

Homework 1

Due 24 January 2007 at 6:00 pm

1. Waves and Phasor Notation

Be sure that you read the following questions carefully. Also, when you convert an expression from phasor to time domain form or vice versa, convert it back, carefully following the rules, to check your answer.

- a. Write the following voltage phasor expression in time domain form $\tilde{V} = 100e^{-j\frac{\pi}{2}}$. Note that this is just a voltage, not a voltage wave. Assume $f = 10\text{MHz}$.

$$\text{Re}\left(\tilde{V} = 100e^{-j\frac{\pi}{2}}\right)e^{j\omega t} = V(t) = \text{Re}\left(100(-j)(\cos\omega t + j\sin\omega t)\right) = 100\sin\omega t$$

$$\omega = 2\pi f = 20\pi 10^6$$

- b. Write the following time domain current wave expression in phasor form.

$$i(z,t) = I_o \cos\left(\omega t - a\omega z - \frac{\pi}{2}\right) = I_o \text{Re}\left(e^{j\left(\omega t - a\omega z - \frac{\pi}{2}\right)}\right) = I_o \text{Re}\left((-j)e^{-ja\omega z} e^{j\omega t}\right). \text{ Thus}$$

$$\text{the phasor is } \tilde{I} = I_o\left((-j)e^{-ja\omega z}\right) = -jI_o e^{-ja\omega z}$$

- c. Is the expression in part b a standing or traveling wave? If it is a traveling wave, what direction does it travel and what is its velocity?

It is a traveling wave since it involves both z and t inside its argument. The

velocity will be $v = \frac{\omega}{\beta} = \frac{\omega}{a\omega} = \frac{1}{a}$ in the positive z direction.

2. Plane Wave Representations

The numbers given in this problem are realistic but not necessarily real. That is, your answers should come out in a reasonable range, but the numbers are not based on a real, commercially available transmission line.

The current on a transmission line is given by $i(z,t) = 0.2\cos(3\pi 10^6 t + 0.015\pi z)$.

- a. Is this a standing wave or a traveling wave? If it is a traveling wave, what direction does it travel and what is its velocity?

$$\text{Traveling. } v = \frac{\omega}{\beta} = \frac{3\pi 10^6}{0.015\pi} = 2 \times 10^8$$

- b. What is the period of this wave T ? What is the wavelength λ ?

$$\text{Period } T = \frac{1}{f} = \frac{1}{1.5 \times 10^6} = 0.67 \mu\text{s} \quad \lambda = \frac{2\pi}{\beta} = \frac{v}{f} = \frac{2\pi}{0.015\pi} = 133.3$$

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- c. Plot this expression as a function of space at $t=0$, $t=T/3$, $t=2T/3$ using Maple or Matlab or some similar program.

The m-file used to plot is

```
% Plot Plane Waves at Different Times
% K. A. Connor 18 January 2007

%Set Wave Parameters
f=1.5e6;b=0.015*pi;w=2*pi*f;lam=2*pi/b;T=1/f;A=0.2;

%Set Range of Position to Plot to be 3 Times the Wavelength
z=[0:.01:1].*lam.*3;

%Select First Time for Plot
t=0;

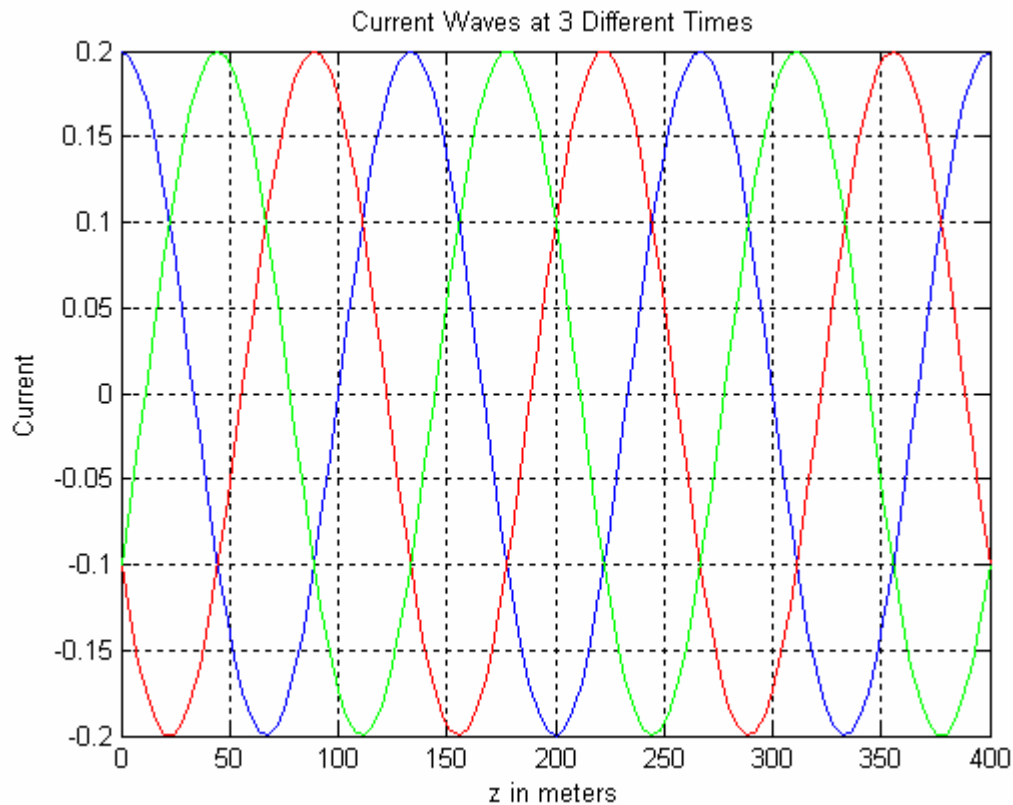
%Evaluate Current Wave
i=A.*cos(w.*t+b.*z);

%Start Plot With the Wave at t=0 (blue color)
plot(z,i,'b');title('Current Waves at 3 Different Times');xlabel('z in
meters');ylabel('Current');hold on;grid;

%Set Other Times & PLOT (red and green chosen for color)
t=T/3; i=A.*cos(w.*t+b.*z);plot(z,i,'r');
t=2*T/3;i=A.*cos(w.*t+b.*z);plot(z,i,'g');
```

The plot looks like the following. Note that the waves are plotted in the order blue, red and green, so that it can be seen that the waves propagate to the left.

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d. Write this current expression in phasor form.

$$i(z,t) = 0.2 \cos(3\pi 10^6 t + 0.015\pi z) = \operatorname{Re}\left(0.2e^{j(3\pi 10^6 t + 0.015\pi z)}\right) \text{ so that the phasor is}$$

$$\tilde{I} = \left(0.2e^{j(0.015\pi z)}\right) = 0.2e^{j0.015\pi z}$$

e. Assume that the transmission line has a capacitance per unit length of 50 pF/m .

Find the characteristic impedance of the line Z_o and then the voltage on the line in phasor form.

The velocity is evaluated above and we know that $v = \frac{1}{\sqrt{lc}}$ so that we can find the

inductance from $l = \frac{1}{v^2 c} = 0.5 \mu\text{H/m}$. We can now find

$$Z_o = \sqrt{\frac{l}{c}} = \sqrt{\frac{0.5 \times 10^{-6}}{50 \times 10^{-12}}} = 100 \Omega. \text{ The voltage on the line is therefore}$$

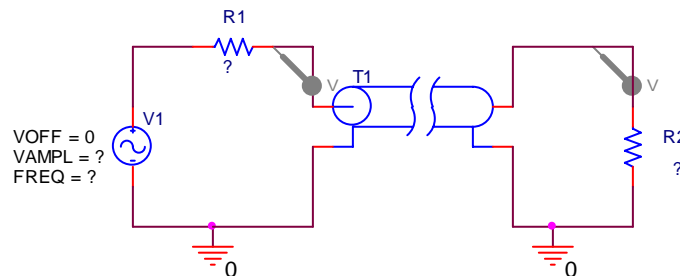
$$V(z,t) = Z_o i(z,t) = 20 \cos(3\pi 10^6 t + 0.015\pi z)$$

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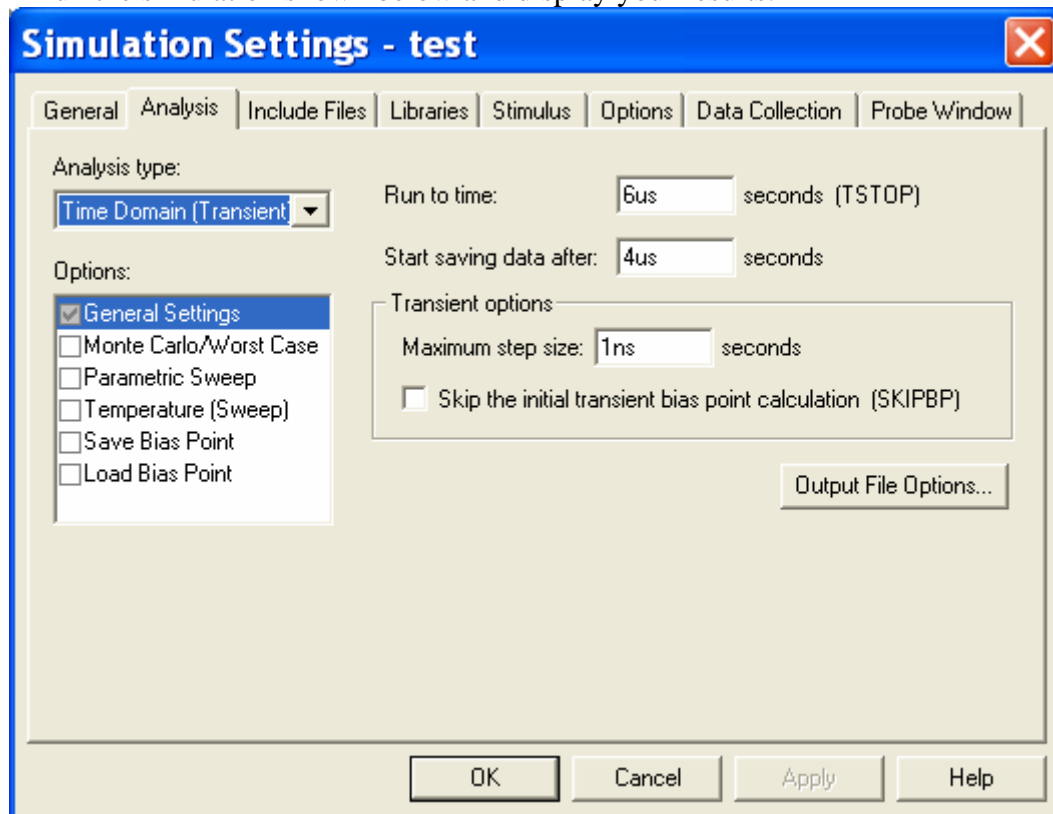
3. PSpice Simulated Experiments

PSpice can show us a lot about how the voltages look at the input and output ends of transmission lines. The simulation can, in effect, replace an experiment by giving us essentially the same results.

- a. To get some practice using PSpice, set up the following simulation using the parameters of your transmission line from problem 2. That is the source, line and load impedances should all be equal to the characteristic impedance of the line you determined in problem 2. The source voltage and frequency should be selected to obtain the current given in problem 2 on the line. Set the offset voltage to zero. Use the lossless line model and assume the length of the line is 40meters. Be careful to use the zero ground, since it is the only one that works with PSpice.



Run the simulation shown below and display your results.

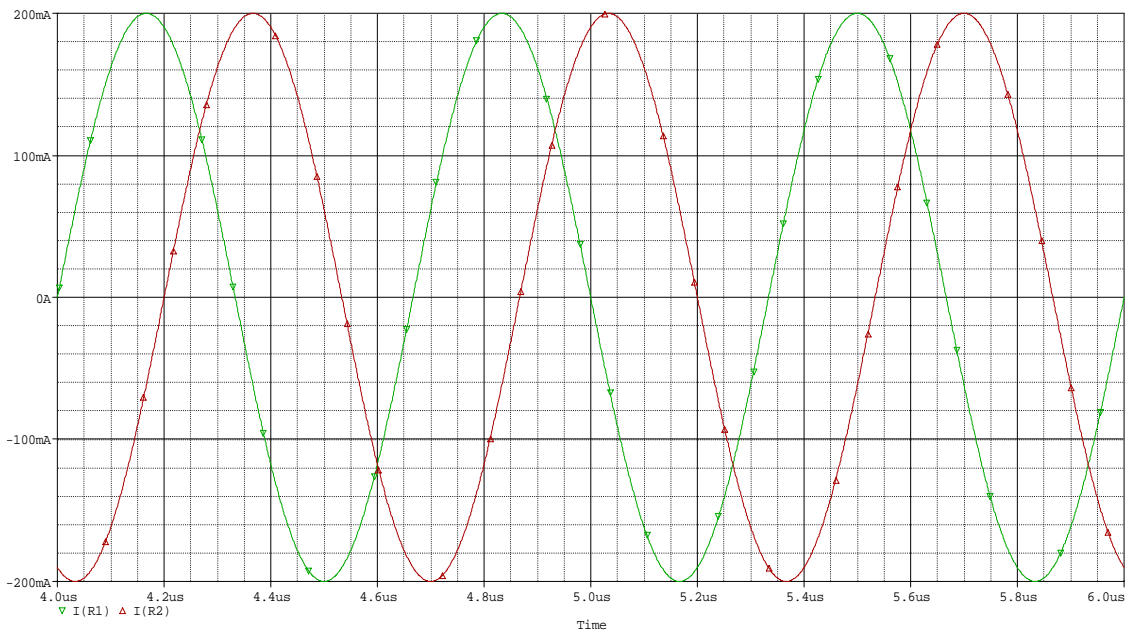
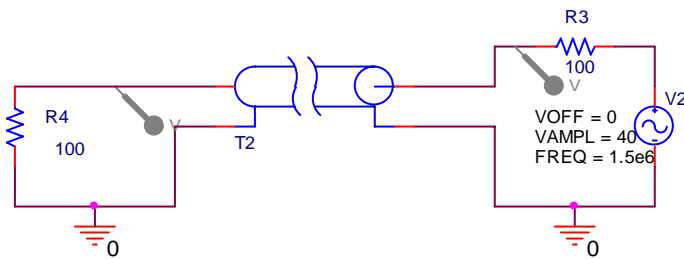
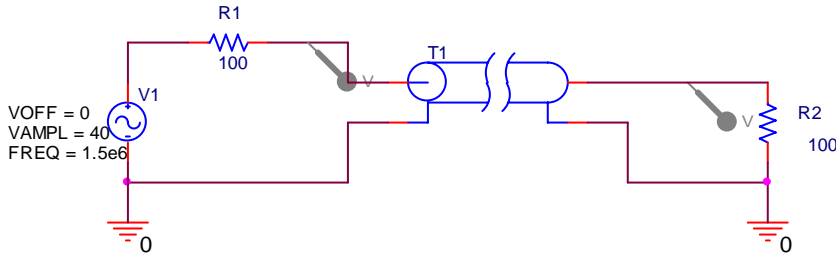


- b. Explain why your result makes sense. Be sure to thoroughly label your plot.

Simulation Results

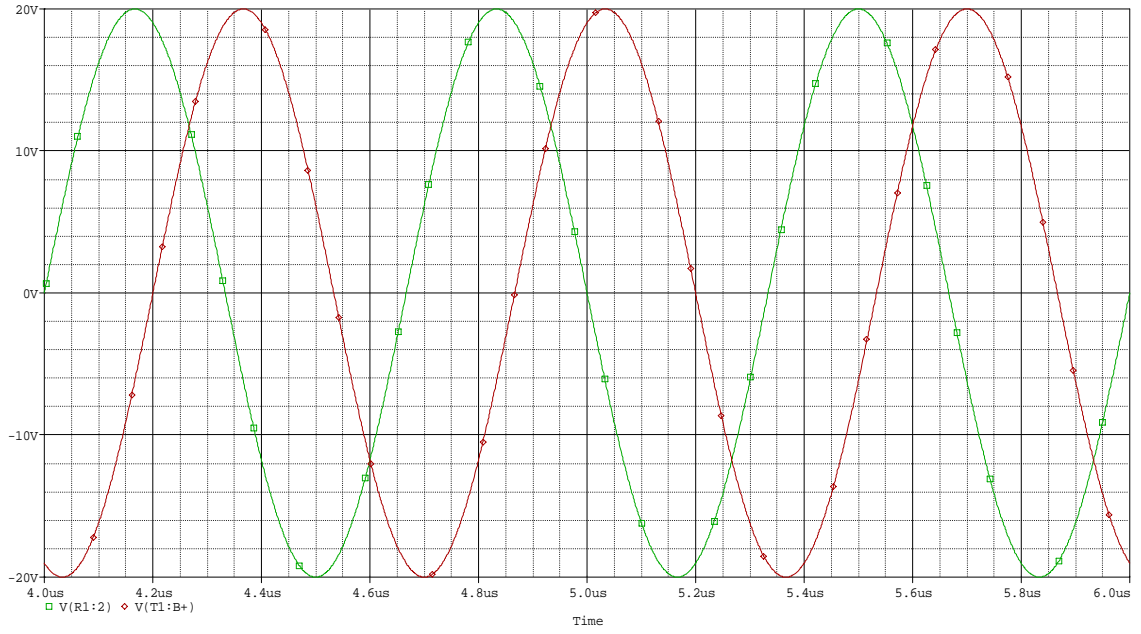
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Since the wave goes in the negative z direction, one can draw the circuit backwards, but this is not really necessary since the wave travels the same in either case. Both options are shown here.



The currents for the two cases sit right on top of one another, but only one set is shown here. The delay is $0.2\mu\text{s}$ as it should be. The voltages for completeness are shown on the next page. Note that the plots look the same except for the amplitude. This is what the calculated answers to the last problem show so our results are consistent.

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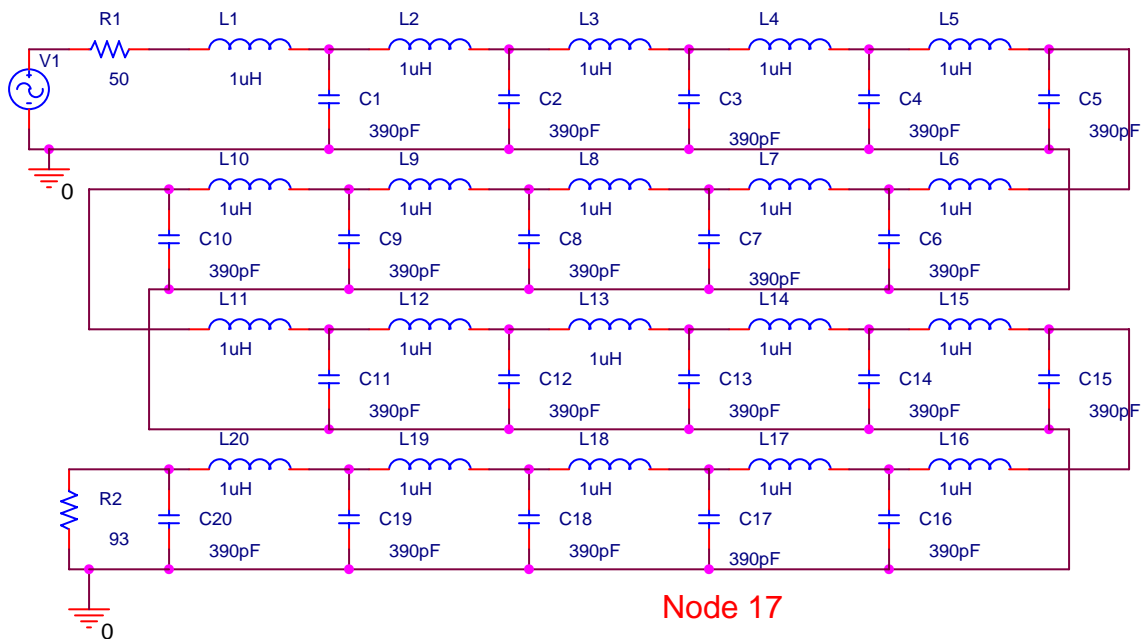


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4. Lumped Transmission Lines

A transmission line can be replaced by a series of lumped circuit elements, under certain conditions. In this exercise, you will compare the response of such a lumped model line with the coil of coax. Obtain a coil of coax, a 50 Ohm terminator and one of the lumped component transmission lines from a TA. Note: we will be analyzing the lumped configuration using PSpice in lecture. Thus, your measurements should be guided by what we obtain there.

- c. First, we will repeat an experiment from the first studio session at a different frequency. Put the 50 Ohm terminator across the output of the coaxial cable and simultaneously measure input and output signals on the oscilloscope. Set the input voltage at $1V_{P-P}$ with a frequency of $3.5MHz$. Measure the time delay between the signals. What else is different about the input and output voltages?
- d. Replace the coaxial cable with the lumped version and repeat the experiment.
- e. You should have observed that the lumped line behaves in a qualitatively similar manner to the spool of coax. Thus, it must represent a similar length of line. We want to determine the actual length of line. Each $L-C$ combination represents some equivalent length of line. Since there are 20 such combinations, we only need to figure out what length each combo represents and multiply it by 20. To understand better how the lumped circuit is configured, look at the diagram below, done with PSpice. (This diagram has a load impedance $R2$ of 93 Ohms, which is not what we are using here.) Note that the inductance and capacitance for each section is indicated. Given your knowledge of the actual capacitance and inductance per unit length for the RG58/U coaxial cable, approximately what length does each section represent?



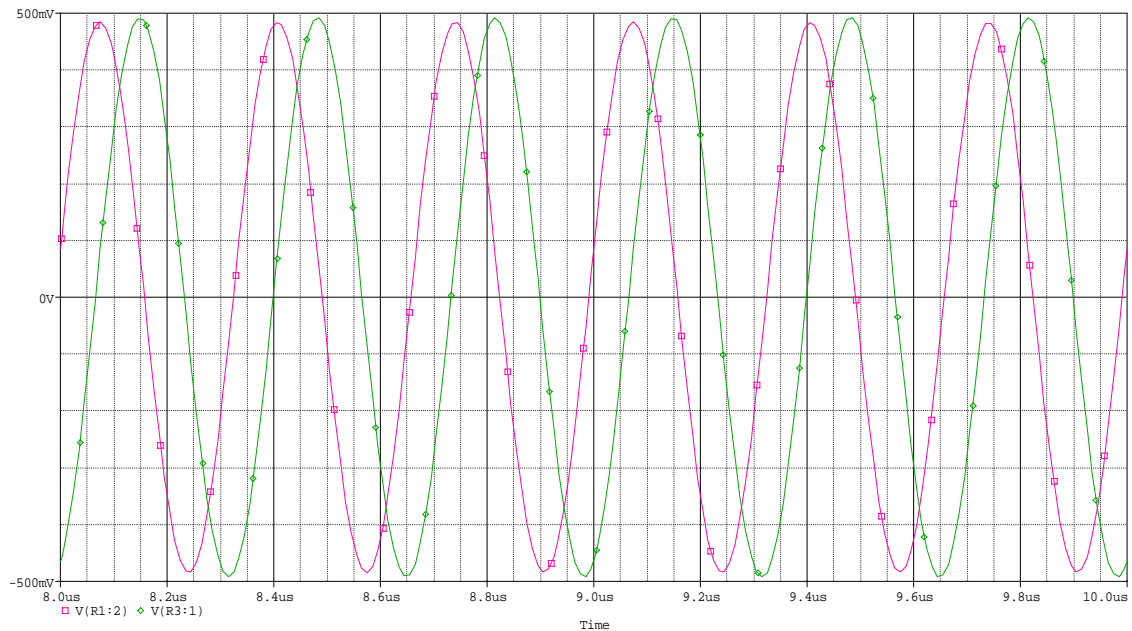
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- f. Remove the load resistor. Measure the voltage at the 17th node. Adjust the frequency somewhere between 3MHz and 4MHz until the voltage at the 17th node is a minimum. It should be somewhere around 100mV. Put the terminating resistor back on as a load. Measure the voltage at each of the nodes for the case where the lumped line is terminated with 50 Ohms. Remove the resistor and repeat for no load (open circuit). Where are the minimas and maximas located? (One minimum should be at node 17.) Plot your results for the measured voltages as a function of distance (using the distance between nodes that you determined above). Plot your results again but now in terms of wavelength rather than meters. (You can use the same plot, if you wish, and provide two sets of labels.)
- g. Show that the maximas and minimas are located where they should be in terms of wavelength.

Note that the lumped lines work slightly differently if different capacitors are used. Thus, please note the color of the capacitors on the board you are using. The correct answer to part d will depend on what capacitors are used on your board.

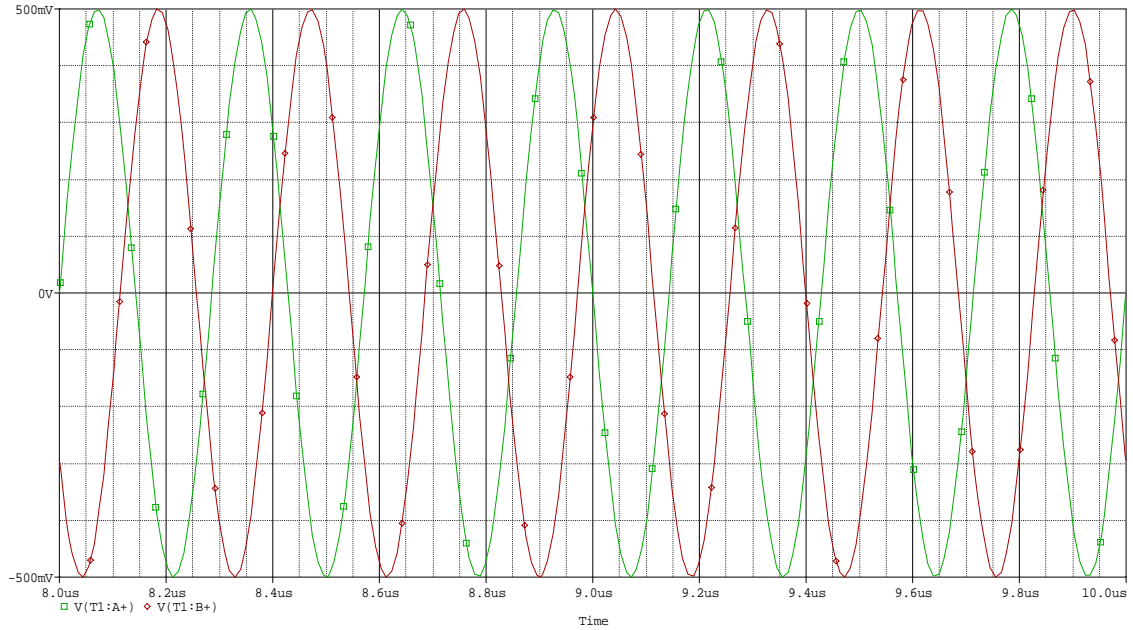
Here we will show the PSpice results. The experimental results should look very similar except that there is loss involved in the experimental results.

The PSpice results for the artificial lumped line (showing only input and output voltages) looks like



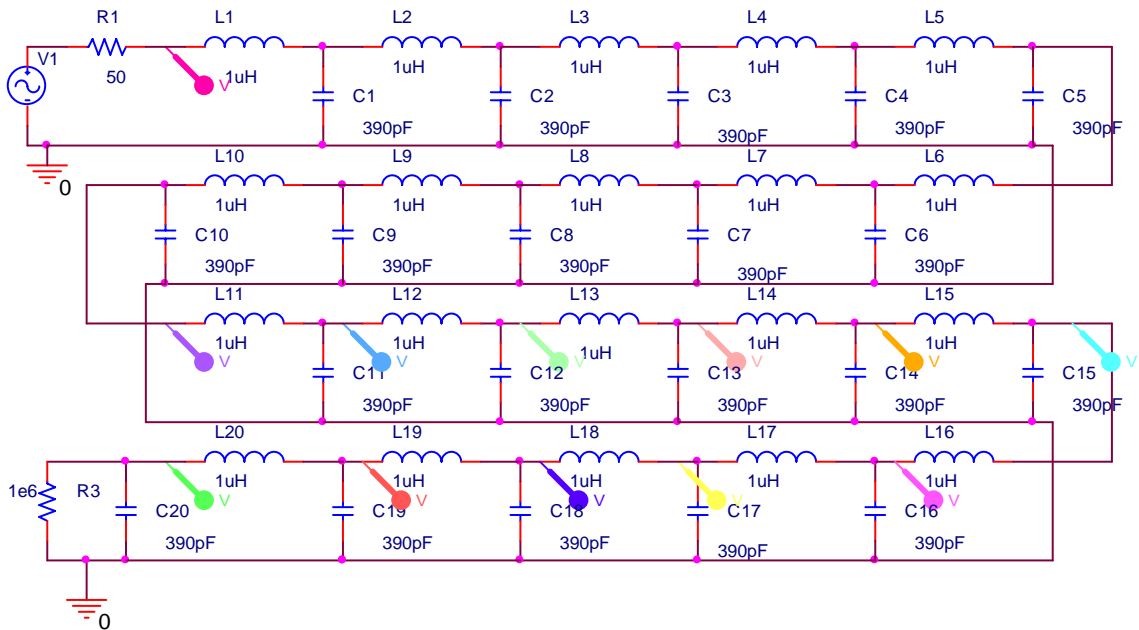
If we do the same thing for a lossless transmission line that is 80 meters long we get something similar. Note that they are not exactly the same since the parameters chosen for the lumped elements are not exactly correct. We have to use standard values.

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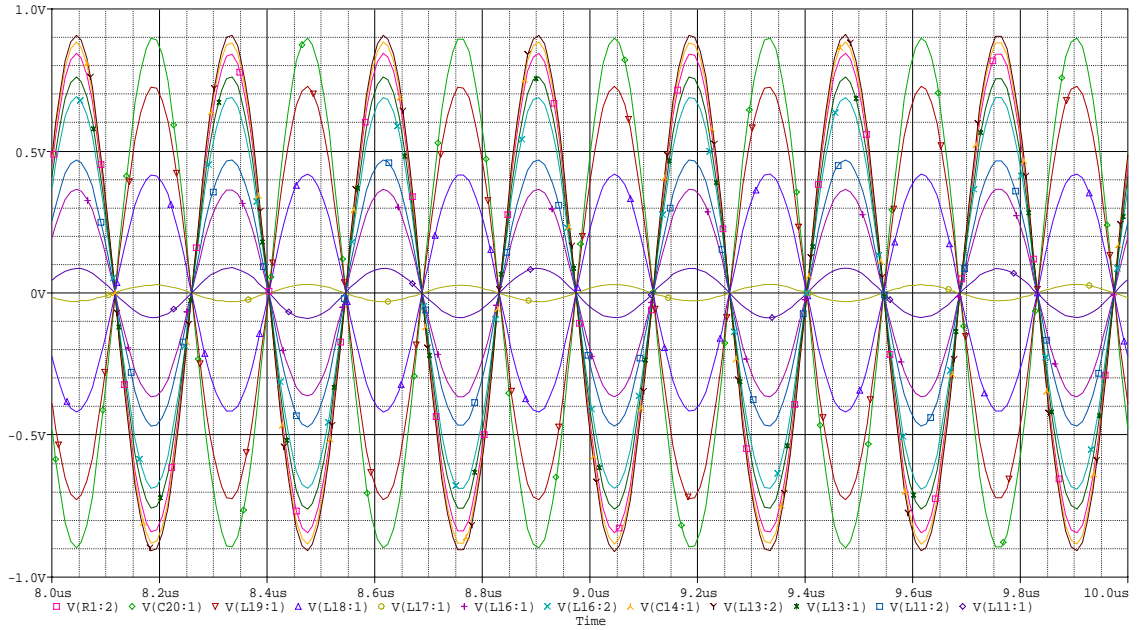
The lumped model already uses the almost correct values for the inductance and capacitance so each lumped element represents 4 meters of cable.

To do the case where the terminator is removed, replace it with the resistance of the scope. Then we put voltage markers at half of the nodes (this number is probably already too hard to read).

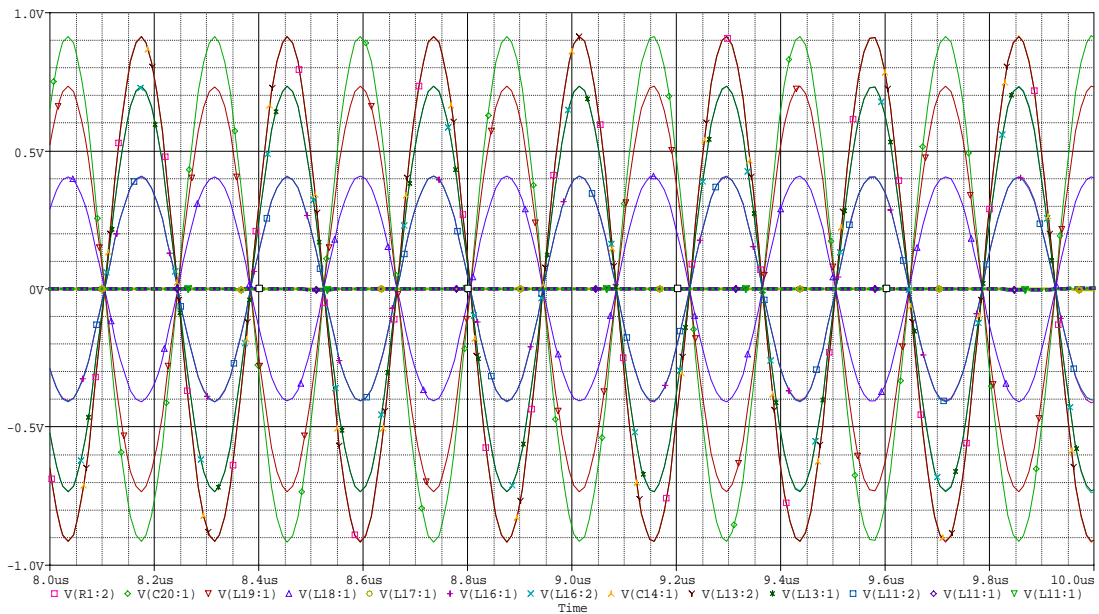


Then the voltages measured at each node will be

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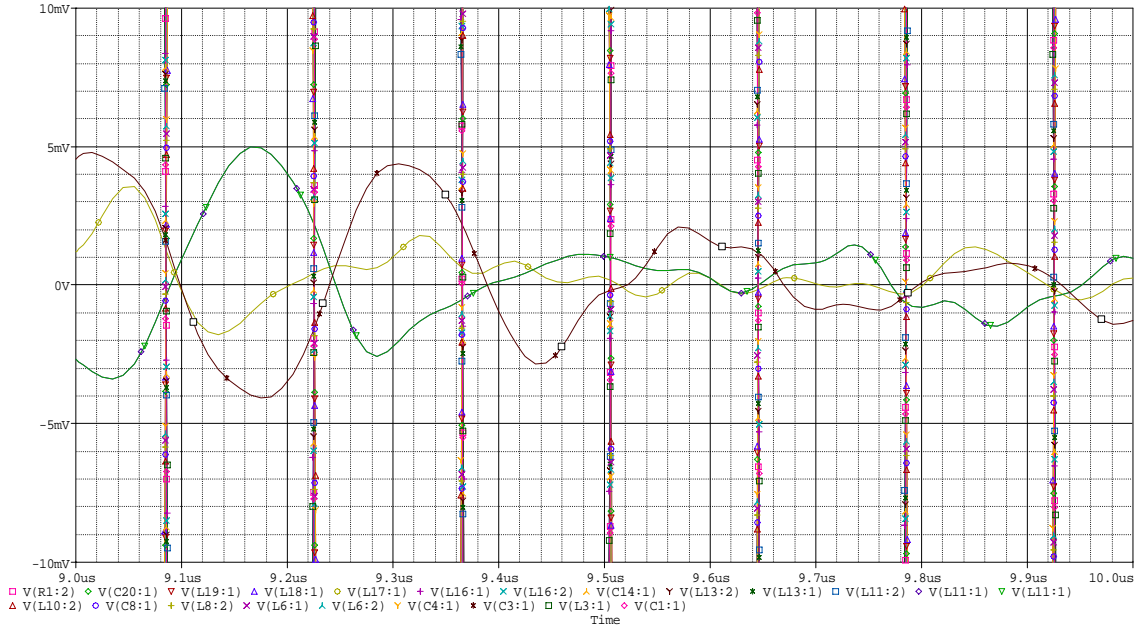
Note that no voltage has gone to zero, so that we should either raise or lower the frequency until we find a condition that gives a true minimum at node 17. The voltage at 17 is quite small already so we won't have to change it much.



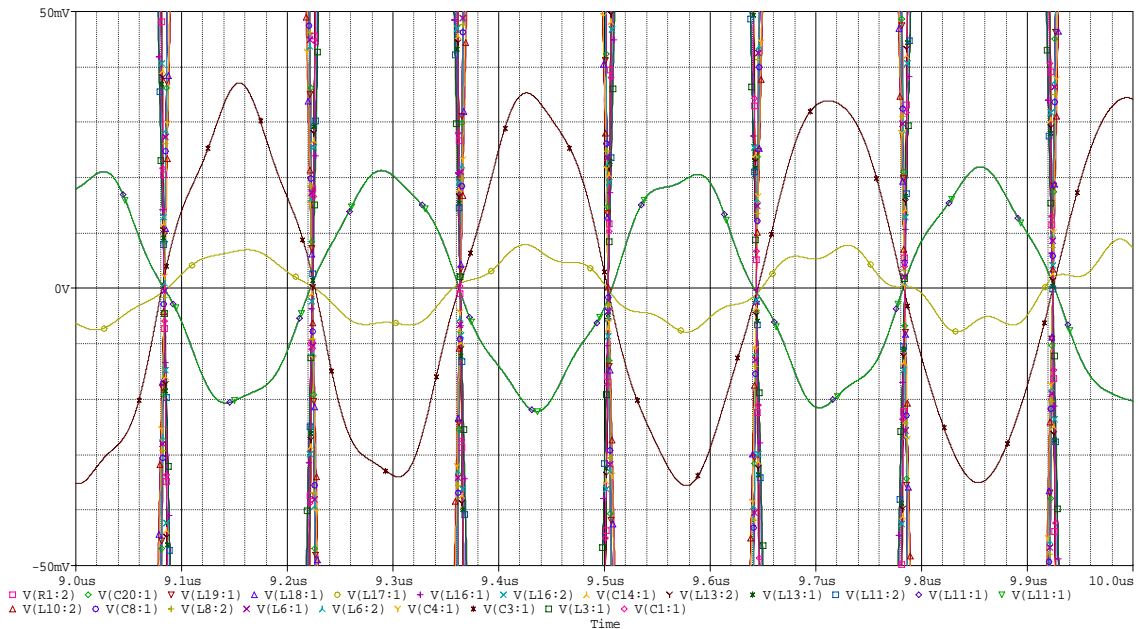
At 3.57MHz we get a true minimum at 17. This is 3 nodes from the end, so that 12 meters will be a quarter wavelength or the wavelength should be about 48 meters. This is not so accurate since the wavelength should be about 56 meters. Note however that the next minimum occurs at node 10 and $7 \times 4 = 28$ should be a half wavelength, which it is. There are some problems with a very small number of lumped elements (like 3) that go away once we use more nodes. Note that this is still not perfectly accurate since the next node (9) is also very small, so there is an error of about 4 meters in the length.

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Note that putting a voltage marker at all 21 nodes, we see that we get another minimum at node 3 as we would expect. The scale has been greatly blown up to be sure we are seeing the correct data points. With this scale, you can also see the errors in the calculation.



If we increase the resolution by reducing the step size, we will see that the data looks cleaner.



The frequency could be adjusted more, but this model is not perfect so we should not get carried away.