Problem Solution #3

Problem 2

$$Z_0 = \sqrt{L'/C'} = 50\Omega$$
$$u = 1/\sqrt{L'C'} = 10^8 m/s$$
$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} = -\frac{2}{3}$$
$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{100 + j50 - 50}{100 + j50 + 50} = 0.447 e^{j26.6^o}$$
$$S = \frac{|\tilde{V}|_{\max}}{|\tilde{V}|_{\min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + 0.447}{1 - 0.447} \approx 2.62$$

$$Z_{in}(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = Z_o \left[\frac{1 + \Gamma e^{j2\beta z}}{1 - \Gamma e^{j2\beta z}} \right]$$

The input impedance at the input of the line at $z = -l$, which is given by

$$Z_{in}(-l) = Z_o \left[\frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}} \right] = Z_o \left(\frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \right)$$

For this problem $\beta = \dfrac{\omega}{u} \approx 1.885 rad/m$

$$Z_{in} = 19.54\angle 10.34^o \Omega$$
$$V_{in} = \frac{Z_{in}}{Z_{in} + Z_s} V_s = 0.664\angle 3.5^o \text{ V}$$
$$V(z) = V_0^+ (e^{-j\beta z} + \Gamma e^{j\beta z})$$

at input end, $z = -l = -1$, $\quad V_0^+ = V_{in} / (e^{j\beta} + \Gamma_L e^{-j\beta})$

at load end, $z = 0$, $\quad V_L = V_0^+ (1 + \Gamma_L) = 1.67\angle -104^o$ V

$$P = \frac{1}{2} \text{Re}\{VI^*\} = \frac{1}{2} \frac{|V|^2}{Z} \cos \angle Z$$

$P_L = 11.1 mW = P_{in} = 11.1 mW$ No power loss.