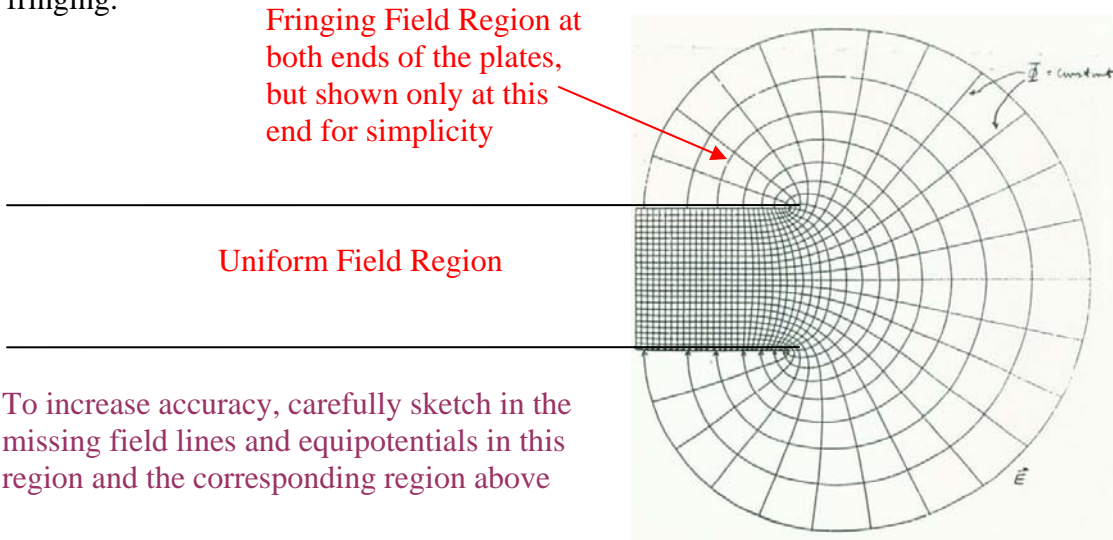


Homework 5

Due 22 March 2005

1. Resistance Measurement It is possible to determine the salt content in a body of water by the resistance between two conducting parallel plates immersed in the water. To assure that the analysis is as accurate as possible, we want to include the effects of fringing.



To increase accuracy, carefully sketch in the missing field lines and equipotentials in this region and the corresponding region above

Some background from <http://lakeaccess.org/russ/conductivity.htm>

Water Body	EC (uS/cm)	TDS (mg/L)
Lake Superior	97	63
Lake Tahoe	92	64
Lake Mead	850	640
Atlantic Ocean	43000	35000
Great Salt Lake	158000	230000

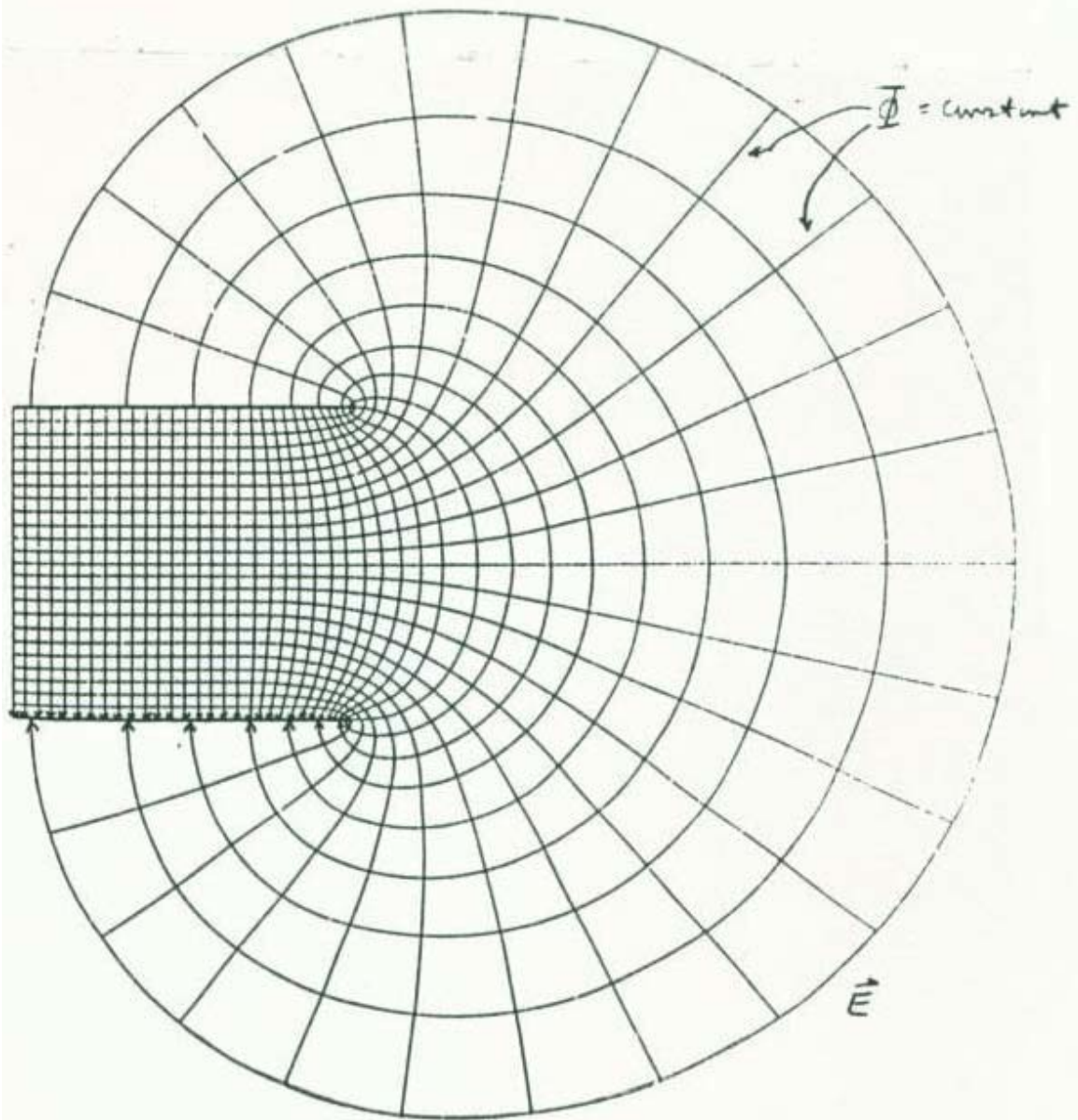
Superior and Tahoe are ultraoligotrophic (nutrient poor) lakes; Mead is an unproductive reservoir (the largest in the U.S.) but has a high TDS (total dissolved salts) due to the salt content of the Colorado River which provides >98% of its water; the Atlantic Ocean overlies the lost Kingdom of Atlantis and possibly Jimmy Hoffa; the Great Salt Lake is an enormous hypersaline lake near Salt Lake City, Utah - it is the relic of what was once a huge inland freshwater sea that dried up, thereby concentrating the remaining salts after the water evaporated.

What in the world are microSiemens per centimeter (µS/cm)?

These are the units for electrical conductivity (EC). The sensor simply consists of two metal electrodes that are exactly 1.0 cm apart and protrude into the water. A constant voltage (V) is applied across the electrodes. An electrical current (I) flows through the water due to this voltage and is proportional to the concentration of dissolved ions in the water - the more ions, the more conductive the water resulting in a higher electrical current which is measured electronically. Distilled or deionized water has very few dissolved ions and so there is almost no current flow across the gap (low EC). As an aside, fisheries biologists who electroshock know that if the water is too soft (low EC) it is difficult to electroshock to stun fish for monitoring their abundance and distribution. Up until about the late 1970's the units of EC were micromhos per centimeter (µmhos/cm) after which they were changed to microSiemens/cm (1µS/cm = 1 µmho/cm). You will

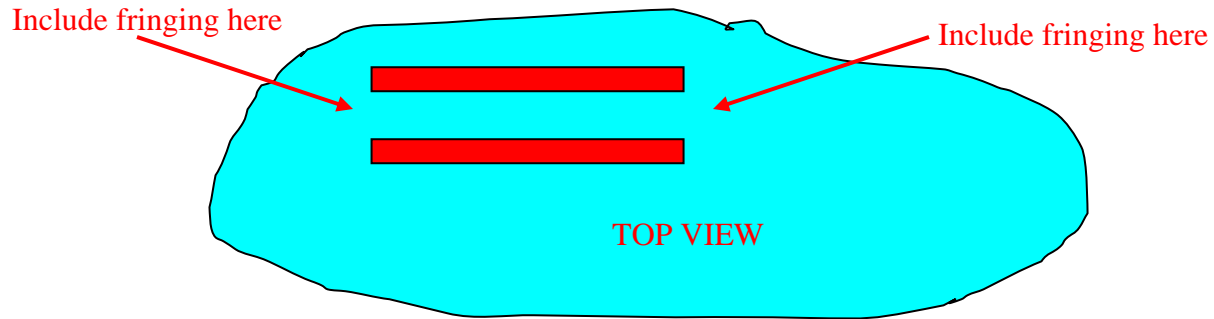
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find both sets of units in the published scientific literature although their numerical values are identical. Interestingly, the unit "mhos" derives from the standard name for electrical resistance reflecting the inverse relationship between resistance and conductivity - the higher the resistance of the water, the lower its conductivity. This also follows from Ohm's Law, $V = I \times R$ where R is the resistance of the centimeter of water. Since the electrical current flow (I) increases with increasing temperature, the EC values are automatically corrected to a standard value of 25°C and the values are then technically referred to as specific electrical conductivity.

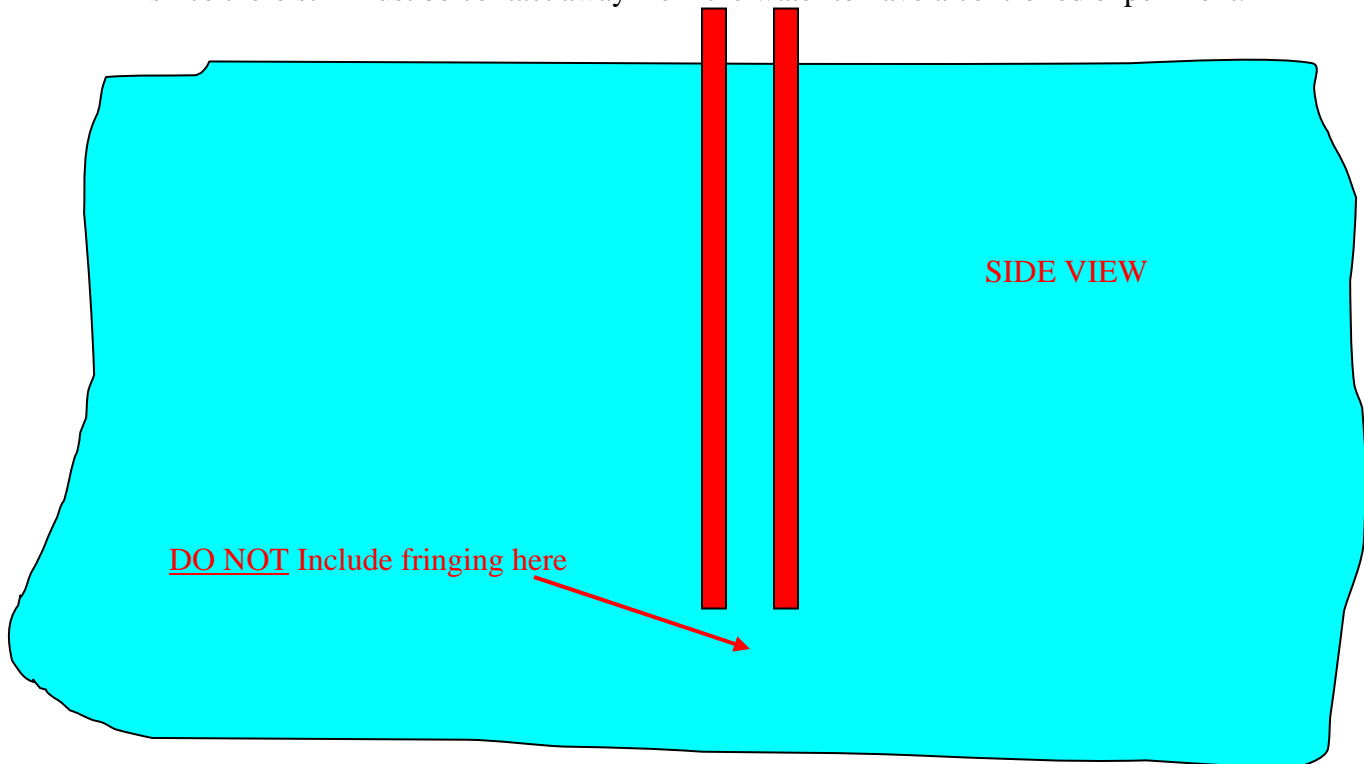


Note that there are 24 equipotential steps between the two plates in this flux plot. As noted below, the separation between the plates will be 1.2 cm so that it is easier to figure out the size of each little box since we do not show all of the boxes in the uniform field region.

Homework 5



Two capacitor plates are immersed in the bodies of water and the resistance between them is measured to determine the salt content. The plates are only partially submerged, since there still must be contact away from the water to have a controlled experiment.



Assume that the field pattern around the plates looks like the pattern given above and that the plates are immersed 50 cm into the water, the plates are 5 cm wide and separated by 1.2 cm. Treat the problem as two dimensional. That is, ignore the fringing at the bottom edge of the plates, only considering it on the two sides, since the plates are much longer than they are wide. Using the graphical information given, and the conductivity data for the water, determine the resistance that would be observed in Lake Superior and the Great Salt Lake. To increase your accuracy, you should sketch some additional field lines and equipotentials for the remainder of the fringing field outside of the plates. Also determine the resistance assuming no fringing as a comparison. Note that the separation is 1.2 cm to make it easy to figure out the size of each little box in the flux plot. Also, be sure that you incorporate the depth of the cells properly when you determine resistance.

EC Meters

The numbers used for the resistance measurement in problem 1 are not realistic, since no one wants such a large meter. However, they make for a more interesting problem for us. Just so that you know a bit more about the actual devices used in the field based on conductivity measurements, here is some additional information. First, some commercially available devices:

BlueLab Truncheon (New Zealand) <http://www.bluelabassist.tx.co.nz/index.html>



Oakton Instruments (Illinois) <http://www.4oakton.com/>



Links from GlobalSpec

http://search.globalspec.com/Industrial/tds/conductivity_and_TDS

Using EC Meters

<http://www.gemplers.com/a/pages/iecmeters.asp>

Homework 5

2. Properties of Magnetic Fields The following expressions characterize magnetic fields. Using Maxwell's equations in either differential or integral form, or Laplace's or Poisson's equations for magnetic fields, demonstrate that these are indeed correct solutions. Draw a picture to go with each question to show that you understand the configuration.

a. $\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{a}_\phi$ for $r > a$ and $\vec{B} = \frac{\mu_0 I r}{2\pi a^2} \hat{a}_\phi$ for $r < a$ are the magnetic fields for a cylindrical wire of radius a carrying a current I uniformly distributed throughout its cross section.

b. $\vec{B} = \frac{\mu_0 N I}{d} \hat{a}_z$ for $r < a$ and $\vec{B} = 0$ for $r > a$ are the magnetic fields for a cylindrical solenoid of radius a , length d , wound with N turns of wire carrying a current I . The current is carried in a single layer of wire, whose thickness is so small that it can be neglected.

3. Field Direction The three parallel current-carrying wires shown below will produce a net magnetic field at each of the four points indicated. Determine the direction of the magnetic flux density \vec{B} and the magnetic vector potential \vec{A} at each of the four points. Assume that the currents marked as \otimes are in the z -direction and those marked with \odot are in the negative z -direction. The other two axes are shown.

