Homework 4

Due 7 March 2005

Note: It is very important that you fully understand the method for determining the capacitance of standard structures like parallel plates, coax and two-wire lines. The first two are discussed in examples 4-11 and 4-12 of Ulaby and in the lecture slides. The two-wire line is also discussed in the lecture slides. Also, read over the handout found at http://hibp.ecse.rpi.edu/%7Econnor/education/Fields/gauss_law.pdf. There are no questions of this type in the homework, since they are done in examples. However, you can expect to see similar questions on tests. If you do not understand this material, ask questions in lecture and during the Tuesday studio sessions.
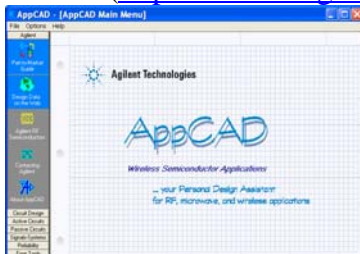
1. Short Questions
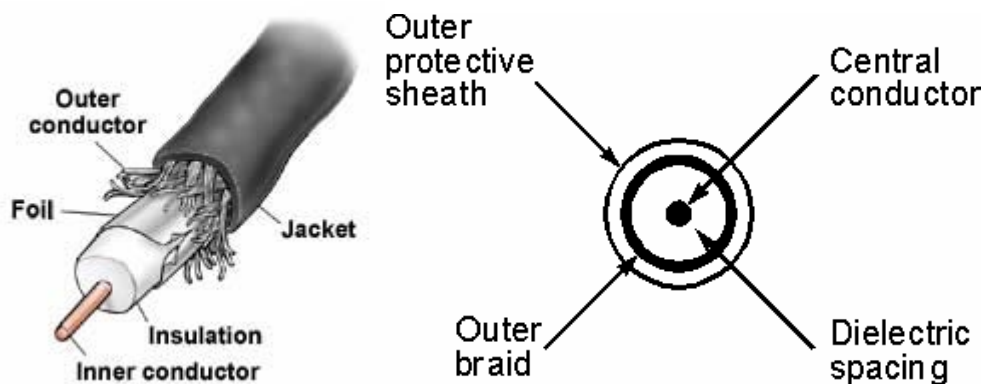
a. When we were studying transmission lines, we learned that commercial coaxial cable specifications generally include velocity and capacitance per unit length. We now know how to determine capacitance and the velocity is determined by the material properties. Assume that we have a cable with velocity equal to 2/3 the speed of light and that the capacitance is 80 pF/m. Determine the inductance and the characteristic impedance.

b. Assume we are free to redesign the cable in part a and we would like its characteristic impedance to be 50 Ohms. To make this happen, will we have to increase or decrease the capacitance per unit length of the cable? We can accomplish the change we desire in Z_o by using a different insulator (change ε) and/or changing the dimensions of the cable. Should ε be increased or decreased? Should the radius of the inner conductor be increased or decreased? *Show your work.*

c. Download and install the AppCAD design tool. The first window looks like the one below. (http://www.avagotech.com/pc/downloadDocument.do?id=4219)

K. A. Connor   Rensselaer Polytechnic Institute   2 March 2005

Select passive circuits, which will bring up a menu with several kinds of transmission lines on it. Scroll down to the Round Coax and select it. When the Round Coax window appears, change the length units to millimeters. Change the outer diameter to 5mm. Select Polyethylene as the insulator material. (The default is free space.) Adjust the inner diameter until you obtain the Z_o you calculated in part a. Using the formula for the capacitance per unit length of a coaxial cable, show that 80pF/m is the correct value for this cable. Then, adjust the inner diameter to obtain $Z_o = 50\Omega$. Is your answer consistent with your response to part b? What is the inner diameter? If you have a screen capture routine, include a copy of the final window with your answer.

d. Return to the main AppCAD menu. Select the Microstrip. Again change the dimensions to millimeters. Select G-10 as the insulator. G-10 is an excellent machinable plastic that can be used in a wide variety of applications. Select a height H of 3mm and a thickness T of the top strip of 0.1mm. Find the width W of the strip that results in $Z_o = 50\Omega$. On the diagram below, sketch 5 equipotentials and several electric field lines. Be careful to account for the conducting (red) and dielectric (blue) material boundaries in your diagram. (It might be good to review example 4-10 and the similar example from the lecture slides.) Using the information in your diagram, explain why AppCAD indicates that the effective dielectric constant ε_{eff} is less than the G-10 dielectric constant $\varepsilon_r = 4.6$.
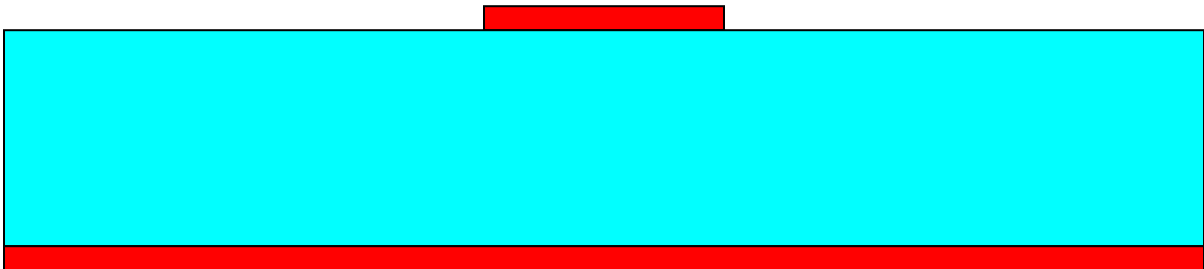
2. Using a Spreadsheet to Find Capacitance: Assume that you have the following two dimensional configuration of conductors and dielectrics. The outer (red) surface is grounded. The inner (yellow) square region is some unknown dielectric material. The upper (blue) region is a conductor connected to some positive voltage. The lower (blue) region is a conductor connected to some negative voltage. We will assume for simplicity that the positive and negative voltages have the same magnitude.
Reference:
http://hibp.ecse.rpi.edu/%7Econnor/education/Fields/SpreadsheetLaplace.PDF

With the exception of the two gray areas, the remainder (white) of the region is a known dielectric material (could be air, for example). The outer box is *50mm* by *50mm*. All other dimensions can be determined by counting cells in the figure. *Hint: Using Excel, it is possible to obtain such square cells by selecting the column width as 3 and the row height as 20.*

K. A. Connor   Rensselaer Polytechnic Institute   2 March 2005

Case 1: Assume that the two gray areas, the inner (yellow) square region and the (white) background region are all an insulating material with $\varepsilon = \varepsilon_o$ (probably free space).
   a.   Use the spreadsheet method to find the capacitance per unit length of this shielded, two wire transmission line.
   b.   Produce a plot showing at least 8 equipotentials. Sketch a representative set of electric field lines on this plot.
   c.   Find the charge per unit length.

Case 2: Assume the same conditions as Case 1, except that the inner (yellow) square region is now filled with water ($\varepsilon = 81\varepsilon_o$).
   d.   Use the spreadsheet method to find the capacitance per unit length of this shielded, two wire transmission line.
   e.   Produce a plot showing at least 8 equipotentials. Sketch a representative set of electric field lines on this plot.
   f.   Find the charge per unit length.

Case 3: Assume the same conditions as Case 1, except that the two gray areas are now conductors connected to the top and bottom plates, respectively. That is, the upper gray area is at the potential of the upper plate and the lower gray area is at the potential of the lower plate.
   g.   Use the spreadsheet method to find the capacitance per unit length of this shielded, two wire transmission line.
   h.   Produce a plot showing at least 8 equipotentials. Sketch a representative set of electric field lines on this plot.
   i.   Find the charge per unit length.

Case 4: Assume the same conditions as Case 3, except that the inner (yellow) square region is now filled with water ($\varepsilon = 81\varepsilon_o$).
   j.   Use the spreadsheet method to find the capacitance per unit length of this shielded, two wire transmission line.
   k.   Produce a plot showing at least 8 equipotentials. Sketch a representative set of electric field lines on this plot.
   l.   Find the charge per unit length.

The purpose of these configurations is to determine when water is present in the inner (yellow) square region. For example, one might want to do this in a basement that is prone to flooding during heavy rains. Compare the percentage capacitance change observed with the two conductor configurations. If we also knew the different costs associated with the two configurations, we could decide which is preferable.

K. A. Connor   Rensselaer Polytechnic Institute   2 March 2005

3. Poisson's Equation and Gauss' Law: The electric scalar potential in a cylindrical region $0 \le r \le a$ is given by $V(r) = V_o\left(1 - \dfrac{r^2}{a^2}\right)$. This region is known to be filled with charge with an unknown density distribution $\rho = \rho(r)$ and is surrounded by a cylindrical conducting shell at $r = a$.

a. Determine the charge distribution responsible for this potential.

b. Determine the electric field $\vec{E}(r)$ in the region $0 \le r \le a$.

c. Since the electric field must end at the conducting shell, there must be a surface charge distribution on the conductor. Determine the density of this surface charge.

K. A. Connor   Rensselaer Polytechnic Institute   2 March 2005