1. Set up the experiment to measure the input and the output voltage. Use two 50 Ohm terminators, one on the lad end and another on the generator end on a T connector. The other 50 Ohms represents the impedance of the function generator. The generator is expecting a load of 50 Ohms and the voltage on the output display is the terminal voltage assuming that the generator sees 50 Ohms. Set the generator to 10V P-P, square wave 1MHz burst mode (we want to keep the pulse length < 300ns).

   The amplitude of the forward wave down the line is found from a voltage divider rule with the line being represented by Z_0. Find the amplitude of the forward wave. For an 80 meter long cable, what is the delay time? The spools are not quite 80 meters long so find the length by looking at the transit time of the wave.

   Repeat the experiment with the 93 Ohm terminator at the load end. Find the reflection coefficients at the load and at the source. Make a "bounce" diagram up to 6T and plot the voltage at the generator end and the load end.

2. A transmission line is connected to a source voltage $V_S(t) = 1.0\cos\omega t$ where the frequency is $f = 30\text{MHz}$. The internal impedance of the source is $R_S = 30\Omega$. The length of the line is 1 meter. The line is lossless and the inductance per meter is $L' = 0.5\,\mu H/m$, and capacitance per meter is $C' = 200\,pF/m$. The line is terminated with a load of $Z_L = 100\Omega$.

   Find the following: characteristic impedance, wave velocity, reflection coefficient at the input end, standing wave ration, input impedance, voltage at the load end and power delivered to the load. Since it is a lossless line, check that the power to the line is equal to the power delivered to the load.

   Characteristic Impedance
   $$Z_0 = \sqrt{\frac{L'}{C'}} = \sqrt{\frac{0.5\,\mu H/m}{200\,pF/m}} = 50\Omega$$

   Wave Velocity
   $$u_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{(0.5\,\mu H/m)(200\,pF/m)}} = 10^8\,m/s$$

   Reflection Coefficient
   $$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{30\Omega - 50\Omega}{30\Omega + 50\Omega} = -\frac{1}{4} \quad \left(\text{Reflection coefficient at sending end}\right)$$
   $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100\Omega - 50\Omega}{100\Omega + 50\Omega} = \frac{1}{3} \quad \left(\text{Reflection coefficient at receiving end}\right)$$

   Standing Wave Ration
   $$SWR = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|} = \frac{1 + \left|\frac{1}{3}\right|}{1 - \left|\frac{1}{3}\right|} = 2$$

Input Impedance

$$\beta = \frac{\omega}{u_p} = \frac{2\pi(30\text{MHz})}{10^8 \text{ }^\text{m}\!/\!_\text{s}} = 1.885 \text{ }^\text{rad}\!/\!_\text{m}$$

$$\tan\beta l = \tan\left[(1.885 \text{ }^\text{rad}\!/\!_\text{m})(1\text{m})\right] = -3.077$$

$$Z_{IN} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right) = (50\Omega)\left(\frac{100\Omega + j(50\Omega)(-3.077)}{50\Omega + j(100\Omega)(-3.077)}\right) = 26.929 + j11.871\Omega$$

Voltage at the load

$$V_{IN} = V_S\frac{Z_{IN}}{Z_{IN} + Z_S} = (1\text{V})\frac{26.929 + j11.871\Omega}{26.929 + j11.871\Omega + 30\Omega} = 0.495 + j0.105\text{V}$$

$$V^+ = \frac{V_{IN}}{e^{j\beta l} + \Gamma_L e^{-j\beta l}} = \frac{0.495 + j0.105\text{V}}{e^{j(1.885\text{ }^\text{rad}\!/\!_\text{m})(1\text{m})} + \left(\frac{1}{3}\right)e^{-j(1.885\text{ }^\text{rad}\!/\!_\text{m})(1\text{m})}} = -0.239 - j0.625\text{V}$$

$$V_L = V^+(1 + \Gamma_L) = -0.319 - j0.833\text{V}$$

Power at the load

$$P_L = \tfrac{1}{2}\Re\{V_L I_L^*\} = \tfrac{1}{2}\Re\left\{V_L\left(\frac{V_L}{Z_L}\right)^*\right\} = \tfrac{1}{2}\Re\left\{(-0.319 - j0.833\text{V})\left(\frac{(-0.319 - j0.833\text{V})}{100\Omega}\right)^*\right\} = 3.981\text{mW}$$

Check power at input

$$P_{IN} = \tfrac{1}{2}\Re\{V_{IN}I_{IN}^*\} = \tfrac{1}{2}\Re\left\{V_{IN}\left(\frac{V_{IN}}{Z_{IN}}\right)^*\right\} = \tfrac{1}{2}\Re\left\{(0.495 + j0.105\text{V})\left(\frac{(0.495 + j0.105\text{V})}{26.929 + j11.871\Omega}\right)^*\right\} = 3.981\text{mW}$$

3. A generator with sinusoidal voltage $V_S = 100\angle 0°$, and $Z_S = 50\Omega$ is connected to a load of $Z_L = 75\Omega$ through a 50Ω lossless line of $l = 0.15\lambda$. Find the input impedance of the line at the generator end. Find the current and voltage at the input end of the line. Find the average power delivered to the line and the average power delivered to the load.

Input Impedance

$$\beta l = \left(\frac{2\pi}{\lambda}\right)(0.15\lambda) = 0.942\text{rad}=54°$$

$$\tan\beta l = \tan(54°) = 1.376$$

$$Z_{IN} = Z_0\left(\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right) = (50\Omega)\left(\frac{75\Omega + j(50\Omega)(1.376)}{50\Omega + j(75\Omega)(1.376)}\right) = 41.251 - j16.347\Omega$$

Current at input end

$$I_{IN} = \frac{V_S}{Z_S + Z_{IN}} = \frac{100}{50\Omega + 41.251 - j16.347\Omega} = 1.062 + j0.19\text{A}$$

Voltage at input end

$$V_{IN} = I_{IN}Z_{IN} = (1.062 + j0.19\text{A})(41.251 - j16.347\Omega) = 46.91 - j9.511\text{V}$$

Average power delivered to the line

$$P_{IN} = \tfrac{1}{2}\Re\left\{V_{IN}I_{IN}^*\right\} = \tfrac{1}{2}\Re\left\{(41.251 - j16.347\text{V})(1.062 + j0.19\text{A})^*\right\} = 24\,\text{W}$$

Average power delivered to the load

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{75\Omega - 50\Omega}{75\Omega + 50\Omega} = 0.2$$

$$V^+ = \frac{V_{IN}}{e^{j\beta l} + \Gamma_L e^{-j\beta l}} = \frac{46.91 - j9.511\text{V}}{e^{j(0.942)} + (0.2)e^{-j(0.942)}} = 29.389 - j40.451\text{V}$$

$$V_L = V^+\left(1 + \Gamma_L\right) = 35.267 - j48.541\text{V}$$

$$I_L = \frac{V^+}{Z_0}\left(1 - \Gamma\right) = \frac{29.389 - j40.451\text{V}}{50\Omega}\left(1 - 0.2\right) = 0.47 - j0.647\text{A}$$

$$P_L = \tfrac{1}{2}\Re\left\{V_L I_L^*\right\} = \tfrac{1}{2}\Re\left\{(35.267 - j48.541\text{V})(0.47 - j0.647\text{A})^*\right\} = 24\,\text{W}$$