## Homework 2

Due 31 January 2006 at 8:00 pm
We want to consider the same lossless transmission line used in different applications. The line to be analyzed is not necessarily realistic, but it does have reasonable parameters. Assume that it has a capacitance per unit length of $c=80 \mathrm{pF} / \mathrm{m}$ and an inductance per unit length of $l=0.2 \mu \mathrm{H} / \mathrm{m}$. The length of the line is 95meters.

## 1. Transmission Line Characteristics

Determine the following characteristics of the line and waves on the line.
The characteristic impedance $Z_{o}=$
ans: $Z_{o}=\sqrt{\frac{0.2 \times 10^{-6}}{80 \times 10^{-12}}}=50 \Omega$
The propagation velocity $u=$
ans: $u=\sqrt{\frac{1}{\left(.2 \times 10^{-6}\right)\left(80 \times 10^{-12}\right)}}=2.5 \times 10^{8} \mathrm{~m} / \mathrm{s}$
The delay time for the line $T=$
ans: $T=\frac{95}{2.5 \times 10^{8}}=380 \mathrm{~ns}$

## 2. PSpice Simulation of the Transmission Line with Sinusoidal Input Voltage

First we will consider how the line behaves with the sinusoidal input voltage $V(t)=10 \cos \omega t$ where the frequency $f=11.11 \mathrm{MHz}$. The voltage source is assumed to be matched to the line. You will have to figure out what resistance to use for the source term from your answer to the previous question. Set up one of the following representations (your choice).


## Homework 2



When setting up your simulation profile, use the following as your first guess. You will be asked to possibly adjust it when you address pulsed sources. Run this simulation and save the output plots for your report.

## Simulation Settings - test


ans: Both versions were used to simulate this. The plots are identical. The magnitude of the larger signal (at the input) is 6.59 V and that of the smaller signal (at the output) is 3.26 V .

## Homework 2



The time delay between the sine waves is (using the plot cursors) is 21.7ns. The delay should be 380ns minus an integral number of periods. One period is
$\frac{1}{f}=\frac{1}{11.11 \mathrm{MHz}}=90 n \mathrm{~ns}$. Four periods is 360ns, so the delay should be about 20ns. Within the accuracy of the simulation, this is correct. One can re-run the simulation with a smaller time step, but this is sufficient.

## 3. PSpice Simulation of the Transmission Line with a Pulsed Source

Set up the following configuration on the same page as your sinusoidal simulation. You can have more than one circuit analyzed by PSpice as long as you don't exceed the limitations of the student version. You can use either the lossless or lossy line model. Again, this is your choice. The delay used is not necessary for this case, because there are no capacitors or inductors in the circuit. However, we will use it to be better prepared for more general problems.


Now run your simulation again, which should simulate both the sinusoidal and pulsed cases simultaneously. You should observe pulses at the input and output ends of the line. It might be helpful to show the sinusoidal and pulsed cases on two separate plots, which PSpice will allow you to do. Now adjust the simulation profile until your plot demonstrates that you are observing all pulses whose voltages exceed 0.5 V . Change the

## Homework 2

simulation conditions only enough to show this. Your plot or plots should show both the sinusoidal case and the pulsed case. Carefully label all waves and pulses, noting where they are observed (input or output end) and their amplitude (since this is not always easy to read from the plots).
ans: First, we show the plot from the original simulation profile. Note that one cannot tell if there is another pulse following the three shown. Also, the pulse plots use somewhat thicker lines for clarity.

Output


Expanding the time of simulation to 2500 ns or 3000 ns shows that these are all the pulses. The magnitudes of the 3 pulses are $5 \mathrm{~V}, 3.333$, and -1.667 V , respectively. These values can be read from the plots or done more accurately with the plot cursor.


## Homework 2

## 4. Input Impedance of a Line

This question relates to the sinusoidal voltage.
a. Determine the input impedance of the line.
b. From the input impedance, determine the input power to the line.
c. From the input power to the line, determine the power delivered to the load.
d. From the power delivered to the load, determine the voltage observed at the load. This should allow you to explain the magnitude of the observed voltages.
e. Explain why the phase difference between the two voltages observed in your plots also makes sense.
ans: To obtain the input impedance, it is first necessary to find
$\beta=\frac{\omega}{u}=\frac{2 \pi 11.11 \times 10^{6}}{2.5 \times 10^{8}}=0.08888 \pi$ Then, the impedance is given by
$Z_{i n}=Z(-d)=Z_{o} \frac{Z_{L}+j Z_{o} \tan \beta d}{Z_{o}+j Z_{L} \tan \beta d}=91.5+j 23.8$
The voltage across the input will be given by $10 \frac{Z_{\text {in }}}{50+Z_{\text {in }}}=6.56+j 0.58$
The magnitude of this voltage is 6.59 which agrees with the input voltage determined using PSpice. The input power to the line is $P_{i n}=\frac{1}{2} \operatorname{Re}\left(\frac{V_{i n}{ }^{2}}{Z_{i n}{ }^{*}}\right)=0.222 \mathrm{~W}$. Since the line is lossless, the output power will be equal to the input power, from which we can determine $V_{\text {out. }} P_{\text {out }}=\frac{1}{2} \operatorname{Re}\left(\frac{V_{\text {out }}^{2}}{25}\right)=0.222 \mathrm{~W}$ so that $V_{\text {out }}=3.333 \mathrm{~V}$ which is, again, very close to the voltage observed at the output. Note that the disagreement is due almost entirely to the accuracy of the simulation. Running the simulation with 100 times better accuracy (which takes a few minutes on a T40) gives almost perfect agreement. Note also that the determination of the output voltage is less complicated than the input voltage since the load is a real resistance.

## 5. Pulses on a Transmission Line

This question relates to the pulsed voltage.
a. Determine the voltage input to the line from the voltage divider relationship that characterizes the initial line voltage.
b. Determine the reflection coefficients at the load and source end for both lines.
c. Generate the bounce diagram for both lines, showing the voltages observed until the level reaches less than 0.5 V .
d. Sketch the voltages observed at the input and output ends of the line.

The voltage input to the lines again involves the voltage divider action with source and input to the line. For transients/pulses, the initial input impedance is the characteristic impedance. This voltage is then $V_{i n}=10 \frac{50}{50+50}=5 \mathrm{~V}$. We will label this voltage $V^{+}$on the bounce diagram.

## Homework 2

The voltage reflection coefficient at the load is $\Gamma_{L}=\frac{Z_{L}-Z_{o}}{Z_{L}+Z_{o}}=\frac{25-50}{25+50}=-0.333$ while the voltage reflection coefficient at the source is $\Gamma_{L}=\frac{Z_{g}-Z_{o}}{Z_{g}+Z_{o}}=\frac{50-50}{50+50}=0$ Note that we expect no reflection from the matched source end.

The bounce diagram:


The voltages look just like the PSpice simulation. The first voltage at the input is 5 V . The only pulse seen at the output is $5-1.667=3.333 \mathrm{~V}$ and the last pulse seen at the input is 1.667 V , as predicted by this bounce diagram.
6. Comparison Between Modeling and Theory
a. Use your results for problem 2 to explain the voltages observed for the sinusoidal source. ans: Done above.
b. User your results for problem 3 to explain the voltages observed for the pulsed source. ans: Done above.

## 7. Experimental Verification

Set up both configurations using one of the reels of coaxial cable in the studio classroom. Be sure to terminate your cable at the scope with 25 Ohms. There are no 25 Ohm terminators, but this can easily be done. The RG58A/U cables do not have the same characteristics as the cable you analyzed. However, you should see the same qualitative results. Capture your results using the Agilent Intuilink software. Compare your experimental results with those from your analysis. That is, identify what is the same and what is different. For the pulses, you probably should terminate the short cable from the function generator to one of the scope channels. Try it both ways to see the effect. Otherwise the setup is the same as the experiment that many of you did the first day of class.

## Homework 2

ans: The cables used in the studio have 50 Ohm characteristic impedances, like the one in this problem. However, the velocity and lengths are only in the same ballpark. Thus, we should expect that the delays observed will be of the same order, but no exact agreement should be observed. Since the impedances are the same, the pulse heights should also be the same. However, the real lines are also a bit lossy and we need to add some short lines to the scope to observe the voltages. The short line changes the impedances observed a bit and the loss causes the pulses to round off. We should only look for qualitative agreement.

The experiment was set up as in the first class, but with the short line to the scope properly terminated with 50 Ohms. To obtain a 25 Ohm load, two 50 Ohm terminators were attached to a Tee since their parallel combination is 25 Ohms. The results for two of the reels of cable (2 and 10) look like the following. The source was not selected to be 10 V , but the default value of 20 mV was used. Using a terminated line to the scope at the input means that the voltage across the line is given by $V_{i n}=20 \frac{25}{50+25} m V=6.666 \mathrm{mV}$. This agrees remarkably well with the experiment.

and

## Homework 2



The second pulse should be about $2 / 3$ as large as the first one, but it is smaller. Observe that the pulse is rising and would achieve a greater value if it was longer. Eventually, it will make it to near 4.44 V but come up a bit short due to loss. The last pulse changes sign as expected and would be equal to the difference between the first two pulses except for loss and the rise time and one more effect. Again, since a terminated cable was used to connect the input to the source, the load reflection coefficient is not zero. Rather, it is again -0.333. Thus, some of the pulse reflects from the source end, which makes the third pulse observed smaller and produces a fourth pulse. This fourth pulse can be largely eliminated by not using a terminated line at the input, but a great deal of ringing will occur. This was discussed in the solution to the Spring 2005 HW 2 problem. It is a good idea to review the solution to that problem.

