Problem Solution #5

Problem 1

$$B = \frac{\mu_0 I}{2\pi r};$$

$$\Phi = \int \vec{B}\cdot d\vec{s} = \frac{\mu_0 I}{2\pi}\int dy \int_{0.3}^{0.8}\frac{1}{x}dx \approx 0.098 I(\mu Wb);$$

$$0.098 I = 50 \Rightarrow I \approx 510 A$$

Problem 2

$$T = k\theta = 3\times 10^{-6}\times 60 = 1.8\times 10^{-4}(Nm);$$

$$T = NIAB = 60\times(15\times 20\times 10^{-6})\times 0.6 I = 108\times 10^{-4} I(Nm);$$

$$108\times 10^{-4} I = 1.8\times 10^{-4} \Rightarrow I \approx 0.017 A$$

Problem 3

$$\nabla\times\vec{F} = \left(\frac{1}{r}\frac{\partial F_z}{\partial\theta} - \frac{\partial F_\theta}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r F_\theta) - \frac{\partial F_r}{\partial\theta}\right]\hat{\mathbf{z}}.$$

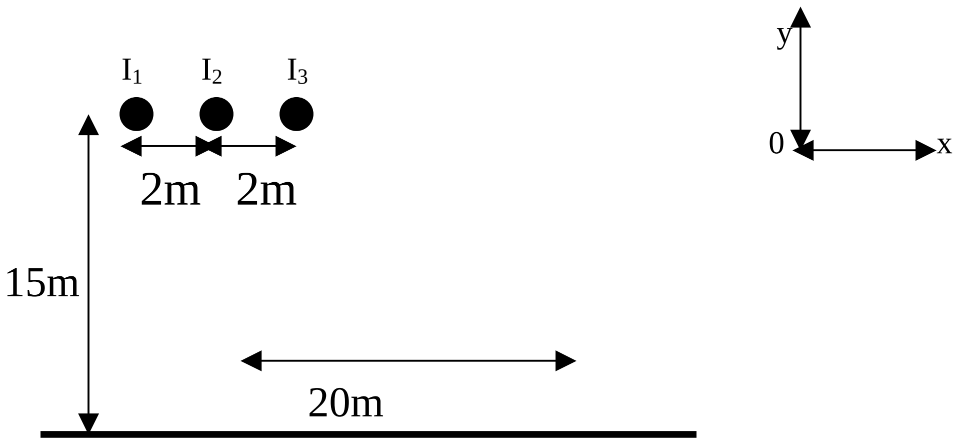
$$\nabla\times\vec{H} = \vec{J} \Rightarrow \vec{J} = \frac{1}{r}\frac{\partial}{\partial r}(r\cdot 3r)\hat{z} \Rightarrow \vec{J} = 6\hat{z}(A/m^2)$$

Problem 4

$$\oint_c \vec{H}\cdot d\vec{l} = H 2\pi r = \int \nabla\times\vec{H}\cdot d\vec{s} = \int \vec{J}\cdot\hat{\mathbf{k}}ds = \int_0^r J_0(1-\frac{r'}{b})2\pi r' dr' = 2\pi J_0(\frac{r^2}{2}-\frac{r^3}{3b}) \Rightarrow \vec{H} = J_0(\frac{r}{2}-\frac{r^2}{3b})\hat{z}$$

Problem 5

a)



$$\oint_c \vec{H}\cdot d\vec{l} = I \quad \Rightarrow \vec{H} = \frac{I}{2\pi r}\hat{\phi}$$

$$H_{1x} = H_1\frac{15}{r} = \frac{15I_1}{2\pi r^2} = -\frac{15\times 500}{2\pi\left(22^2+15^2\right)} \qquad H_{1y} = H_1\frac{22}{r} = \frac{22I_1}{2\pi r^2} = -\frac{22\times 500}{2\pi\left(22^2+15^2\right)}$$

$$H_{2x} = H_2\frac{15}{r} = \frac{15I_2}{2\pi r^2} = \frac{15\times 1000}{2\pi\left(20^2+15^2\right)} \qquad H_{2y} = H_2\frac{20}{r} = \frac{20I_2}{2\pi r^2} = \frac{20\times 1000}{2\pi\left(20^2+15^2\right)}$$

$$H_{3x} = H_3\frac{15}{r} = \frac{15I_3}{2\pi r^2} = -\frac{15\times 500}{2\pi\left(18^2+15^2\right)} \qquad H_{3y} = H_3\frac{18}{r} = \frac{18I_3}{2\pi r^2} = -\frac{18\times 500}{2\pi\left(18^2+15^2\right)}$$

$$H_x = H_{1x}+H_{2x}+H_{3x} \approx \boxed{-0.038A/m} \qquad H_y = H_{1y}+H_{2y}+H_{3y} \approx \boxed{0.015A/m}$$

$$\boxed{\vec{B} = \mu_0\vec{H} = \left(-4.79\hat{x}+1.83\hat{y}\right)(\times 10^{-8}A/m)}$$

b) $\vec{H} = \dfrac{Ia^2}{2(a^2+z^2)^{3/2}}\hat{z} = \dfrac{1\times 0.025^2}{2(0.025^2+1^2)^{3/2}}\hat{z} \approx 0.312\hat{z}(mA/m)$

$$\boxed{\vec{B} = \mu_0\vec{H} = 3.92\times 10^{-10}\,\hat{z}(A/m)}$$

c) The force on the center conductor carrying I_2 is zero.

The force per meter on each of the other two conductors is

$$\left|BI\right| = \frac{\mu_0}{2\pi}\times 500\times\left(\frac{1000}{2}-\frac{500}{4}\right) = 0.0375(N/m)$$

The direction of the force on I_1 is left while the direction of the force on I_3 is right.