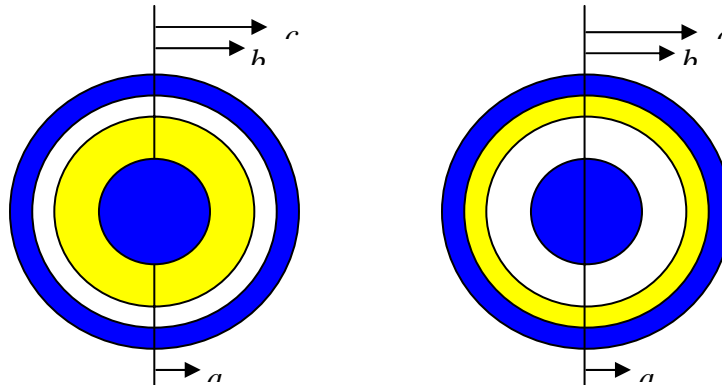




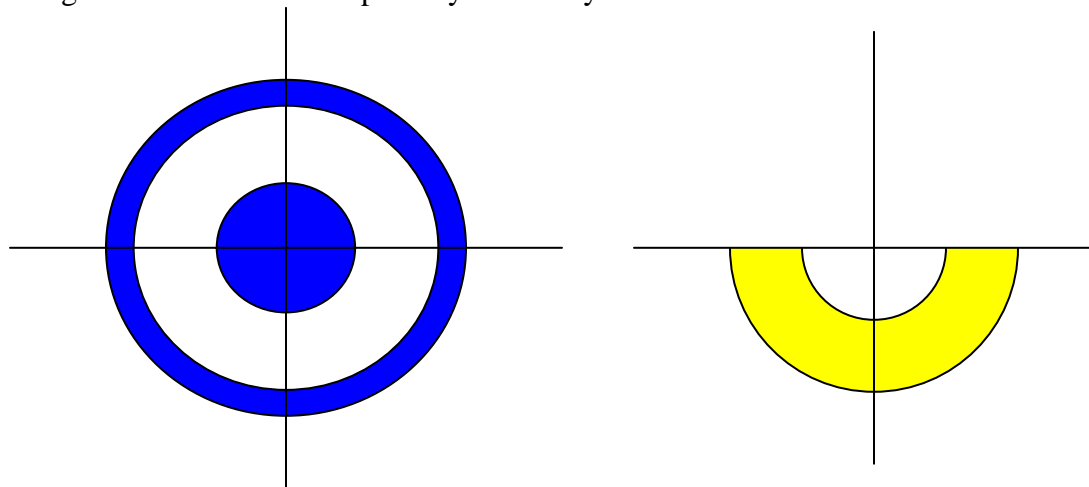
Due 5 March 2008 at 6:00 pm (at the end of Wednesday)

Before beginning this homework assignment, it is a very good idea to read over previous HW 4 assignments from the last few spring terms. Also, since the first problem is based on HW3, be sure that you understand the solution to that assignment. Also, remember that, while the geometry in problems 1 and 2 below is spherical, you should be able to analyze similar geometries for parallel plate (rectangular) and coaxial (cylindrical) cases.

1. **Spherical Capacitor** Assume that we have the final configuration addressed in HW3, so that the region inside  $r = a$  and outside  $r = c$  are conductors. The outer conductor has a small thickness  $t$ . Determine the capacitance for both cases considered. That is, find  $C$  for the geometry where the plastic is in the region  $a \leq r \leq b$  and the region  $b \leq r \leq c$  is empty and where the region  $a \leq r \leq b$  is empty and the plastic is in the region  $b \leq r \leq c$ . Assume that  $c = 3a$ ,  $b = 2.41a$ , and  $\epsilon = 4\epsilon_0$ .

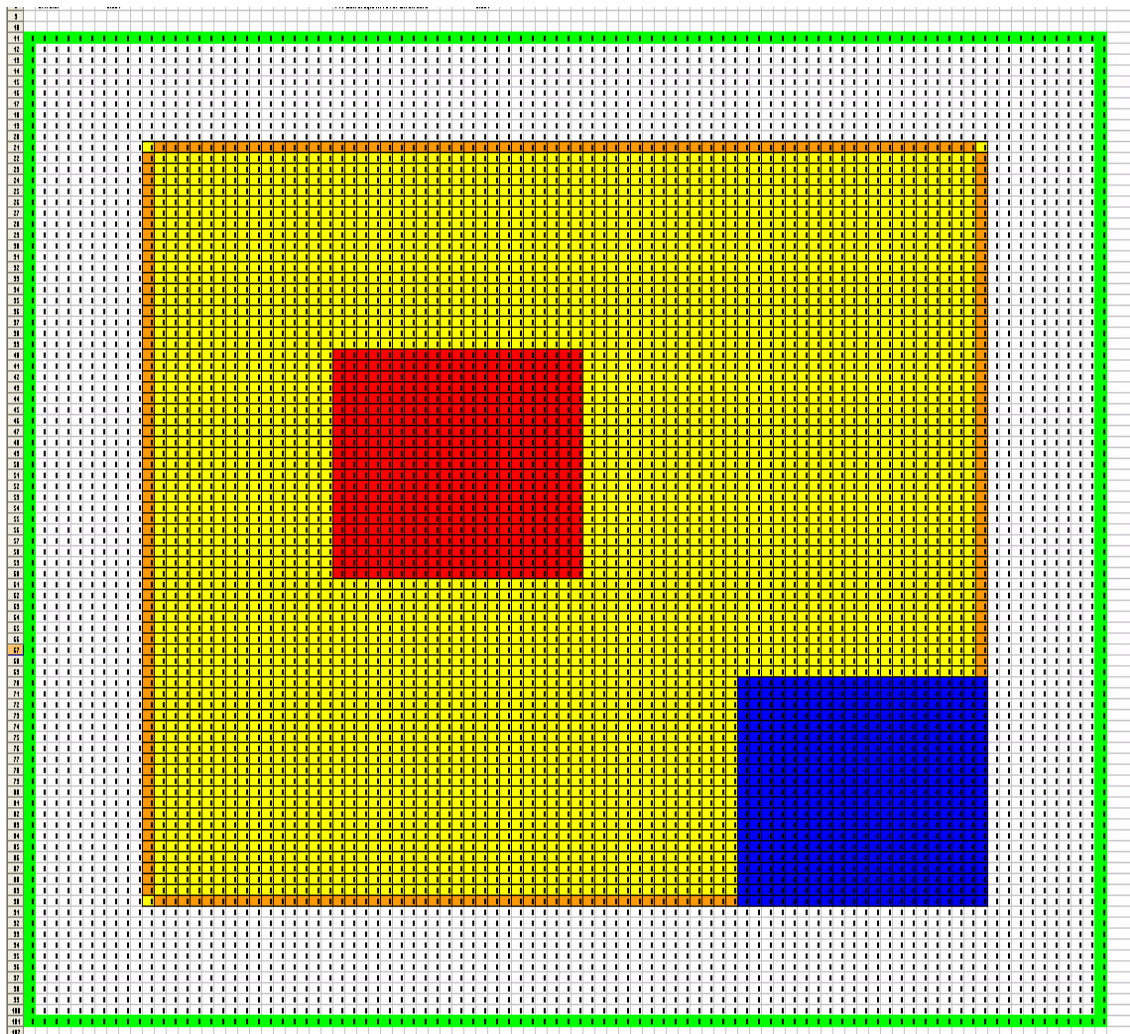


2. **Spherical Capacitor** Now consider a somewhat different configuration. Here the region between the conductors is split so that the bottom half is plastic and the upper half is empty. Otherwise, it is the same as above. Find the capacitance. The two parts of the configuration were drawn separately for clarity.





3. **Capacitance Using a Spreadsheet** Shown below is the spreadsheet setup for a configuration that addresses how capacitance can be determined using a variety of techniques based on the same basic analysis. For this configuration, assume that the structure is 91 cells wide and 91 cells high and that each cell is 1mm by 1mm. The positive electrode is shown in red and the negative electrode is shown in blue. Both are 21 cells wide and 21 cells high. The yellow region is an insulator. The dielectric constant used is 10 (which is relatively large). Since the red and blue cells are electrodes, they contain fixed voltages. Also the white and yellow cells contain the formula for the average of the near neighbors. The orange cells on the boundary of the insulating region contain the formula for the boundary condition between two dielectric media. The yellow region is 71 cells wide and 70 cells high. There are 9 cells between the yellow region and the outer green region on the top and two sides, but 10 cells on the bottom. Thus, corners of the yellow region (actually shown in orange to indicate the boundary cells) are upper left (cell K21), upper right (CC21), lower left (K90) and lower right (CC90). Note that the top row (green) is row 11.





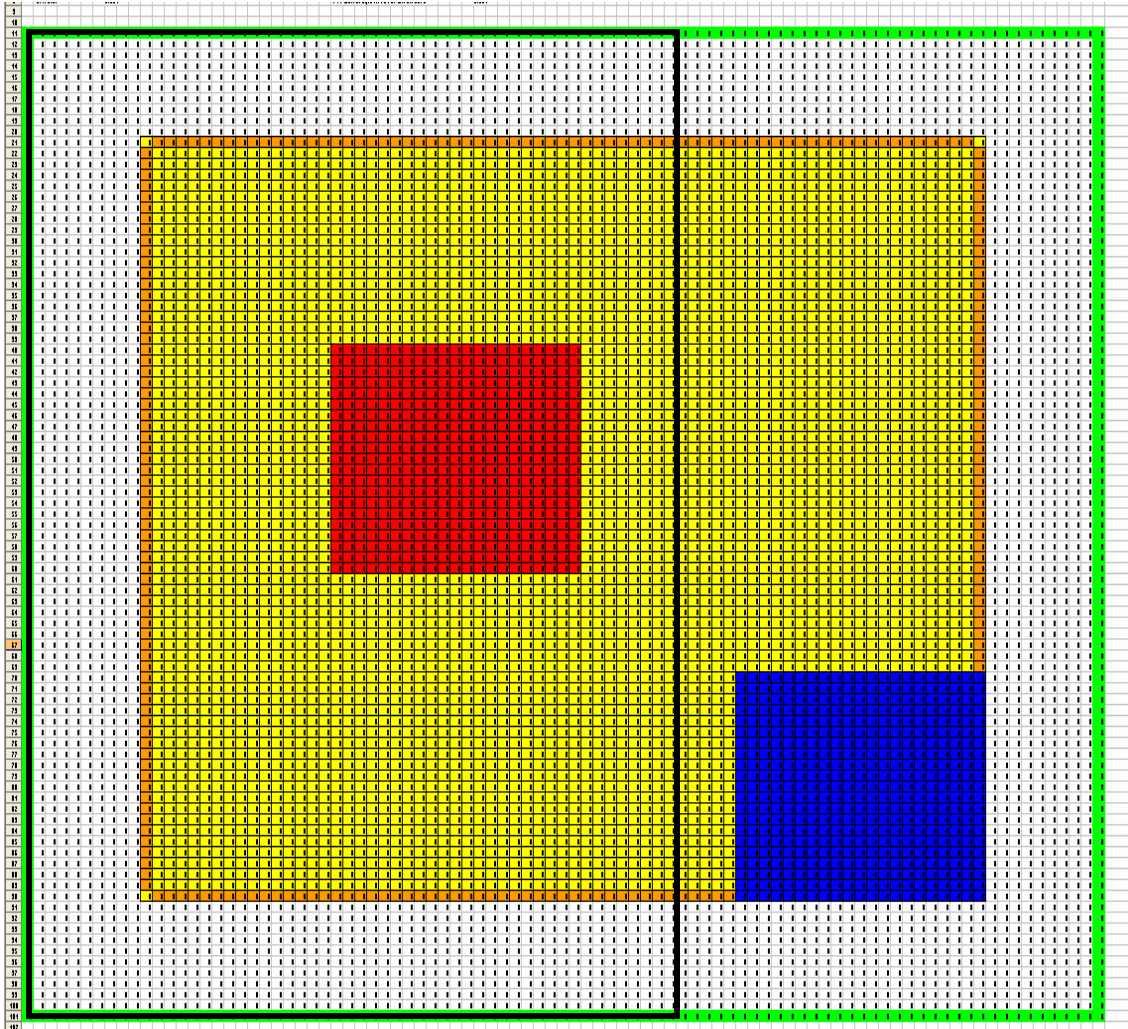
The shield region (in green) was not put at the top of the spreadsheet to permit some room for other information there. One can place the region of interest anywhere in a spreadsheet.

This is a three electrode configuration, since the shield is also an electrode. Thus, the capacitance is not found quite as simply as it is with a coaxial cable or other standard two electrode configurations. There are several reasons for considering this geometry, one of which is that it permits us to see that the shield can affect the capacitance. However, the primary reason this configuration was chosen is that it permits us to see that there are several ways to find the capacitance and none are completely accurate.

- 1) First, set up this configuration using Excel or some other spreadsheet, putting fixed voltages in the electrode regions and the appropriate formulas in the dielectric regions. Assume that the dielectric is air so that  $\epsilon = \epsilon_0$  except in the conducting regions. Let Excel run for a sufficient period of time to give reasonable accuracy. Given the number of cells, you should expect this to possibly take a few minutes. Do not choose a large number for the total iterations because your computer may be tied up for a very long time. It usually works better to choose a number like 100 and then push the F9 button enough times to get good accuracy.
- 2) Determine the charge on each of the three electrodes by first finding the average voltage in the immediately adjacent cells. Then follow the standard procedure for finding the charge. For simplicity, express your answers in terms of  $\epsilon_0$  rather than fully evaluating the numbers.
- 3) Repeat part b for the case where the yellow region has a permittivity of  $\epsilon = 4\epsilon_0$ . If you have not already put the appropriate formulas in the boundary cells, you must do so now. Remember that you also must use the appropriate value of  $\epsilon$  when you evaluate the displacement vector  $\vec{D}$ .
- 4) Next, find the capacitance for the two cases considered in b and c. You should find that the charges on the red and blue electrodes are not equal and opposite and that there is some finite charge on the shield. To see the significance of the differences in the charges, evaluate the capacitance twice, once using the charge on the red electrode and once using the charge on the blue electrode.
- 5) One of the problems in using this method is that there is net charge on the shield. However, there are other sources of error. One is that the variation in the field is greatest near the electrodes, which is exactly where we are using the solution. Thus, we are using the least accurate part of the spreadsheet. Let us consider a second approach to finding the charge on the red electrode. Evaluate the left hand side of Gauss' Law to find the charge on the electrode, but use one of the largest possible surfaces, as follows. The surfaces on which you are to find the average voltage are:



- i) The left hand side of the outer shield plus the top of the shield from the upper left corner to column BC, column BC, and the bottom of the shield from the lower left corner to column BC.
- ii) The cells just inside the left hand side of the shield, the cells just below the top of the shield from the left side to column BB, column BB, and the cells just above the bottom shield from the left side to column BB.



The two surfaces are located approximately where the black rectangle is shown above. Note that this surface surround the red electrode, so the electric flux passing through it should be equal to the charge on the electrode. Find the charge this way, evaluate the capacitance and compare your result with your numbers based on the cells next to the electrodes. Again, remember to use the appropriate value of  $\epsilon$  when you find the flux.



4. **Capacitance Measurements** Repeat the experiment done the first day of class where you made a small capacitor from two twisted wires on a protoboard. Calculate the capacitance of your configuration (you will have to carefully measure the dimensions of the wires and their separation distance) and compare with the measurement using the impedance bridge. You don't need to do the protoboard part of the experiment. You only need to make the capacitor and measure its properties. Remember to take into account the capacitance of the cable you use to connect to your wires.