Homework 1

Due 24 January 2006 at 8:00 pm

1. Waves and Phasor Notation

Be sure that you read the following questions carefully. Also, when you convert an expression from phasor to time domain form or vice versa, convert it back, carefully following the rules, to check your answer.

a. Write the following current phasor expression in time domain form $\tilde{I} = I_o e^{j\pi}$. Note that this is just a current, not a current wave. *ans:* $\operatorname{Re}\left(\tilde{I} = I_o e^{j\pi}\right)e^{j\omega t} = I_o \operatorname{Re}\left((-1)e^{j\omega t}\right) = -I_o \cos\omega t$

b. Write the following time domain current wave expression in phasor form $i(z,t) = I_o \sin\left(\omega t + \frac{\omega}{u_o} z\right)$. *ans: to easily convert a sine function to phasor form, note that* $e^{jx} = \cos x + j\sin x$ *so that* $-je^{jx} = -j\cos x + \sin x$ *so and thus* $\sin x = \operatorname{Re}(-je^{jx})$. *Applying this to our expression, we have* $i(z,t) = I_o \sin\left(\omega t + \frac{\omega}{u_o} z\right) = I_o \operatorname{Re}\left(-je^{j\left(\omega t + \frac{\omega}{u_o} z\right)}\right) = I_o \operatorname{Re}\left(e^{-j\frac{\pi}{2}} e^{j\left(\omega t + \frac{\omega}{u_o} z\right)}\right)$. *The phasor is what is left after we remove the time exponential, so* $\tilde{I} = I_o e^{-j\frac{\pi}{2} + j\frac{\omega}{u_o} z}$

c. Is the expression in part b a standing or traveling wave? If it is a traveling wave, what direction does it travel and what is its velocity? *ans: It is a traveling wave, moving in the negative z direction at a velocity of u_o. The answer can also be given as traveling in the z direction at a velocity of $-u_o$ or we can use vector notation.*

2. Plane Wave Representations

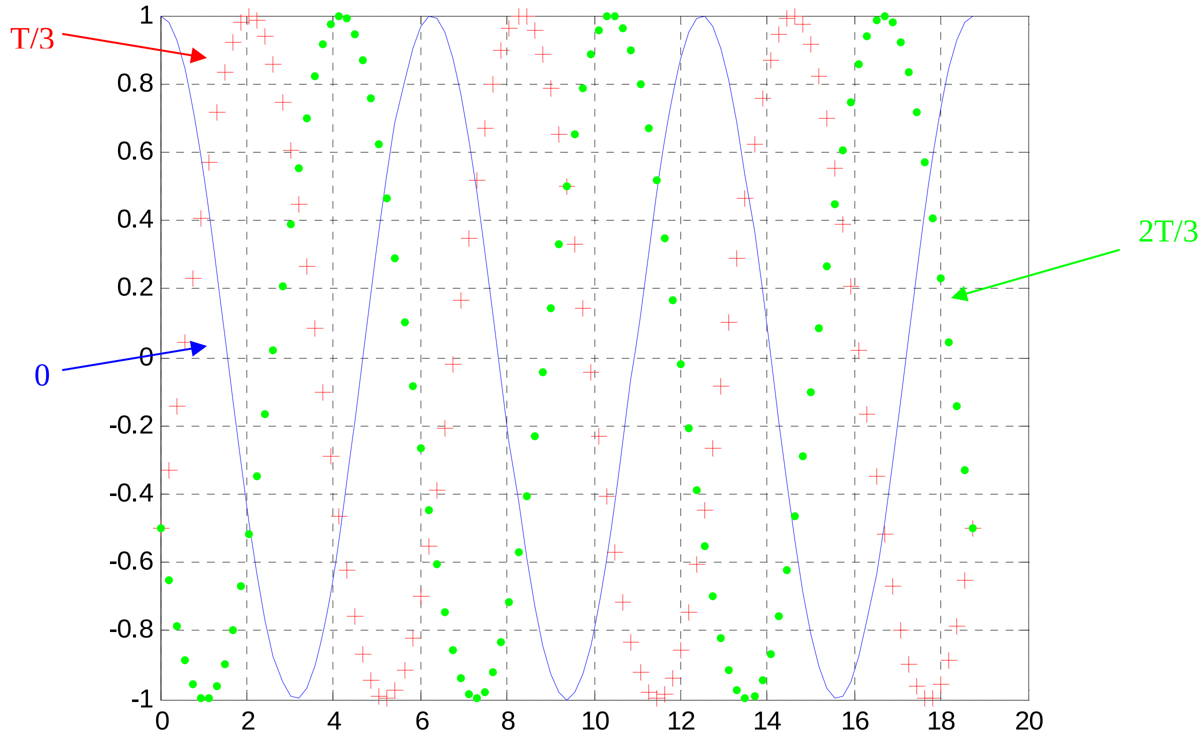
The numbers given in this problem are realistic but not real. That is, your answers should come out in a reasonable range, but the numbers are not based on a real, commercially available transmission line.

The voltage on a transmission line is given by $v(z,t) = 20\cos\left(8\pi 10^7 t - 0.32\pi z\right)$.

a. Is this a standing wave or a traveling wave? If it is a traveling wave, what direction does it travel and what is its velocity *u*? *ans:* $u = \frac{\omega}{\beta} = \frac{8\pi 10^7}{.32\pi} = 2.5x10^8$

b. What is the period of this wave *T*? What is the wavelength λ? *ans:* $T = \frac{1}{4x10^7} = 2.5x10^{-8}$ *and* $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.32\pi} = 6.25 meter$

c. Plot this expression as a function of space at *t=0, t=T/3, t=2T/3* using Maple or Matlab or some similar program.



d. Write this voltage expression in phasor form. *ans:* $v(z,t) = 20\,\mathrm{Re}\, e^{j\left(8\pi 10^7 t - 0.32\pi z\right)}$ *so* $\tilde{V} = 20e^{j(-0.32\pi z)} = 20e^{-j0.32\pi z}$

e. Assume that the transmission line has a capacitance per unit length of $100\,{}^{pF}\!/_{m}$. Find the characteristic impedance of the line Z_o and then the current on the line in phasor form. *ans:* $Z_o = \sqrt{\frac{l}{c}}$ *and* $u = \sqrt{\frac{1}{lc}}$ *so* $Z_o = \frac{1}{uc} = 40\Omega$

3. Verification of the Wave Solution

The time domain and phasor forms of the voltage waves in the previous problem must satisfy the corresponding form of the wave equation. Since you have determined all of the wave and transmission line parameters, you should be able to show that these expressions are correct.

a. Verify that the time domain voltage $v(z,t)$ satisfies $\frac{\partial^2 v(z,t)}{\partial z^2} = lc\frac{\partial^2 v(z,t)}{\partial t^2}$ by evaluating the second derivatives as shown and using the values for inductance and capacitance per unit length you obtained above. *ans:*

$$\frac{\partial}{\partial t}v(z,t) = \frac{\partial}{\partial t}20\cos\left(8\pi 10^7 t - 0.32\pi z\right) = -20(8\pi 10^7)\sin\left(8\pi 10^7 t - 0.32\pi z\right)$$

$$\frac{\partial^2}{\partial t^2}v(z,t) = \frac{\partial}{\partial t}\left(-20(8\pi 10^7)\sin\left(8\pi 10^7 t - 0.32\pi z\right)\right) = -20(8\pi 10^7)^2\cos\left(8\pi 10^7 t - 0.32\pi z\right)$$

$$lc\frac{\partial^2}{\partial t^2}v(z,t) = -\frac{1}{\left(2.5x10^8\right)^2}20(8\pi 10^7)^2\cos\left(8\pi 10^7 t - 0.32\pi z\right)$$

$$\frac{\partial}{\partial z}v(z,t) = \frac{\partial}{\partial z}20\cos\left(8\pi 10^7 t - 0.32\pi z\right) = -20(-.32\pi)\sin\left(8\pi 10^7 t - 0.32\pi z\right)$$

$$\frac{\partial^2}{\partial z^2}v(z,t) = \frac{\partial}{\partial z}\left(-20(-.32\pi)\sin\left(8\pi 10^7 t - 0.32\pi z\right)\right) = -20(-.32\pi)^2\cos\left(8\pi 10^7 t - 0.32\pi z\right)$$

Note that the cosine terms are the same so we only need to check the amplitude term. $-\frac{1}{\left(2.5x10^8\right)^2}20(8\pi 10^7)^2 = -20\pi^2\frac{(8x10^7)^2}{\left(2.5x10^8\right)^2} = -20\pi^2\left(.32\right)^2$ *so it checks.*

b. Also, verify that the phasor voltage $\tilde{V}$ you obtained in part d above satisfies $\frac{\partial^2\tilde{V}}{\partial z^2} = -\omega^2 lc\tilde{V}$ *ans:* $\tilde{V} = 20e^{-j0.32\pi z}$

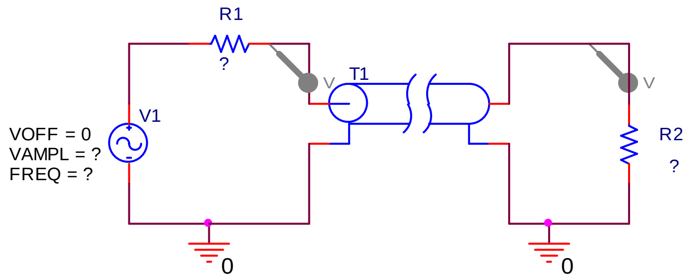
$$\frac{\partial}{\partial z}\tilde{V} = 20\frac{\partial}{\partial z}e^{-j0.32\pi z} = 20(-j0.32\pi)e^{-j0.32\pi z}$$

$\frac{\partial^2}{\partial z^2}\tilde{V} = 20(-j0.32\pi)^2 e^{-j0.32\pi z} = -20(0.32\pi)^2 e^{-j0.32\pi z}$ *again we only need to check the amplitude since the exponential term is the same. First recall that* $lc = \frac{1}{u^2} = \left(\frac{\beta}{\omega}\right)^2$ *so that* $-\omega^2 lc = -\frac{\omega^2}{u^2} = -\omega^2\left(\frac{\beta}{\omega}\right)^2 = -\beta^2 = -\left(0.32\pi\right)^2$ *so it checks.*

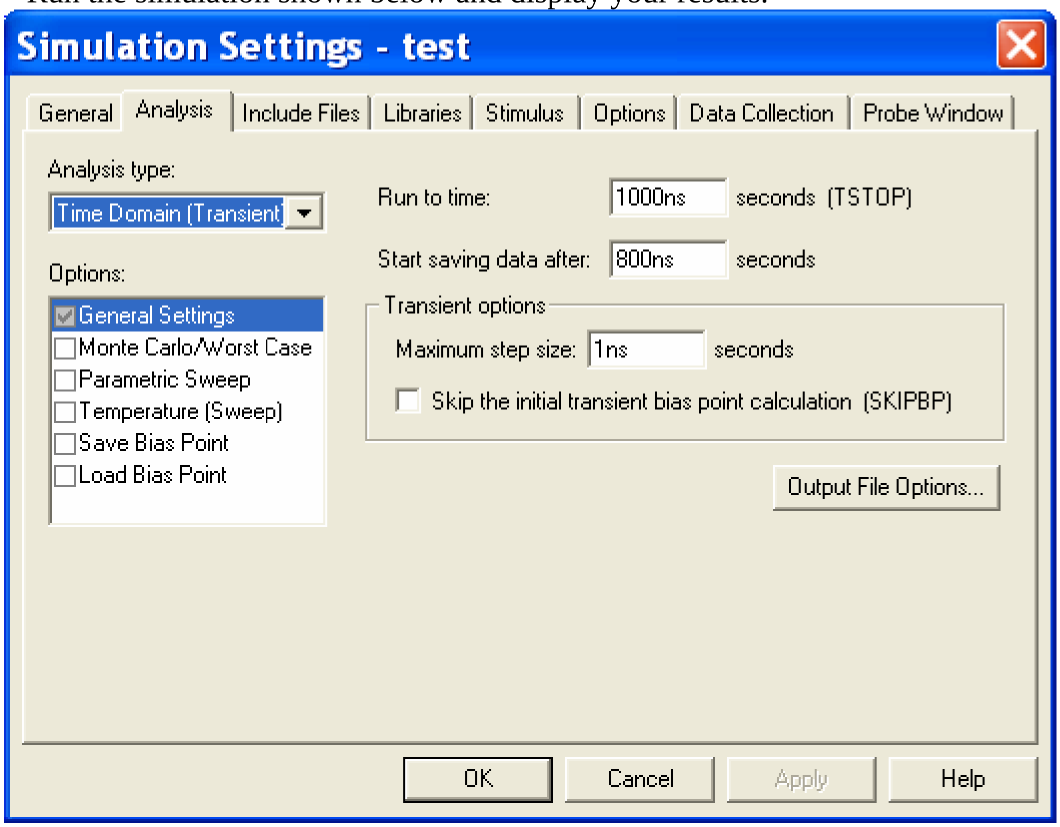
4. PSpice Simulated Experiments

PSpice can show us a lot about how the voltages look at the input and output ends of transmission lines. The simulation can, in effect, replace an experiment by giving us essentially the same results.

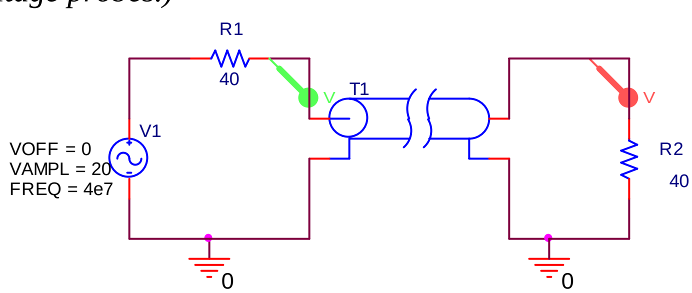
a. To get some practice using PSpice, set up the following simulation using the parameters of your transmission line from problem 2. That is the source, line and load impedances should all be equal to the characteristic impedance of the line you determined in problem 2. The source voltage and frequency should be selected to obtain the voltage given in problem 2 on the line. Set the offset voltage to zero. Use the lossless line model and assume the length of the line is 92.5meters. Be careful to use the zero ground, since it is the only one that works with PSpice.



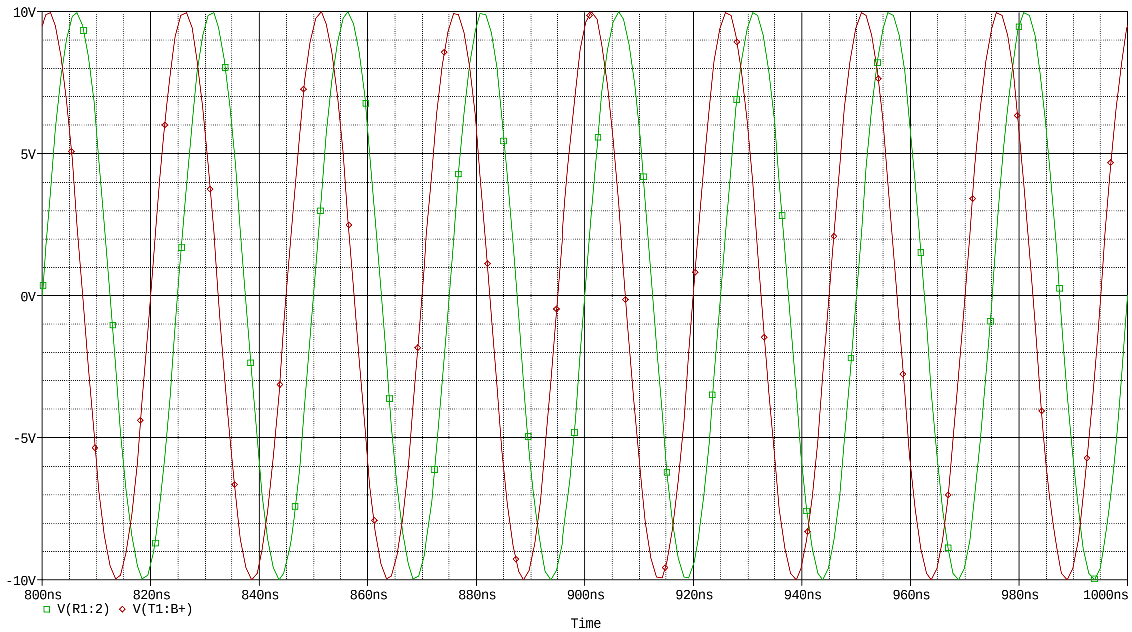
Run the simulation shown below and display your results.



Ans: First just input the 40 Ohms for each of the impedances and add the voltage and frequency for the voltage source. Also, the delay of 370ns has to be used, which is calculated below.(The following picture is from after the simulation since it shows the colors of the voltage probes.)



Then running the simulation, we obtain:



b. Explain why your result makes sense.

Ans: There are really only two characteristics to observe: the magnitude of the voltages and their position in time. First, note that the voltages are the same at each end of the line. That is because the line is lossless and properly matched. Either loss or mismatch would cause a difference. Next, the amplitude is 10V. This is due to the voltage divider action at the input end. The 20V source divides equally between the internal 50Ohm resistor and the 50Ohm input impedance of the line. Finally, what is the delay between the sine waves? For a 92.5m line with a velocity of 2.5x10^{8}m/s, the time to propagate from one end of the line to the other is

$\Delta t = 92.5/(2.5x10^8) = 370ns$ *which is the delay shown in the plot and, hopefully, the delay one has used for a transmission line parameter. To confirm that the delay is correct, one can use the cursor feature in PSpice. On the plot above, it appears that the delay is 20ns. In fact, at first glance, it looks like the output appears before the input, which is counter intuitive. Why does this occur? Recall that the period of the wave is 25ns. Any delay equal to some integer times 25ns will appear as no delay at all. Thus, the first 350ns of the delay does not matter since 350=14x25 or 14 full periods. What we see then is 370-350=20ns. Thus, the plot is correct.*