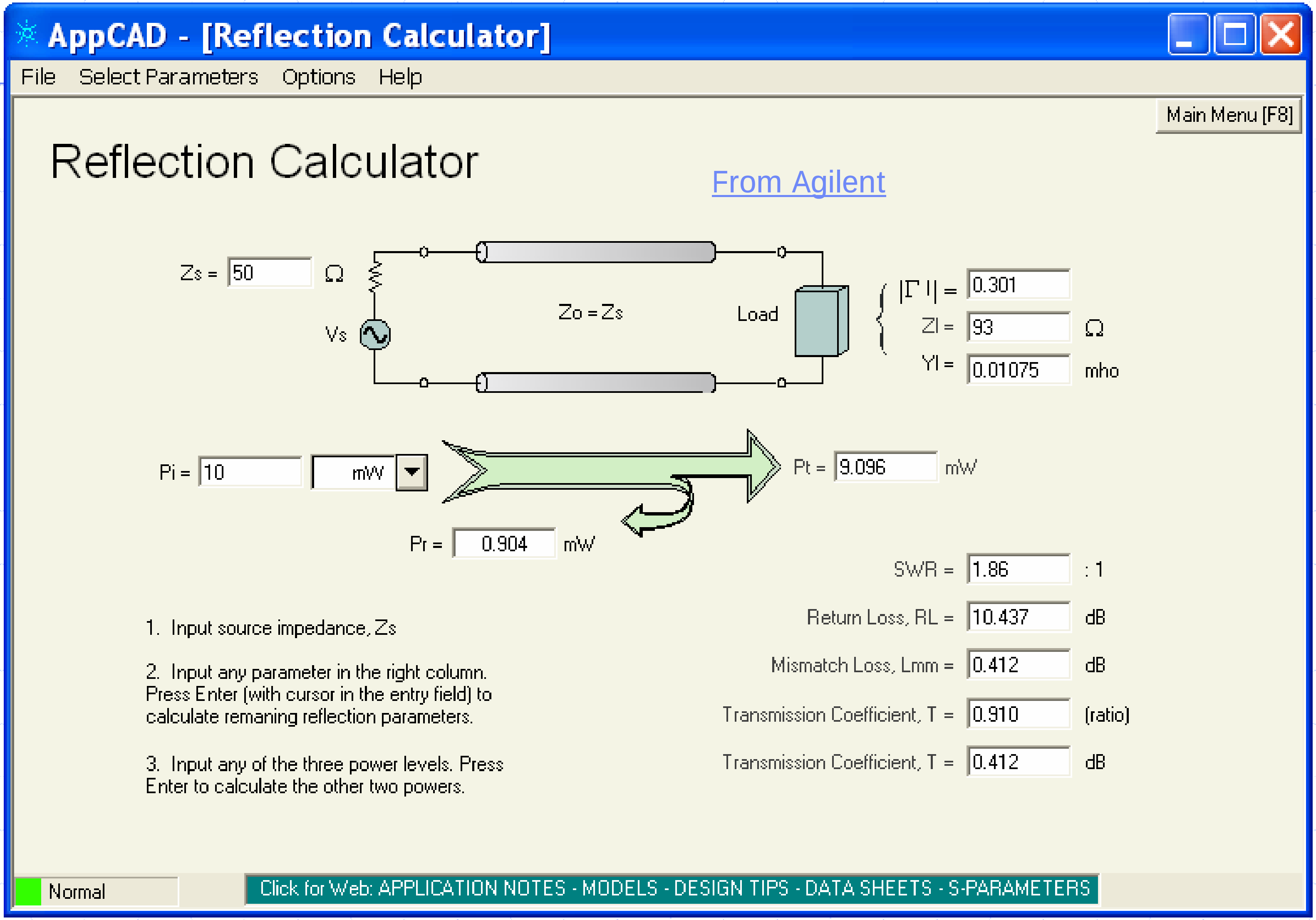
We will see shortly



Standing Wave

Besser Associates

Standing Waves



$\beta = \omega\sqrt{lc}$

Transmission Lines - Standing Wave Derivation

$(\omega t - \beta z)$

$e^{j\omega t}$

Phasor Form of the Wave Equation:

$$\frac{\partial^2 V}{\partial z^2} = l \cdot c \cdot \frac{\partial^2 V}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 V}{\partial z^2} = -\omega^2 \cdot l \cdot c \cdot V$$

where:

$$V = V^{\mp} \cdot e^{\pm j \cdot \beta \cdot z}$$

General Solution:

$$V = V^{+} e^{-j \cdot \beta \cdot z} + V^{-} e^{+j \cdot \beta \cdot z}$$

Workspace

$$\beta^2 = \omega^2 l c$$

$$\frac{\partial^2 \tilde{V}}{\partial z^2} = -\omega^2 l c \tilde{V}$$

$$\tilde{V} = \tilde{V}^+ e^{-j\beta z} + \tilde{V}^- e^{+j\beta z}$$

$$\frac{\partial \tilde{V}}{\partial z} = \tilde{V}^+ (-j\beta) e^{-j\beta z}$$

$$\frac{\partial^2 \tilde{V}}{\partial z^2} = \tilde{V}^+ (-j\beta)^2 e^{-j\beta z} = -\tilde{V}^+ \beta^2 e^{-j\beta z}$$

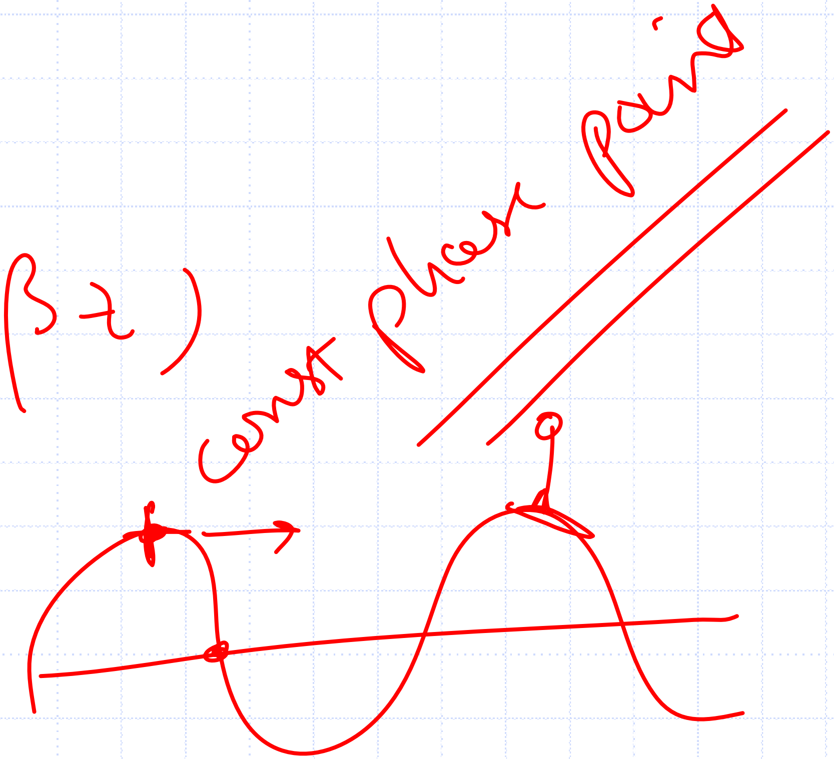
$$= -\beta^2 \tilde{V}$$

Workspace

$$\cos(\omega t - \beta z)$$

$$\frac{d}{dt}(\omega t - \beta z) = \omega - \beta \frac{dz}{dt} = 0$$

$$\frac{dz}{dt} = \frac{\omega}{\beta}$$



Transmission Lines - Standing Wave Derivation

$$V = V^{+}e^{-j\cdot\beta\cdot z} + V^{-}e^{+j\cdot\beta\cdot z}$$

Forward Wave

$$\cos(\omega\cdot t - \beta\cdot z)$$

Backward Wave

$$\cos(\omega\cdot t + \beta\cdot z)$$

TIME DOMAIN

V_{max} occurs when Forward and Backward Waves are in Phase

➡ *CONSTRUCTIVE INTERFERENCE*

V_{min} occurs when Forward and Backward Waves are out of Phase

➡ *DESTRUCTIVE INTERFERENCE*

Transmission Lines Formulas

- Fields and Waves I Quiz Formula Sheet
- In the class notes $v(z) = V_+ e^{-j\beta z} + V_- e^{+j\beta z}$

$$i(z) = \frac{V_+ e^{-j\beta z} - V_- e^{+j\beta z}}{Z_o} = \frac{V_+}{Z_o} e^{-j\beta z} - \frac{V_-}{Z_o} e^{+j\beta z}$$

$$V_+ = V^+ = V_m^{\ +} \qquad V_- = V^- = V_m^{\ -}$$

- Note:

All are used in various handouts, texts, etc. There is no standard notation.

RG58/U Cable

$$\frac{2\pi(1.5\times10^{6})}{2\times10^{8}}$$

- Assume 2 $V_{P\text{-}P}$ 1.5MHz sine wave is launched on such a line. Find $\beta = \frac{\omega}{u} = \omega\sqrt{lc} = \omega\sqrt{\mu\varepsilon} = \frac{2\pi}{\lambda}$ and λ

$$u = \frac{2}{3}c \triangleq 2\times10^{8}\ m/s$$

- Answers?

$$l = .25\times10^{-6}\ H/m$$

$$c = 100\times10^{-12}\ F/m$$

$$\frac{3\pi}{200}$$

Short Circuit Load

- For $Z_L=0$, we have $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{0 - Z_o}{0 + Z_o} = -1$

$$v(z) = V^+ e^{-j\beta z} + \Gamma_L V^+ e^{+j\beta z} = V^+\left(e^{-j\beta z} - e^{+j\beta z}\right)$$

$$e^{+j\beta z} = \cos\beta z + j\sin\beta z$$

$$e^{-j\beta z} = \cos\beta z - j\sin\beta z$$

$$\Rightarrow \quad v(z) = -V^+\left(j2\sin\beta z\right)$$

Short Circuit Load

- Convert to space-time form

$$v(z,t) = \mathrm{Re}\left(v(z)e^{j\omega t}\right) = \mathrm{Re}\left(V^{+}\left(-j2\sin\beta z\right)e^{j\omega t}\right)$$

$$\mathrm{Re}\left(\left(-j2\sin\beta z\right)e^{j\omega t}\right) = \mathrm{Re}\left(-2\sin\beta z\left(j\cos\beta z - \sin\beta z\right)\right)$$

$$v(z,t) = 2V^{+}\sin\beta z\sin\omega t$$

- This is a standing wave

Review

- Traveling Waves & Standing Waves
- Phasor Notation
- General Representation of Waves