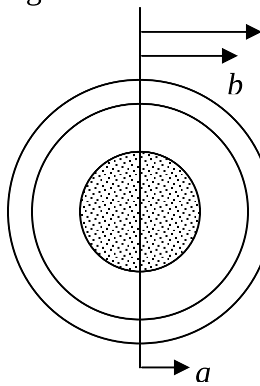
Due 28 February 2008 at 4:00 pm (at the beginning of Thursday's Lecture)

Before beginning this homework assignment, read over HW3 from Spring 2006, especially the discussion in the first 3 pages, and HW3 from Spring 2007. Also be sure that you are keeping up with the reading (both from the textbook and the course notes posted on WebCT) listed on the course handout page. Of course, reviewing the lecture slides is also a good idea, especially since some of this assignment is done there. Also, read through the entire assignment so that you understand what is being asked for, before you begin working on it.



1. **Gauss' Law Methodology** Assume that we have a uniform sphere of charge with density $\rho = \rho_o$ in the region $0 \le r \le a$. There are no other charges in this problem. The sphere of charge is surrounded by a hollow spherical plastic shell in the region $a \le r \le b$. For now, assume that the plastic has a permittivity given by ε_o so it has no impact on the solution. However, it will have to be included in the next problem. For this problem, then, the permittivity will be ε_o everywhere.

    a. What coordinate system should you use to solve this problem?

    b. Sketch the charge distribution in two and three dimensions.

    c. Solve for the electric field $\vec{E}(\vec{r})$ for all values of *r*. Remember that the answer to this question is a vector function, so be sure you express it as such. Begin by simplifying the expression for the field as much as possible and then identifying the appropriate Gaussian surface.

    d. Check your answer for $\vec{E}(\vec{r})$ by evaluating $\nabla \cdot \vec{E}$ for all values of *r*. That is, show that $\nabla \cdot \vec{E} = \rho / \varepsilon_o$.

    e. Assuming that the voltage is referenced to zero at $r = c$, find the voltage (also known as the electric potential) $V(\vec{r})$ for all values of *r*. Remember that the answer to this question is a scalar function, so be sure that you express it as such. Assume that $c > b$.

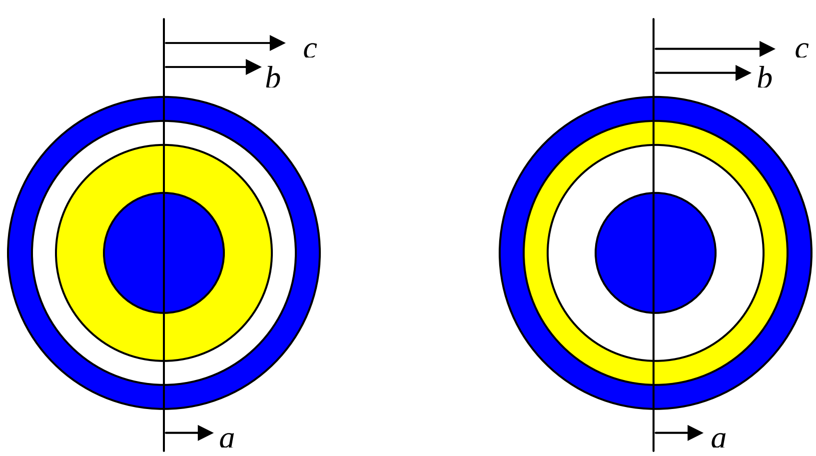
f. Check your answer for $V(\vec{r})$ by evaluating $\nabla^2 V(\vec{r})$ for all values of *r*. That is, show that $\nabla^2 V(\vec{r}) = -\rho/\varepsilon_o$.

g. **Extra Credit** Repeat steps a – f for a uniform cylindrical distribution of charge with density $\rho = \rho_o$ in the region $0 \le r \le a$. Note that this is not required.

2. Repeat the steps of problem 1 for the spherical case when the permittivity of the plastic shell is ε.

3. Now assume that the charges are free to move and thus try to get as far away from one another as possible. Because of the plastic spherical shell, they can move out to *r* = *a* where they will be found in an infinitesimally thin shell. That is, there will be a surface charge distribution ρ_S at *r* = *a*. To have the same total charge as before, find the relationship between ρ_S and ρ_o. In addition, assume there is also a second surface charge distribution ρ_{SC} at *r* = *c*, and the total charge in the two surface charges is zero. Find the relationship between ρ_{SC} and ρ_S.

4. Finally, assume that the region inside *r* = *a* and outside *r* = *c* are conductors. The outer conductor has a small thickness *t*. Remember that conductors with a voltage difference between them will have surface charges on them, so the charge distribution will be the same as in the previous problem. Find the voltage as a function of position and the voltage difference between the two conductors. Repeat both calculations for the geometry where the plastic is in the region $b \le r \le c$ and the region $a \le r \le b$ is empty and assume that $c = 3a$ and $b = 2.41a$. Compare the voltage difference of the two choices for location of the plastic. That is, imagine that you have a fixed amount of plastic and you have the choice of locating it in $a \le r \le b$ or $b \le r \le c$. Which will produce the larger voltage difference for the same charges? Also assume that $\varepsilon = 4\varepsilon_o$.



5. **Extra credit** Which of the two locations for plastic will produce the largest electric field at any location for the same voltage difference between the two conductors? To answer this question, you must find the electric field at all locations in space, express it in terms of the voltage difference and then find the maximum value for the field. Finding the maximum value for the electric field allows us to determine where the configuration is most likely to have breakdown problems.