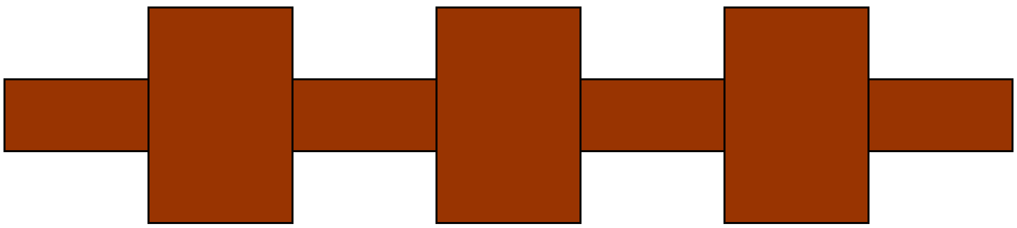
Homework 5

Due 21 March 2006

1. **Resistance Measurement**

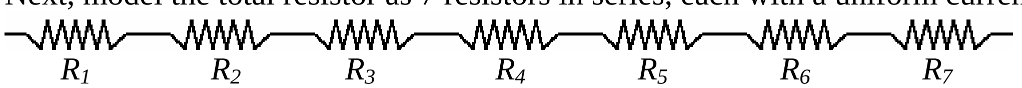
A resistor is deposited on the surface of a printed circuit board in the pattern shown below. The thickness of the layer is *0.2mm*.



The width of the narrow regions is *3mm* while the wider regions are *9mm* in width. Each region is *6mm* long, so the total length of the resistor is *42mm*. Note, these are not really typical dimensions.

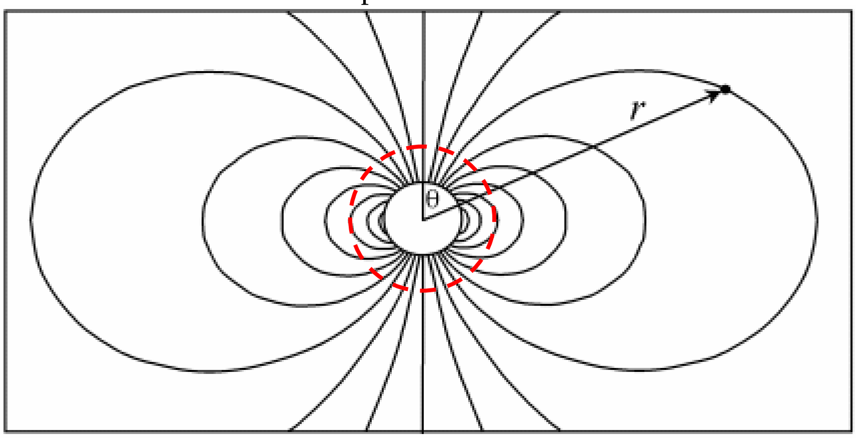
a. Determine the resistance of the region if the resistive material is nichrome. Data on this material can be found at http://www.8886.co.uk/ref/resistivity_values.htm First, model the total resistor as one resistor with the same total length whose width is equal to the average width, taking into account all seven regions.

Next, model the total resistor as 7 resistors in series, each with a uniform current density.



b. The current does really flow uniformly since it does not turn sharply at the boundary between regions. Rather, the flow pattern looks something like a standing wave. To better model the actual current, assume that the width varies with position according to $w(z) = w_o\left(1 - 0.5\sin\frac{2\pi z}{d}\right)$ where w_o and d are both *6mm*. This will produce a smoothly varying width that is similar to the step changes in the diagram above. For a variable width resistor, we need to use an expression like the one on page VIII-4 of *Connor & Salon* (as indicated in the reading for Lecture 14): $R = \int \frac{dl}{\sigma(l)S(l)}$ which allows both the conductivity and the area of the resistor to vary along its length. For this problem, we have constant conductivity (all of the resistor material is nichrome) and the thickness is also constant so only the width varies. Determine the resistance using this model.

2. **Properties of Magnetic Fields** The following expression characterizes the magnetic field of a dipole, but is valid only at distances large compared with the radius of the dipole. Using Maxwell's equations in differential form for magnetic fields, you are to demonstrate that this is indeed a correct solution. We can use the form of the field expression given in Ulaby section 5-2.2. $\vec{B} = \frac{\mu_o m}{4\pi R^3}\left(\hat{R}2\cos\theta + \hat{\theta}\sin\theta\right)$ where $m = I\pi a^2$ is the magnetic moment of the current loop with radius *a*.

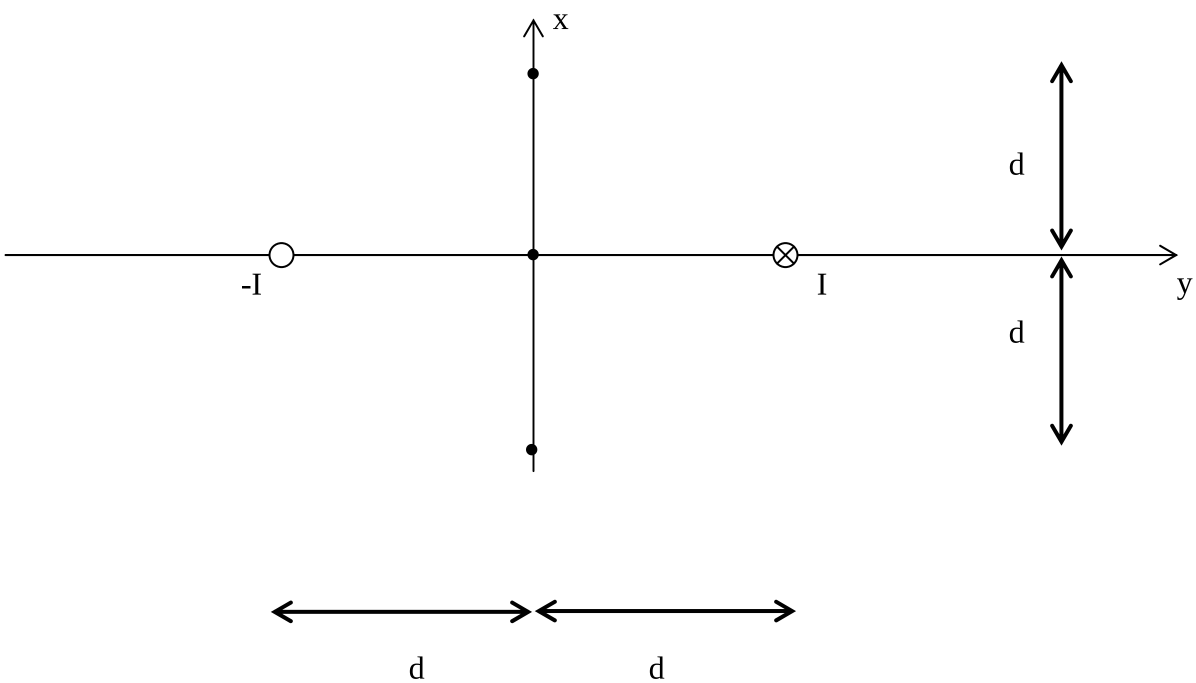


The above figure should help to understand this configuration.

a. Show that the divergence and curl of this expression have the expected values far from the dipole. That is, evaluate the expressions and explain why your answer is correct.

b. Evaluate the flux of the magnetic field through a sphere of radius $b >> a$. That is evaluate the integral $\oint \vec{B} \cdot d\vec{S} = ?$ for the surface shown in red above. First, provide the expression for the surface element $d\vec{S}$ and then evaluate the integral. Explain why your answer is correct.

3. **Field Direction** The two parallel current-carrying wires shown below will produce a net magnetic field at each of the three points indicated. Determine the direction of the magnetic flux density $\vec{B}$ at each of the three points. Assume that the currents marked as $\otimes$ are in the z-direction and those marked with $\bigcirc$ are in the negative z-direction. The other two axes are shown.



Find the expression for the magnetic flux density $\vec{B}(x)$ everywhere on the *x-axis*. Remember that this expression is a vector, so you will need both the magnitude and the direction. Your result should be a function of *x*.