Homework 3

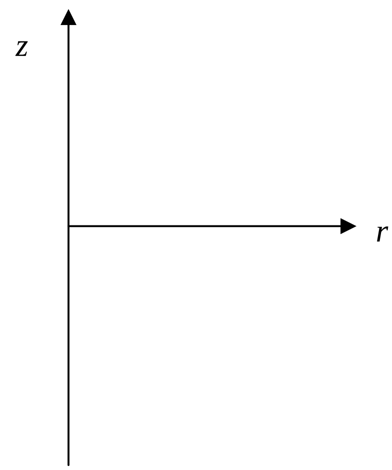
Due 1 March 2005

Some general advice on doing basic vector mathematics in Fields and Waves I.

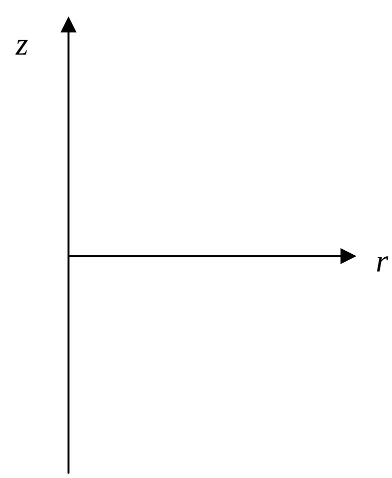
- Always draw as many views of the problem you are considering as you find necessary to fully understand the configuration.
- Always write out the full expression for the line, surface or volume element before attempting any integrals. Then, for line or surface integrals, take any dot products before doing anything else. This will usually reduce the problem to a more manageable scalar integral.
- Simplify the mathematical expressions before you try to solve them. Usually the math, once simplified, will be relatively simple.
- When doing surface integrals, it is usually possible to check one's answer against Maxwell's equations or, if the integrals are used to find a field expression, the differential forms of Maxwell's equations can be used to check answers.

1. Flux Integrals

a. The electric field due to a point charge is given by $\vec{E} = \dfrac{q}{4\pi\varepsilon_o R^2}\hat{a}_R$ where $\vec{R}$ is the vector from the charge to the observation point. Assume that the charge is located at the origin of a cylindrical coordinate system $(r,\phi,z) = (0,0,0)$. Determine the total electric flux $\int \vec{E}\cdot d\vec{S}$ passing through the surface $z = d$. Begin by drawing a diagram showing the point charge and the surface in the *r-z* plane below. Also indicate the value of $d\vec{S}$. Recall that the $z = d$ surface goes from $r = 0$ *to infinity.*

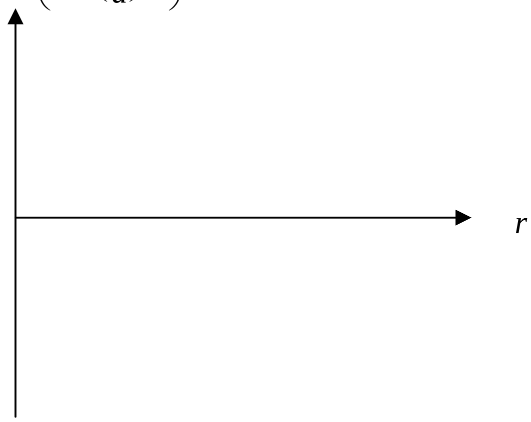


b. The magnetic field outside of a long straight wire of radius $r = a$, carrying a current I is given by $\vec{B} = \dfrac{\mu_o I}{2\pi r}\hat{a}_\phi$. Determine the total magnetic flux passing through the surface defined by $a \le r \le b$ and $0 \le z \le d$. Begin by drawing a picture of the surface in the *r-z* plane. Also indicate the value of $d\vec{S}$. What is the solution for the specific case where $b \to \infty$?



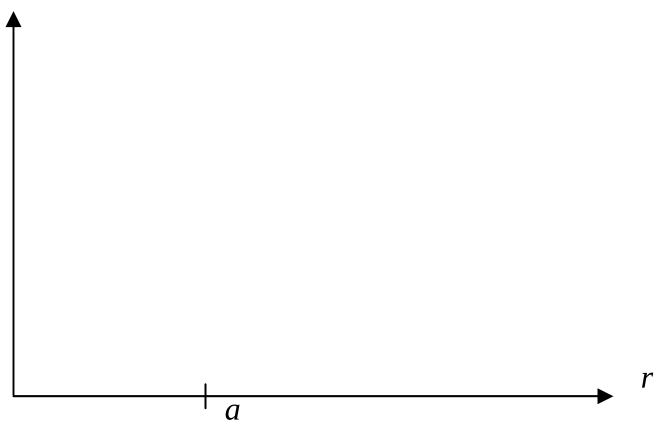
2. The Electric Field due to a Volume Charge Distribution

a. Assume that there is a volume charge distribution in the spherical region $0 \le r \le a$ given by $\rho = \rho_o \left(1 - \left(\frac{r}{a} \right)^{33} \right)$. First, plot this expression as a function of *r*.



Next, determine the total amount of charge in this distribution. Repeat for a uniform distribution in the same region, that is, for $\rho = \rho_o$. Compare your results for the two cases.

b. Using Gauss' Law in integral form, determine the electric field $\vec{E}$ for all values of radius for both charge distributions. Plot the magnitude of the electric field as a function of radius $|\vec{E}|$.



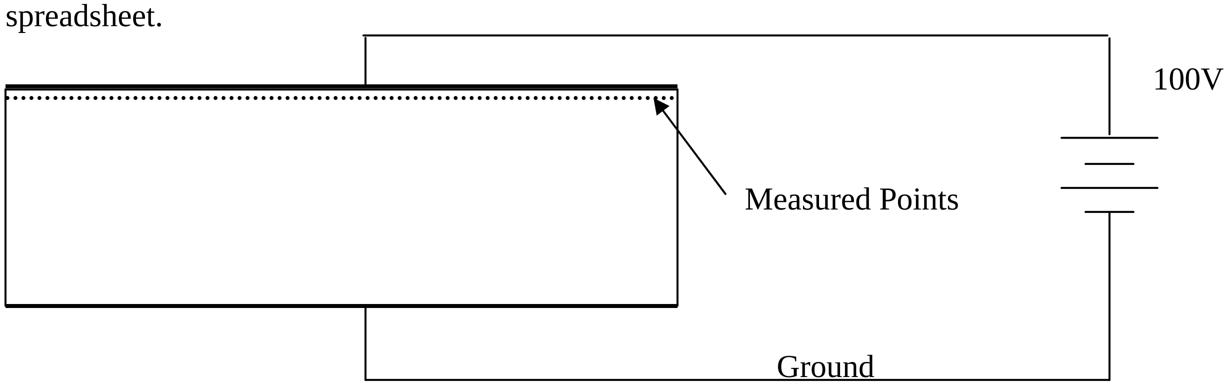
c. Use Gauss' Law in differential form to check your answer for both cases.

3. Electric Scalar Potential

For both cases in problem 2, find the electric scalar potential as a function of position $V = V(r)$ and then also evaluate the potential at the origin $V = V(0)$.

4. Charge on a Capacitor Plate

A parallel plate capacitor is connected to a 100V DC voltage source, as shown. Little is known about how the capacitor is constructed so we do not know enough to calculate the capacitance from first principles. However, somehow, we are able to measure the voltage at an array of points located *1mm* inside the top surface of the capacitor. The measured voltages are given in the Excel spreadsheet *vdata.xls* found next to this assignment on the *Handout* webpage. Each plate of the capacitor is *30cm* by *30cm.* The voltages are measured at the edge points and points every *cm* to form the *31x31* grid given in the spreadsheet.



a. Determine the average value of the electric field in the region between the measured points and the top plate.
b. Assume the dielectric constant in the region where the voltage is measured is $\varepsilon = \varepsilon_r \varepsilon_o = 4\varepsilon_o$. Determine the average value of the electric flux density.
c. Determine the total charge on the top plate.
d. Find the capacitance.