Homework 1

Due 25 January 2005

1. Plane Wave Representations

The numbers given in this problem are realistic but not real. That is, your answers should come out in a reasonable range, but the numbers are not based on a real, commercially available transmission line.

The voltage on a transmission line is given by $v(z,t) = 10\cos\left(8.8\pi 10^7 t + \frac{2\pi}{5} z\right)$.

a. Is this a standing wave or a traveling wave? If it is a traveling wave, what direction does it travel and what is its velocity u?
b. What is the period of this wave T? What is the wavelength λ?
c. Plot this expression as a function of space at $t=0$, $t=T/3$, $t=2T/3$ using Maple or Matlab or some similar program.
d. Write this voltage expression in phasor form.
e. Assume that the transmission line has a capacitance per unit length of $100\, {pF}/{m}$. Find the characteristic impedance of the line Z_o and then the current on the line in phasor form.

2. Reflection of Plane Waves

The waves in the previous problem can exist if the transmission line is properly matched to its load. That is, the load impedance would have to be equal to Z_o. In this problem, we will consider what happens when there is no load (open circuit).

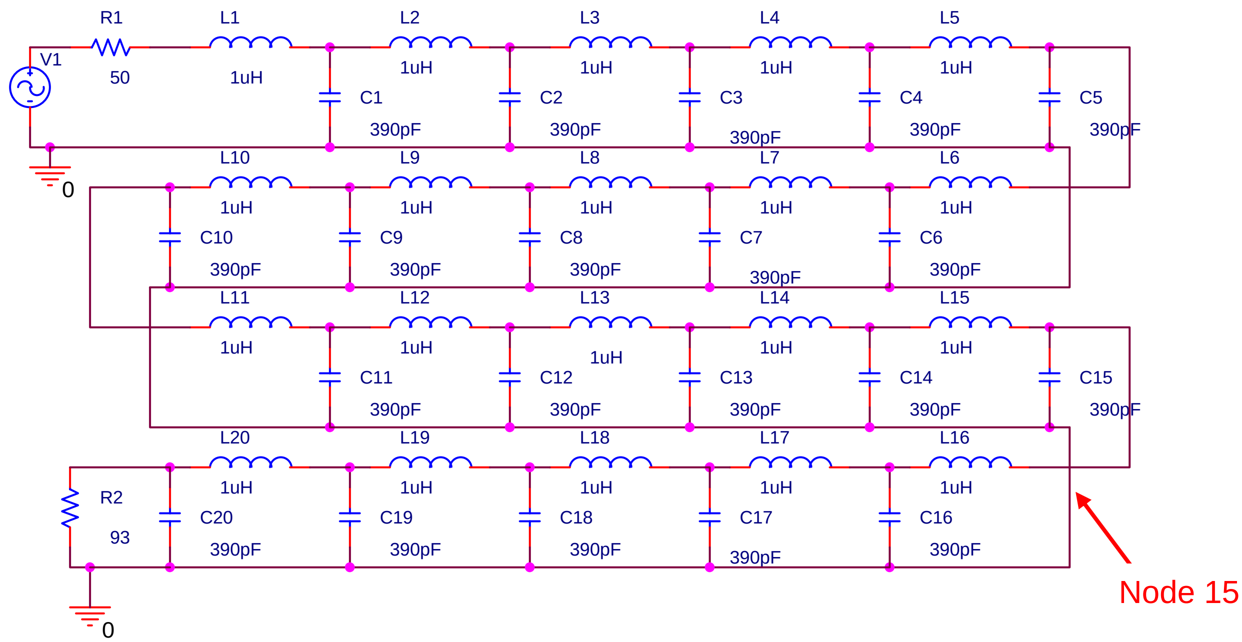
a. If the load is an open circuit, what is the reflection coefficient at the load Γ_L?
b. Find the reflected voltage and current waves in phasor form.
c. Plot the standing wave pattern for both the current and voltage.
d. What is the standing wave ratio in this case?
e. Is the voltage standing wave a maximum, minimum or neither at the load? Is the current standing wave a maximum, minimum or neither at the load?
f. What is the distance to the closest minimum to the load for both the voltage and current standing waves? That is, where is the first minimum located that is not at the load? Give your answer both in meters and in fractions of a wavelength.

3. Lumped Transmission Lines

A transmission line can be replaced by a series of lumped circuit elements, under certain conditions. In this exercise, you will compare the response of such a lumped model line with the coil of coax. Obtain a coil of coax, a 50 Ohm terminator and one of the lumped component transmission lines from a TA.

a. First, we will repeat an experiment from the first studio session at a different frequency. Put the 50 Ohm terminator across the output of the coaxial cable and simultaneously measure input and output signals on the oscilloscope. Set the input voltage at $1V_{P-P}$ with a frequency of *2MHz*. Measure the time delay between the signals. What else is different about the input and output voltages?

b. Replace the coaxial cable with the lumped version and repeat the experiment.
c. You should have observed that the lumped line behaves in a qualitatively similar manner to the spool of coax. Thus, it must represent a similar length of line. We want to determine the actual length of line. Each *L-C* combination represents some equivalent length of line. Since there are 20 such combinations, we only need to figure out what length each combo represents and multiply it by 20. To understand better how the lumped circuit is configured, look at the diagram below, done with PSpice. (This diagram has a load impedance R2 of 93 Ohms, which is not what we are using here.) Note that the inductance and capacitance for each section is indicated. Given your knowledge of the actual capacitance and inductance per unit length for the RG58/U coaxial cable, what length does each section represent?



d. Remove the load resistor. Measure the voltage at the 15^{th} node. Adjust the frequency somewhere between *2MHz* and *2.5MHz* until the voltage at the 15^{th} node is a minimum. It should be somewhere around *100mV*. Put the terminating resistor back on as a load. Measure the voltage at each of the nodes for the case where the lumped line is terminated with 50 Ohms. Remove the resistor and repeat for no load (open circuit). Where are the minimas and maximas located? (One minimum should be at node 15.) Plot your results for the measured voltages as a function of distance (using the distance between nodes that you determined above). Plot your results again but now in terms of wavelength rather than meters. (You can use the same plot, if you wish, and provide two sets of labels.)
e. Show that the maximas and minimas are located where they should be in terms of wavelength.

Note that the lumped lines work slightly differently if different capacitors are used. Thus, please note the color of the capacitors on the board you are using. The correct answer to part d will depend on what capacitors are used on your board.