ECSE 2100
Fields & Waves I
Fall 2007
Homework #1 Solutions

1. For the following wave expressions, indicate if the wave is standing or traveling. If the wave is traveling, find the direction of propagation and the velocity.
   a. $\sin(377t + 0.3x)$
      Traveling in the $-x$ direction.
      $$u = \frac{\omega}{\beta} = \frac{377\ \text{rad}/\text{s}}{0.3\ \text{rad}/\text{m}} = 1256.67\ \text{m}/\text{s}$$
   b. $\cos\left(2\times10^{5}t - 6\times10^{-1}z\right)$
      Traveling in the $+z$ direction.
      $$u = \frac{\omega}{\beta} = \frac{2\times10^{5}\ \text{rad}/\text{s}}{6\times10^{-1}\ \text{rad}/\text{m}} = 3.3\times10^{5}\ \text{m}/\text{s}$$
   c. $\cos(120t)\sin(10x)$
      Standing wave.

2. Find the phasor representation of the following expressions:
   a. $v(t) = 8\cos\left(\omega t - \frac{2\pi}{3}\right)$
      $$\tilde{V} = 8e^{-j\frac{2\pi}{3}}$$
   b. $v(t) = 1.5\sin\left(\omega t + \frac{\pi}{3}\right)$
      $$v(t) = 1.5\cos\left(\omega t + \frac{\pi}{3} - \frac{\pi}{2}\right) = 1.5\cos\left(\omega t - \frac{\pi}{6}\right)$$
      $$\tilde{V} = 1.5e^{-j\frac{\pi}{6}}$$
   c. $v(t) = 6\sin\left(\omega t + \frac{2\pi}{3}\right) + 8\cos\left(\omega t - \frac{\pi}{3}\right)$
      $$v(t) = 6\cos\left(\omega t + \frac{2\pi}{3} - \frac{\pi}{2}\right) + 8\cos\left(\omega t - \frac{\pi}{3}\right) = 6\cos\left(\omega t + \frac{\pi}{6}\right) + 8\cos\left(\omega t - \frac{\pi}{3}\right)$$
      $$\tilde{V} = 6e^{j\frac{\pi}{6}} + 8e^{-j\frac{\pi}{3}} = 6\left[\cos\left(\frac{\pi}{6}\right) + j\sin\left(\frac{\pi}{6}\right)\right] + 8\left[\cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)\right]$$
      $$\tilde{V} = \left(3\sqrt{3} + 4\right) - j\left(3 - 4\sqrt{3}\right) = 9.196 - j3.928 = 10e^{-j0.404\,\text{rad}} = 10e^{-j23.13^\circ}$$

3. Find the time domain expression for the following phasors:
   a. $\tilde{V} = 3 + j1.5\text{V}$

   $\tilde{V} = 3.35e^{j26.6^\circ} = 3.35e^{j0.464\,\text{rad}}$

   $v(t) = 3.35\cos(\omega t + 26.6^\circ) = 3.35\cos(\omega t + 0.464\,\text{rad})$

   b. $\tilde{V} = 2.0e^{j\frac{\pi}{4}}\text{V}$

   $v(t) = 2\text{Re}\left\{e^{j\frac{\pi}{4}}e^{j\omega t}\right\} = 2\cos\left(\omega t + \frac{\pi}{4}\right)$

4. A wave is described by $v(t,z) = 35e^{-\alpha z}\sin(2\pi \times 10^8 t - 12\pi z)\text{V}$. Find the frequency, wavelength, and velocity. At $z = 2\text{m}$ the magnitude is measured as 1V. Find the attenuation constant.

$\omega = 2\pi \times 10^8 \text{ rad/s}$

$$f = \frac{\omega}{2\pi} = \frac{2\pi \times 10^8 \text{ rad/s}}{2\pi} = 10^8\,\text{Hz}$$

$\beta = 12\pi \text{ rad/m}$

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{12\pi \text{ rad/m}} = \frac{1}{6}\text{m}$$

$$u = \frac{\omega}{\beta} = \frac{2\pi \times 10^8 \text{ rad/s}}{12\pi \text{ rad/m}} = 1.667 \times 10^7 \text{ m/s}$$

$$35e^{-\alpha(2\text{m})} = 1\text{V} \longrightarrow \alpha = -\frac{1}{2}\ln\left(\frac{1}{35}\right)$$

$\alpha = 1.777 \text{ Np/m}$