Homework #3
Due 28 February

Before beginning this homework assignment, read over HW3 from Spring 2006, especially the discussion in the first 3 pages. Also be sure that you are keeping up with the reading posted on the course handout page. Of course, reviewing the lecture slides is also a good idea.

1. **Gauss' Law Methodology** This problem was really done in class and in the lecture notes. Your purpose here is to fully reproduce the solution while simultaneously developing the general set of steps necessary (written in your own words) to solve such problems. Assume that we have a uniform cylinder of charge with density $\rho = \rho_o$ in the region $0 \le r \le a$. There are no other charges in this problem.

    a. What coordinate system should you use to solve this problem?

    b. Sketch the charge distribution in two and three dimensions.

    c. Solve for the electric field $\vec{E}(\vec{r})$ for all values of *r*. Remember that the answer to this question is a vector function, so be sure you express it as such.

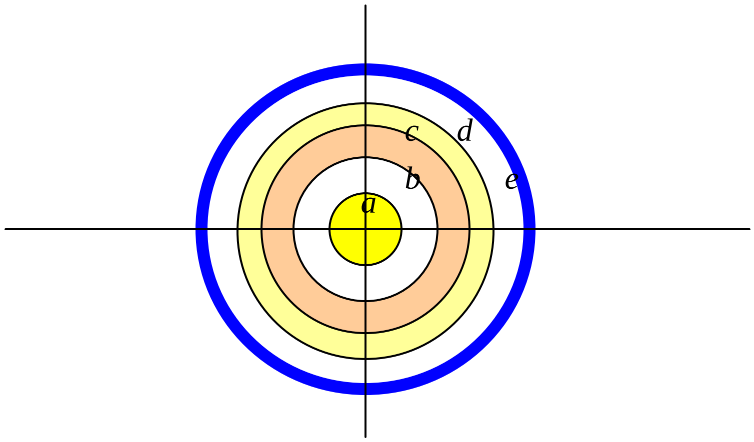
    d. Check your answer for $\vec{E}(\vec{r})$ by evaluating $\nabla \cdot \vec{E}$ for all values of *r*. That is, show that $\nabla \cdot \vec{E} = \rho / \varepsilon_o$.

    e. Assuming that the voltage is referenced to zero at *r* = *b* where $b > a$, find the voltage (also known as the electric potential) $V(\vec{r})$ for all values of *r*. Remember that the answer to this question is a scalar function, so be sure that you express it as such.

    f. Check your answer for $V(\vec{r})$ by evaluating $\nabla^2 V(\vec{r})$ for all values of *r*. That is, show that $\nabla^2 V(\vec{r}) = -\rho / \varepsilon_o$.

    g. Read over the discussion in the handout on Gauss' Law. Also, the reading in the textbook and class notes on Gauss' Law. In the handout, there is a list of steps to follow to use Gauss' Law to solve for fields. Put these steps in your own words and expand them to include instructions on how to find the voltage function and to check your solutions.

2. **Application of Gauss' Law Methodology** The following charge distribution exists in a cylindrical system: $\rho = \rho_{1o}$ in the region $b \le r \le c$ and $\rho = \rho_{2o}$ in the region $c \le r \le d$. The voltage is referenced to zero at $r = e$. There are no other charges in this problem.



a. Determine the total charge per unit length.

b. Now assume that the outer charge density is negative and that the total charge per unit length is zero. Find the relationship between the inner and outer charge densities. That is solve for one in terms of the other.

For the remainder of this problem, we will assume that the total charge per unit length is equal to zero <u>and</u>, for simplicity, that $b = 2a$, $c = 4a$, $d = 5a$, and $e = 6a$. Thus, the only geometric parameter that should appear in your solutions should be *a*.

c. Solve for the electric field $\vec{E}(\vec{r})$ for all values of *r*.

d. Check your answer for the previous question

e. Find the voltage $V(\vec{r})$ for all values of *r*.

f. Check your answer for the previous question

g. Now assume that *a = 5mm* and that the voltage at *r = a* is *V(a)=10 Volts.* Determine the values of the two charge densities. Plot the electric field and voltage as a function of radius for these conditions.

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3. **Application of Gauss' Law Methodology & Capacitance** For the same geometry as the previous problem, assume that we have placed a conductor at *r* = *a* and at *r* = *e*. Also, assume that the charges that were previously free volume charges have been moved to the surfaces of the two conductors. That is, assume that there is a positive surface charge ρ_{sa} at *r* = *a* and a negative surface charge ρ_{se} at *r* = *e* = *6a*. The region between *a* and *e* is now empty. We still assume that the total charge per unit length is zero.

    a. Find the relationship between the inner and outer surface charge densities. That is, find ρ_{se} in terms of ρ_{sa}.

    b. Solve for the electric field $\vec{E}(\vec{r})$ for all values of *r*.

    c. Find the voltage $V(\vec{r})$ for all values of *r*. We are still using *r* = *e* as the location of the zero reference for voltage.

    d. Evaluate the voltage at *r* = *a*. Using this voltage, determine the capacitance per unit length (The question is not required.)

    e. Using your answer for part b, determine the energy stored per unit length. From this answer, determine the capacitance per unit length and compare with your answer to part c. (This question is not required.)

    h. Now assume that *a* = *5mm*, *e* = *6a*, and that the voltage at *r* = *a* is *V(a)=10 Volts*. Determine the values of the two surface charge densities. Plot the electric field and voltage as a function of radius for these conditions.