Homework #3
Due 28 February

Before beginning this homework assignment, read over HW3 from Spring 2006, especially the discussion in the first 3 pages. Also be sure that you are keeping up with the reading posted on the course handout page. Of course, reviewing the lecture slides is also a good idea.

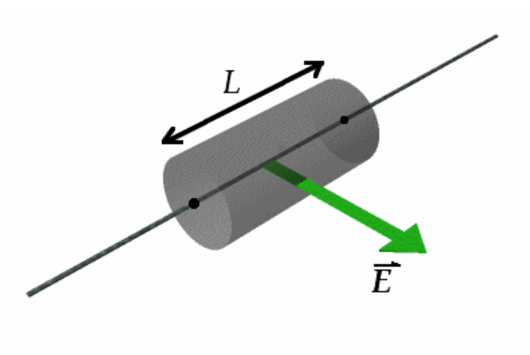
1. **Gauss' Law Methodology** This problem was really done in class and in the lecture notes. Your purpose here is to fully reproduce the solution while simultaneously developing the general set of steps necessary (written in your own words) to solve such problems. Assume that we have a uniform cylinder of charge with density $\rho = \rho_o$ in the region $0 \le r \le a$. There are no other charges in this problem.

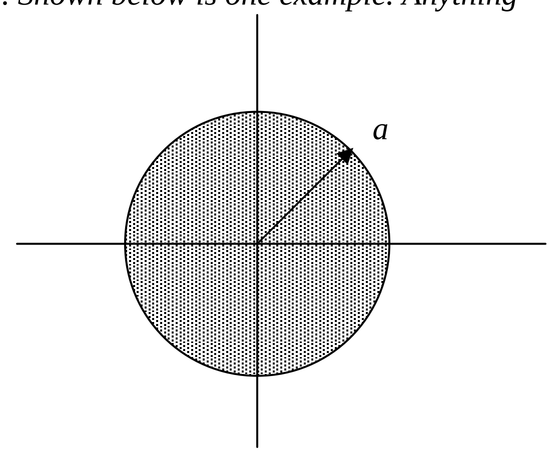
    a. What coordinate system should you use to solve this problem?

    Cylindrical

    b. Sketch the charge distribution in two and three dimensions.

There are various ways of drawing a 3D cylinder. Shown below is one example. Anything similar is fine





    c. Solve for the electric field $\vec{E}(\vec{r})$ for all values of *r*. Remember that the answer to this question is a vector function, so be sure you express it as such.

The first step is to note, from symmetry, the simplified form of the electric field. For this case, since the source only depends on the cylindrical radius, we also know that the electric field is most simply written as $\vec{E}(\vec{r}) = \hat{a}_r E_r(r)$. *Then, we choose a Gaussian surface that is a closed cylinder of radius r and length L, as shown in the 3D image above. To keep things simple, we will show the 2D plot with dashed lines where the Gaussian surface can go (see below). Note that we have to consider both the case where r is less than a and where r is greater than a (i.e. for all r).*

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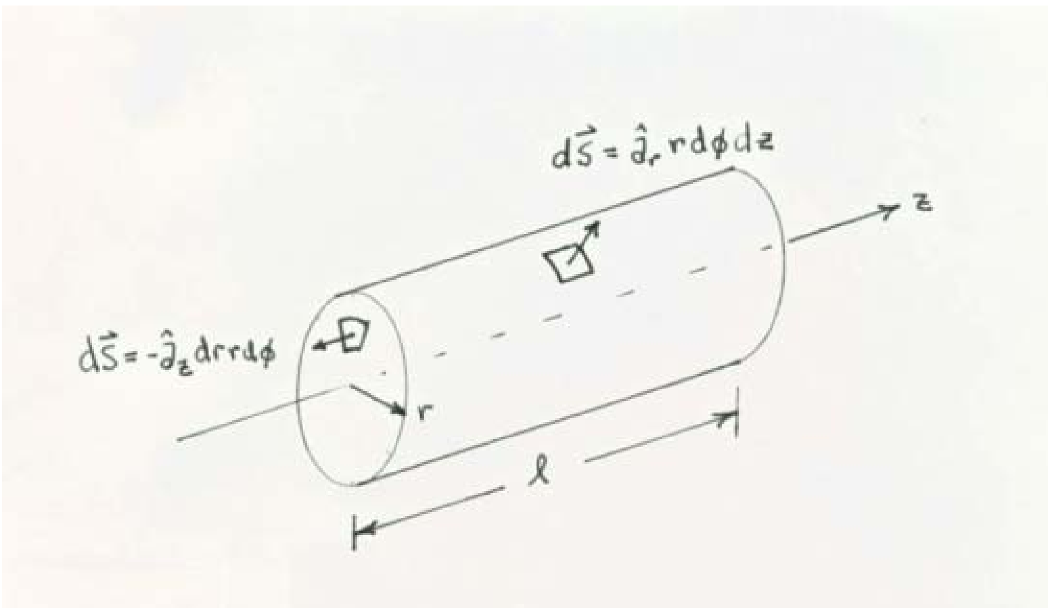
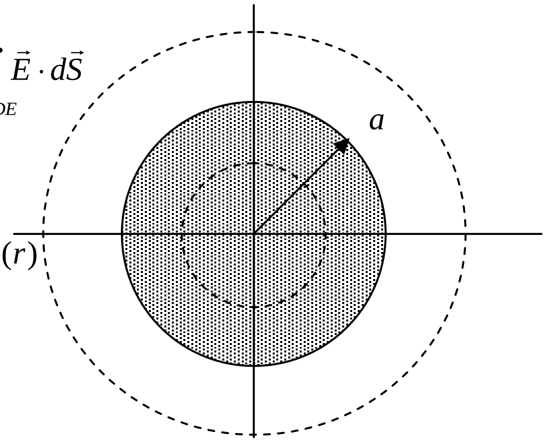
From the class notes, page II-8, we have a helpful picture showing the surface elements on the cylindrical Gaussian surface. Note that the surfaces on the end caps of the cylinder are perpendicular to the direction of the electric field and, thus, there is no contribution to the flux integral from the ends.



Then we have for the left hand side of Gauss' Law

$$\oint \vec{E}\cdot d\vec{S} = \int_{LEFT-END} \vec{E}\cdot d\vec{S} + \int_{RIGHT-END} \vec{E}\cdot d\vec{S} + \int_{SIDE} \vec{E}\cdot d\vec{S} = \int_{SIDE} \vec{E}\cdot d\vec{S}$$



$$\oint \vec{E}\cdot d\vec{S} = \int_{SIDE} \vec{E}\cdot d\vec{S} = \int E_r(r)\hat{a}_r \cdot \hat{a}_r r d\phi dz = 2\pi L r E_r(r)$$

For the right hand side of Gauss' Law,

the answer depends on whether we are inside our outside the charge.

For inside the charge, $\frac{1}{\varepsilon_o}\int \rho dv = \frac{1}{\varepsilon_o} 2\pi L \int_0^r \rho_o r dr = \frac{1}{\varepsilon_o}\pi r^2 L\rho_o$ *while for outside the charge*

$\frac{1}{\varepsilon_o}\int \rho dv = \frac{1}{\varepsilon_o} 2\pi L \int_0^a \rho_o r dr = \frac{1}{\varepsilon_o}\pi a^2 L\rho_o$. *Setting the two sides equal to one another, we obtain the solution for the electric field both inside and outside the charge.*

$E_r(r) = \frac{\rho_o r}{2\varepsilon_o}$ *for r<a and* $E_r(r) = \frac{\rho_o a^2}{2\varepsilon_o r}$ *for r>a.*

d. Check your answer for $\vec{E}(\vec{r})$ by evaluating $\nabla\cdot\vec{E}$ for all values of *r*. That is, show that $\nabla\cdot\vec{E} = \rho/\varepsilon_o$.

Checking the answer is straight forward, since it only requires direct application of the divergence in cylindrical coordinates. There are usually three terms for the divergence, but, in this case, we only need to consider one since there is only an r-directed component $E_r(r)$ *for the electric field.*

$\nabla\cdot\vec{E} = \frac{1}{r}\frac{\partial}{\partial r} rE_r(r) = \frac{1}{r}\frac{\partial}{\partial r} r\frac{\rho_o a^2}{2\varepsilon_o r} = 0$ *outside the charge, as it should*

$\nabla\cdot\vec{E} = \frac{1}{r}\frac{\partial}{\partial r} rE_r(r) = \frac{1}{r}\frac{\partial}{\partial r} r\frac{\rho_o r}{2\varepsilon_o} = \frac{1}{r}\frac{\rho_o 2r}{2\varepsilon_o} = \frac{\rho_o}{\varepsilon_o}$ *inside the charge, as it should.*

e. Assuming that the voltage is referenced to zero at $r = b$ where $b > a$, find the voltage (also known as the electric potential) $V(\vec{r})$ for all values of r. Remember that the answer to this question is a scalar function, so be sure that you express it as such.

$V(r) - V(b) = -\int_b^r E_r dr = -\frac{\rho_o a^2}{2\varepsilon_o}\int_b^r \frac{1}{r}dr = \frac{\rho_o a^2}{2\varepsilon_o}\ln\frac{b}{r}$ *for r>a. Also V(b) = 0. At r = a, we have* $V(a) = \frac{\rho_o a^2}{2\varepsilon_o}\ln\frac{b}{a}$ *which we can use to find the voltage inside the charge, using this as the reference. Note that it is not a zero reference. Then*

$$V(r) - V(a) = -\int_a^r E_r dr = -\frac{\rho_o}{2\varepsilon_o}\int_a^r r dr + \frac{\rho_o a^2}{2\varepsilon_o}\ln\frac{b}{a} = -\frac{\rho_o}{4\varepsilon_o}\left(r^2 - a^2\right) + \frac{\rho_o a^2}{2\varepsilon_o}\ln\frac{b}{a}$$

Note that this expression is found on slide 25 of Lecture 10.

f. Check your answer for $V(\vec{r})$ by evaluating $\nabla^2 V(\vec{r})$ for all values of r. That is, show that $\nabla^2 V(\vec{r}) = -\rho/\varepsilon_o$.

$\nabla^2 V(\vec{r}) = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} V(\vec{r}) = \frac{\rho_o a^2}{2\varepsilon_o}\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}(\ln b - \ln r) = -\frac{\rho_o a^2}{2\varepsilon_o}\frac{1}{r}\frac{\partial}{\partial r} r \frac{1}{r} = 0$ *outside the charge. Inside the charge,*

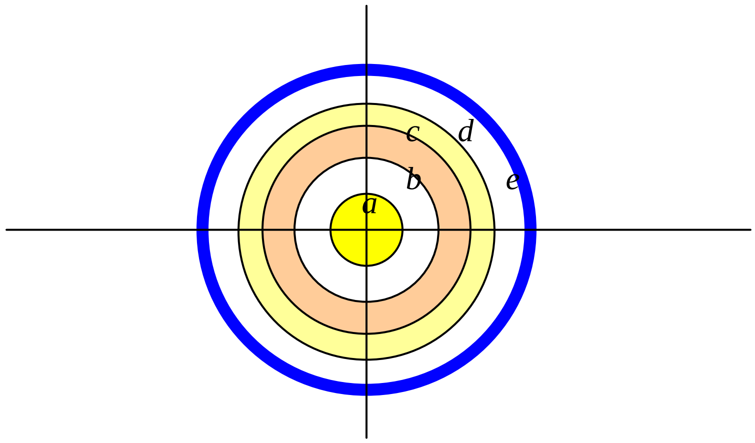
$\nabla^2 V(\vec{r}) = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} V(\vec{r}) = -\frac{\rho_o}{4\varepsilon_o}\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} r^2 = -\frac{\rho_o}{4\varepsilon_o}\frac{1}{r}\frac{\partial}{\partial r} 2r^2 = -\frac{\rho_o}{4\varepsilon_o}\frac{1}{r}4r = -\frac{\rho_o}{\varepsilon_o}$ *as it should.*

g. Read over the discussion in the handout on Gauss' Law. Also, the reading in the textbook and class notes on Gauss' Law. In the handout, there is a list of steps to follow to use Gauss' Law to solve for fields. Put these steps in your own words and expand them to include instructions on how to find the voltage function and to check your solutions.

Many different sets of instructions are acceptable here. It is only necessary that the steps be clear and complete. Please check the stops listed in the handout. After that one must be sure to

1. *identify where the zero reference is for the potential and then*
2. *integrate* $V(r_2) - V(r_1) = -\int_{r_1}^{r_2} E_r dr$ *, for example (this is only an example, since not all systems are cylindrical or spherical. Note that the appropriate form for the electric field must be used in each region*
3. *For completeness, one should also check one's answers as shown above with the differential form.*

2. **Application of Gauss' Law Methodology** The following charge distribution exists in a cylindrical system: $\rho = \rho_{1o}$ in the region $b \le r \le c$ and $\rho = \rho_{2o}$ in the region $c \le r \le d$. The voltage is referenced to zero at $r = e$. There are no other charges in this problem.



   a. Determine the total charge per unit length.

$$\int \rho dv = 2\pi L \int_b^c \rho_{1o} r dr + 2\pi L \int_c^d \rho_{2o} r dr = \pi\left(c^2 - b^2\right) L \rho_{1o} + \pi\left(d^2 - c^2\right) L \rho_{2o}$$

so that the charge per unit length is $\pi\left(c^2 - b^2\right)\rho_{1o} + \pi\left(d^2 - c^2\right)\rho_{2o}$

   b. Now assume that the outer charge density is negative and that the total charge per unit length is zero. Find the relationship between the inner and outer charge densities. That is solve for one in terms of the other.

For the total charge to be zero $\pi\left(c^2 - b^2\right)\rho_{1o} + \pi\left(d^2 - c^2\right)\rho_{2o} = 0$ *and*

$$\rho_{2o} = -\frac{\pi\left(c^2 - b^2\right)}{\pi\left(d^2 - c^2\right)}\rho_{1o} = -\frac{\left(c^2 - b^2\right)}{\left(d^2 - c^2\right)}\rho_{1o}$$

For the remainder of this problem, we will assume that the total charge per unit length is equal to zero <u>and</u>, for simplicity, that $b = 2a$, $c = 4a$, $d = 5a$, and $e = 6a$. Thus, the only geometric parameter that should appear in your solutions should be *a*.

$$\rho_{2o} = -\frac{\left(4^2 - 2^2\right)}{\left(5^2 - 4^2\right)}\rho_{1o} = -\frac{4}{3}\rho_{1o}$$

   c. Solve for the electric field $\vec{E}(\vec{r})$ for all values of *r*.

LHS of Gauss' Law is the same as in problem 1. $\oint \vec{E} \cdot d\vec{S} = 2\pi L r E_r(r)$

For the RHS, things are slightly different.

For the region 0<r<b, $\frac{1}{\varepsilon_o}\int \rho dv = \frac{1}{\varepsilon_o} 2\pi L \int_0^r 0 r dr = 0$.

For b<r<c, $\frac{1}{\varepsilon_o}\int \rho dv = \frac{1}{\varepsilon_o} 2\pi L \int_b^r \rho_{1o} r dr = \frac{1}{\varepsilon_o}\pi\left(r^2 - b^2\right)L\rho_{1o}$.

For d>r>c,

$$\frac{1}{\varepsilon_o}\int \rho dv = \frac{1}{\varepsilon_o}\pi\left(c^2 - b^2\right)L\rho_{1o} + \frac{1}{\varepsilon_o} 2\pi L \int_c^r \rho_{2o} r dr = \frac{1}{\varepsilon_o}\pi\left(c^2 - b^2\right)L\rho_{1o} + \frac{1}{\varepsilon_o}\pi\left(r^2 - c^2\right)L\rho_{2o}.$$

For e>r>d, $\frac{1}{\varepsilon_o}\int \rho dv = 0$ *since the total charge per unit length is zero.*

Setting the two sides of Gauss' Law equal, we have $E_r(r) = 0$ *for 0<r<b & d<r, since no charge is enclosed in these regions.*

For b<r<c $\oint \vec{E} \cdot d\vec{S} = 2\pi L r E_r(r) = \frac{1}{\varepsilon_o}\int \rho dv = \frac{1}{\varepsilon_o}\pi\left(r^2 - b^2\right)L\rho_{1o}$ *so that*

$$E_r(r) = \frac{1}{\varepsilon_o}\frac{\left(r^2 - b^2\right)}{2r}\rho_{1o}$$

For c<r<d $\oint \vec{E} \cdot d\vec{S} = 2\pi L r E_r(r) = \frac{1}{\varepsilon_o}\int \rho dv = \frac{1}{\varepsilon_o}\pi\left(c^2 - b^2\right)L\rho_{1o} + \frac{1}{\varepsilon_o}\pi\left(r^2 - c^2\right)L\rho_{2o}$ *so that* $E_r(r) = \frac{1}{\varepsilon_o}\frac{\left(c^2 - b^2\right)}{2r}\rho_{1o} + \frac{1}{\varepsilon_o}\frac{\left(r^2 - c^2\right)}{2r}\rho_{2o}$

d. Check your answer for the previous question

Trivially, for 0<r<b & d<r, $\nabla \cdot \vec{E} = \frac{1}{r}\frac{\partial}{\partial r} r E_r(r) = \frac{1}{r}\frac{\partial}{\partial r} r 0 = 0$ *as it should since there is no charge there.*

For b<r<c,

$$\nabla \cdot \vec{E} = \frac{1}{r}\frac{\partial}{\partial r} r E_r(r) = \frac{1}{r}\frac{\partial}{\partial r} r \frac{1}{\varepsilon_o}\frac{\left(r^2 - b^2\right)}{2r}\rho_{1o} = \frac{1}{r}\frac{\partial}{\partial r}\frac{1}{\varepsilon_o}\frac{\left(r^2 - b^2\right)}{2}\rho_{1o} = \frac{1}{r}\frac{1}{\varepsilon_o}\frac{2r}{2}\rho_{1o} = \frac{\rho_{1o}}{\varepsilon_o}$$

as it should.

For b<r<c, $\nabla\cdot\vec{E}=\frac{1}{r}\frac{\partial}{\partial r}rE_r(r)=\frac{1}{r}\frac{\partial}{\partial r}r\left(\frac{1}{\varepsilon_o}\frac{\left(c^2-b^2\right)}{2r}\rho_{1o}+\frac{1}{\varepsilon_o}\frac{\left(r^2-c^2\right)}{2r}\rho_{2o}\right)$

$\nabla\cdot\vec{E}=\frac{1}{r}\frac{\partial}{\partial r}rE_r(r)=\frac{1}{r}\frac{\partial}{\partial r}\left(\frac{1}{\varepsilon_o}\frac{\left(c^2-b^2\right)}{2}\rho_{1o}+\frac{1}{\varepsilon_o}\frac{\left(r^2-c^2\right)}{2}\rho_{2o}\right)=\frac{1}{r}\left(\frac{1}{\varepsilon_o}\frac{2r}{2}\rho_{2o}\right)=\frac{\rho_{2o}}{\varepsilon_o}$ *as it should.*

e. Find the voltage $V(\vec{r})$ for all values of r.

Since the zero reference is at r = e and there is no E field in the region r>d, the voltage at r = d is also zero. We will use this as the reference then.

For c<r<d, $V(r)-V(d)=-\int_d^r E_r dr=-\int_d^r\left(\frac{1}{\varepsilon_o}\frac{\left(c^2-b^2\right)}{2r}\rho_{1o}+\frac{1}{\varepsilon_o}\frac{\left(r^2-c^2\right)}{2r}\rho_{2o}\right)dr$

For simplicity, note that there are two kinds of integrals here. $\int\frac{1}{r}dr=\ln r$ *and* $\int r\,dr=\frac{r^2}{2}$. *Thus,* $V(r)=\left(\frac{1}{\varepsilon_o}\frac{\left(c^2-b^2\right)}{2}\rho_{1o}-\frac{1}{\varepsilon_o}\frac{\left(c^2\right)}{2}\rho_{2o}\right)\ln\frac{d}{r}+\frac{1}{\varepsilon_o}\frac{\left(r^2-d^2\right)}{4}\rho_{2o}$. *For the next region, we need this at r = c:*

$$V(c)=\left(\frac{1}{\varepsilon_o}\frac{\left(c^2-b^2\right)}{2}\rho_{1o}-\frac{1}{\varepsilon_o}\frac{\left(c^2\right)}{2}\rho_{2o}\right)\ln\frac{d}{c}+\frac{1}{\varepsilon_o}\frac{\left(c^2-d^2\right)}{4}\rho_{2o}.$$

Then, for b<r<c, $V(r)-V(c)=-\int_c^r E_r dr=-\int_c^r\left(\frac{1}{\varepsilon_o}\frac{\left(r^2-b^2\right)}{2r}\rho_{1o}\right)dr$ *and*

$V(r)-V(c)=-\int_c^r E_r dr=-\frac{\rho_{1o}}{2\varepsilon_o}\left(\frac{\left(r^2-c^2\right)}{2}+b^2\ln\frac{c}{r}\right)$. *Finally, for the region 0<r<b, the voltage is a constant*

$$V(r)=-\frac{\rho_{1o}}{2\varepsilon_o}\left(\frac{\left(b^2-c^2\right)}{2}+b^2\ln\frac{c}{b}\right)-\left(\frac{1}{\varepsilon_o}\frac{\left(c^2-b^2\right)}{2}\rho_{1o}-\frac{1}{\varepsilon_o}\frac{\left(c^2\right)}{2}\rho_{2o}\right)\ln\frac{d}{c}+\frac{1}{\varepsilon_o}\frac{\left(c^2-d^2\right)}{4}\rho_{2o}$$

All of the above expressions can be simplified for the given values of b, c, d, e in terms of a.

f. Check your answer for the previous question

Again, for the regions where there is no charge

$$\nabla^2 V(\vec{r}) = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} V(\vec{r}) = \frac{\rho_o a^2}{2\varepsilon_o}\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} 0 = 0$$

For c<r<d

$$\nabla^2 V(\vec{r}) = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} V(\vec{r}) = -\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\left(\left(\frac{1}{\varepsilon_o}\frac{\left(c^2 - b^2\right)}{2}\rho_{1o} - \frac{1}{\varepsilon_o}\frac{\left(c^2\right)}{2}\rho_{2o}\right)\ln\frac{d}{r} + \frac{1}{\varepsilon_o}\frac{\left(r^2 - d^2\right)}{4}\rho_{2o}\right)$$

Again, for simplicity, there are two types of derivatives here and they were both done in problem 1. $\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\left(\ln\frac{const}{r}\right) = \frac{1}{r}\frac{\partial}{\partial r} r \frac{1}{r} = 0$ *and* $\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}\left(\frac{r^2}{4}\right) = \frac{1}{r}\frac{\partial}{\partial r}\frac{2r^2}{4} = 1$. *Thus,*

$\nabla^2 V(\vec{r}) = -\left(\frac{1}{\varepsilon_o}\rho_{2o}\right)$ *as it should.*

g. Now assume that *a = 5mm* and that the voltage at *r = a* is *V(a)=10 Volts.* Determine the values of the two charge densities. Plot the electric field and voltage as a function of radius for these conditions.

First we will simplify all expressions above, using only a instead of the other dimensions and also eliminate one of the charge densities. $E_r(r) = \frac{1}{\varepsilon_o}\frac{\left(r^2 - 4a^2\right)}{2r}\rho_{1o}$ *and*

$$E_r(r) = \frac{1}{\varepsilon_o}\frac{\left(6a^2\right)}{r}\rho_{1o} + \frac{1}{\varepsilon_o}\frac{\left(r^2 - 16a^2\right)}{2r}\rho_{2o} = \frac{1}{\varepsilon_o}\frac{\left(50a^2\right)}{3r}\rho_{1o} - \frac{2}{\varepsilon_o}\frac{r}{3}\rho_{1o}$$

$V(r) = \left(\frac{1}{\varepsilon_o}6a^2\rho_{1o} - \frac{1}{\varepsilon_o}8a^2\rho_{2o}\right)\ln\frac{d}{r} + \frac{1}{\varepsilon_o}\frac{\left(r^2 - 25a^2\right)}{4}\rho_{2o}$ *and plugging in for the charge*

$V(r) = \left(\frac{1}{\varepsilon_o}\frac{\left(50a^2\right)}{3}\rho_{1o}\right)\ln\frac{5a}{r} - \frac{1}{\varepsilon_o}\frac{\left(r^2 - 25a^2\right)}{3}\rho_{1o}$ *and ,*

$$V(c) = \left(\frac{1}{\varepsilon_o}\frac{\left(50a^2\right)}{3}\rho_{1o}\right)\ln\frac{5a}{4a} - \frac{1}{\varepsilon_o}\frac{\left(16a^2 - 25a^2\right)}{3}\rho_{1o}$$

$$V(c) = \frac{\rho_{1o}a^2}{3\varepsilon_o}\left(50\ln\frac{5}{4} + 9\right)$$

$$V(r) = +V(c) - \frac{\rho_{1o}}{2\varepsilon_o}\left(\frac{\left(r^2 - c^2\right)}{2} + b^2 \ln\frac{c}{r}\right) \text{ for } c>r>b$$

$$V(r) = +V(c) - \frac{\rho_{1o}}{2\varepsilon_o}\left(\frac{\left(r^2 - 16a^2\right)}{2} + 4a^2 \ln\frac{4a}{r}\right)$$

$$V(r) = \frac{\rho_{1o}a^2}{3\varepsilon_o}\left(50\ln\frac{5}{4} + 9\right) - \frac{\rho_{1o}}{2\varepsilon_o}\left(\frac{\left(r^2 - 16a^2\right)}{2} + 4a^2 \ln\frac{4a}{r}\right)$$

$$V(b) = \frac{\rho_{1o}a^2}{3\varepsilon_o}\left(50\ln\frac{5}{4} + 9\right) - \frac{\rho_{1o}}{2\varepsilon_o}\left(\frac{\left(4a^2 - 16a^2\right)}{2} + 4a^2 \ln\frac{4a}{2a}\right)$$

$$V(b) = \frac{\rho_{1o}a^2}{3\varepsilon_o}\left(50\ln\frac{5}{4} + 9\right) - \frac{\rho_{1o}}{2\varepsilon_o}\left(\frac{\left(12a^2\right)}{2} + 4a^2 \ln 2\right)$$

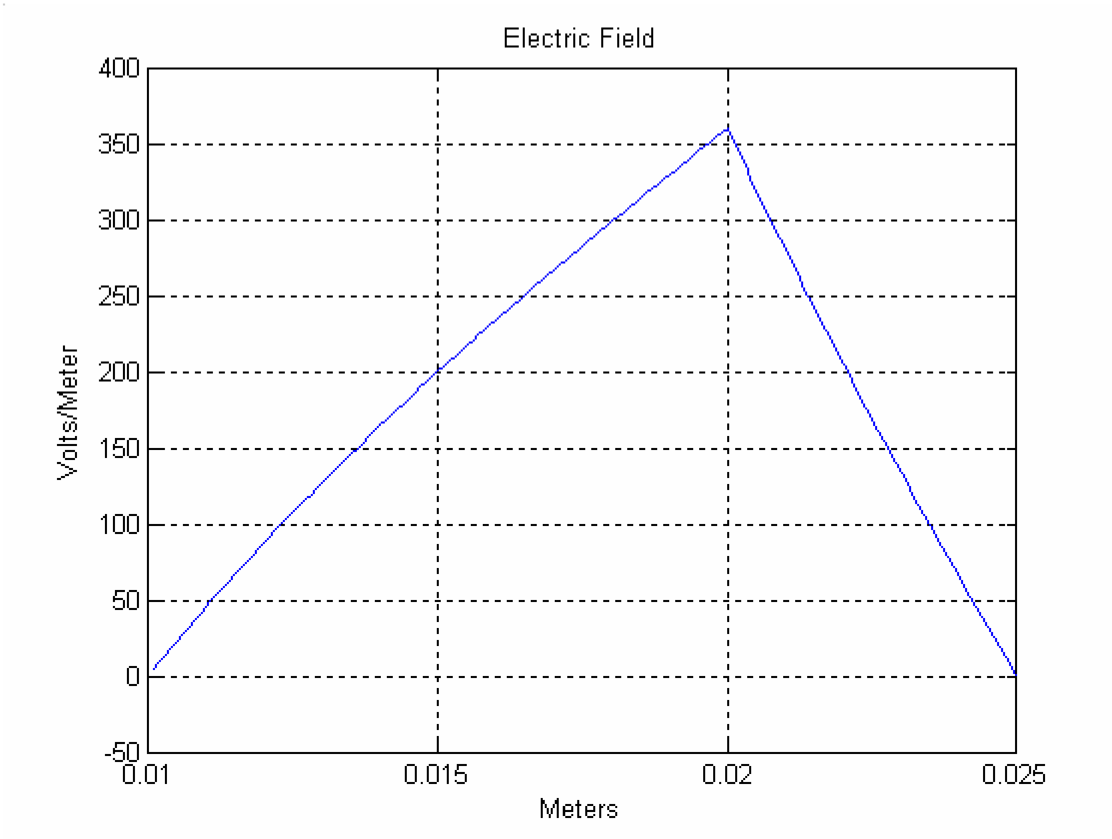
$$V(b) = \frac{\rho_{1o}}{\varepsilon_o}\left(\frac{a^2}{3}\left(50\ln\frac{5}{4} + 9\right) + \frac{1}{2}\left(\frac{\left(12a^2\right)}{2} - 4a^2 \ln 2\right)\right)$$

Since V(b)=10, we can solve for the charge densities once we plug in a = 5mm. It is easiest to do this with Matlab by first solving for the charge density, plug in all values and then plot the results for V and E.

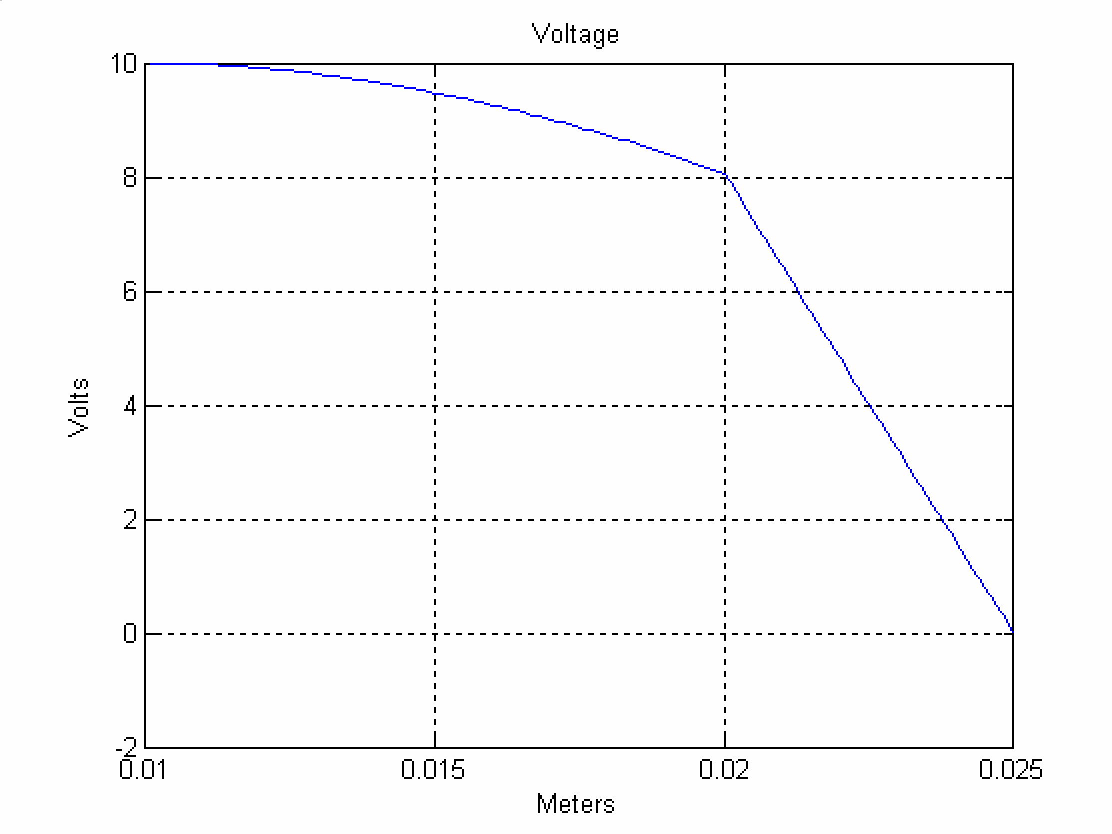
The Matlab M-File:

```
% Problem # 2

% Dimensions

a=.005;

% Radius Vector

 r2=[1.01:.01:2].*2*a;

 r3=[2.01:.01:2.5].*2*a;

 r=[r2 r3];

% Voltage at inner region used to find the charge density

 den=(a^2/3)*(50*log(5/4)+9)+0.5*((6*a^2)-4*a^2*log(2));

 pse=10/den;
```

```
V2=(pse.*(a.^2)./3).*(50.*log(5./4)+9)-(pse./2).*(((r2.^2-
16.*a.^2)/2)+4.*a.^2.*log((4.*a)./r2));

V3=(pse./3).*((50.*a.^2).*log(5.*a./r3)-(r3.^2-25.*a.^2));

V=[V2 V3];

% Plotting

plot(r,V)

grid;title('Voltage');xlabel('Meters');ylabel('Volts');

figure;

% Electric Field

E2=pse.*(r2.^2-4.*a.^2)./(2.*r2);

E3=pse.*(((50.*a.^2)./(3.*r3))-(2.*r3./3));

E=[E2 E3];

% Plotting

plot(r,E);

grid;

title('Electric Field');xlabel('Meters');ylabel('Volts/Meter');
```



Note that the electric field is zero both outside and inside the charge distribution. The voltage goes to 10 Volts at the inner radius of the inner charge distribution as it should.


Voltage
10
8
6
4
2
0
-2
Volts
0.01
0.015
0.02
0.025
Meters

3. **Application of Gauss' Law Methodology & Capacitance** For the same geometry as the previous problem, assume that we have placed a conductor at $r = a$ and at $r = e$. Also, assume that the charges that were previously free volume charges have been moved to the surfaces of the two conductors. That is, assume that there is a positive surface charge ρ_{sa} at $r = a$ and a negative surface charge ρ_{se} at $r = e = 6a$. The region between a and e is now empty. We still assume that the total charge per unit length is zero.

   a. Find the relationship between the inner and outer surface charge densities. That is, find ρ_{se} in terms of ρ_{sa}.

   b. Solve for the electric field $\vec{E}(\vec{r})$ for all values of r.

   c. Find the voltage $V(\vec{r})$ for all values of r. We are still using $r = e$ as the location of the zero reference for voltage.

   d. Evaluate the voltage at $r = a$. Using this voltage, determine the capacitance per unit length

   e. Using your answer for part a, determine the energy stored per unit length. From this answer, determine the capacitance per unit length and compare with your answer to part c.

   h. Now assume that $a = 5mm$, $e = 6a$, and that the voltage at $r = a$ is $V(a)=10$ *Volts*. Determine the values of the two surface charge densities. Plot the electric field and voltage as a function of radius for these conditions.

The answers to all parts of this problem are also found in the reading since this is just a coaxial cable, except for the last part where the results are plotted. The M-File created for this is:

```
% Parameters

a=.005;e=6*a;

r=[1:.01:6].*a;

% Charge Density divided by epsilon zero

den=a*log(e/a);

pse=10/den;
```

% Voltage

*V=pse.*a.*log(e./r);*

plot(r,V);grid;

title('Voltage');xlabel('Meters');ylabel('Volts');

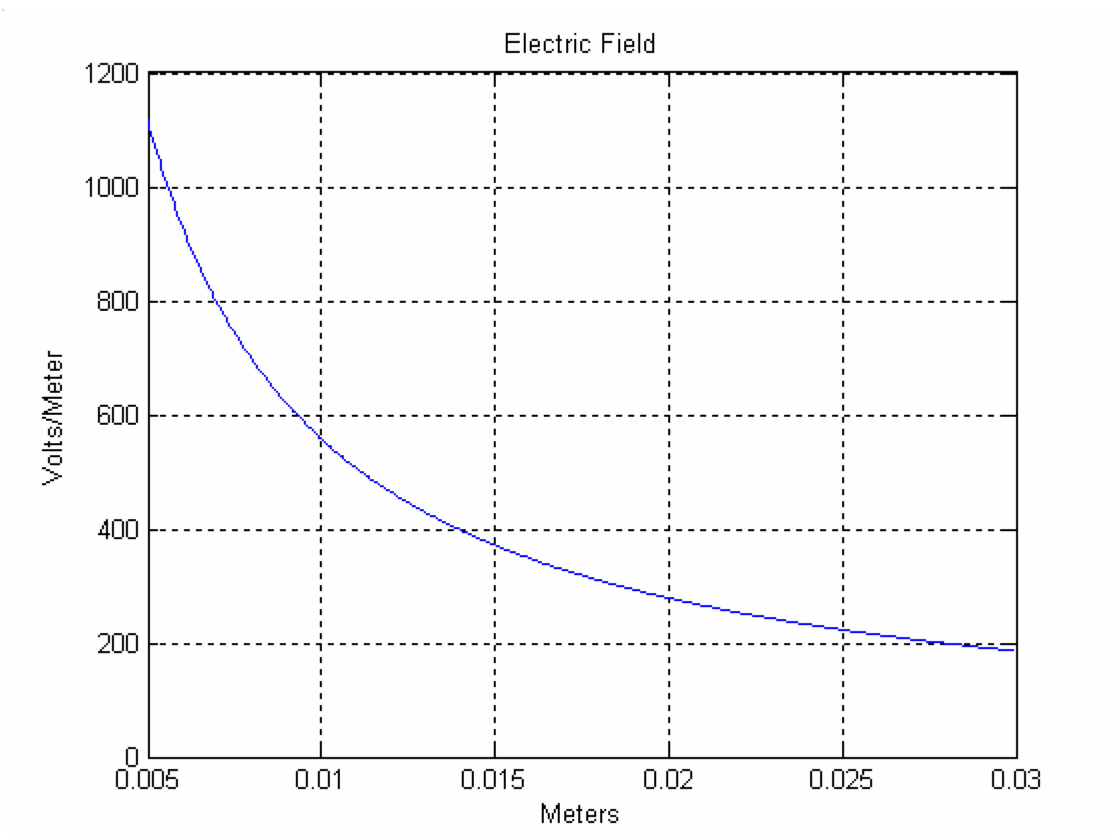
figure;

% E Field

*E=(pse.*a)./r;*

plot(r,E);grid;

title('Electric Field');xlabel('Meters');ylabel('Volts/Meter');



Note that the E field has a finite value at both r = a and r = e, which it should since there is now a surface charge at both locations. We can check these values to see if the boundary conditions are being met. E(a)=1.12e3 and E(e)=186. At the inner conductor, the Electric field is supposed to be the surface charge density divided by epsilon zero, *which it is. The surface charge density at the outer surface should be 1/6 as large because the outer surface area is 6 times larger, which it is.*