Due 28 March 2008 at 12:00 noon (Friday at lunch time)

The first part of this assignment is a continuation of Quiz 2 in that it addresses the issue of determining the resistance of a complex structure. In this case, it is the resistance of a grounding rod, driven into the soil to provide a solid ground for a home or business. To address this issue, you will also be making use of an IEEE Standard, which is something you likely will also be asked to do on the job. There are standards on many, many issues and you have full access to them through the RPI Library. You can find them at http://ieeexplore.ieee.org/xpl/standards.jsp, but you must access this site from campus or use the Library Proxy. I have downloaded the most important standard and posted it at http://hibp.ecse.rpi.edu/~connor/education/Fields/IEEEStd142_2007.pdf. Figure 4-1 from this document is shown below which provides the configuration of a buried conducting rod.

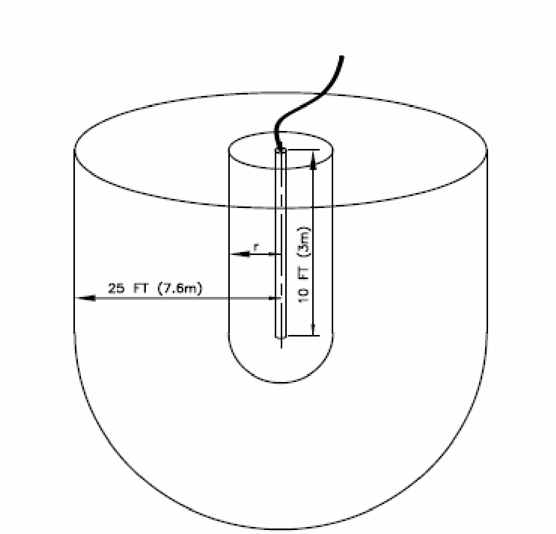
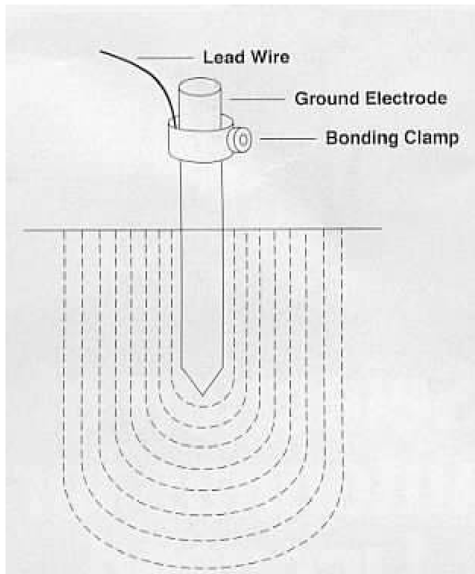


Figure 4-1—Electrode resistance development



Some other references:

Ground Rods: http://www.hubbellpowersystems.com/powertest/tips_news/pdfs_best/05-2002.pdf

Deep Earth Grounding: http://www.cpccorp.com/deep.htm

Grounding Fundamentals:
http://www.usda.gov/rus/telecom/publications/word_files/1751f802.doc

It is also helpful to read over the information available largely through the Fields and Waves I website on Flownets, curvilinear squares, flux and a few other topics.

From the class notes (Connor and Salon), read over II-33 to II-35, III-11 to III-12, IV-17 to IV-20. Also review the discussion in section 2-2 of Ulaby in which he notes that for TEM transmission lines, $\frac{G}{C} = \frac{\sigma}{\varepsilon}$ so that if we know how to find the capacitance of some configuration, we can determine the conductance by just replacing ε by σ. We will be using this in some calculations.

From Wikipedia http://en.wikipedia.org/wiki/Flux for flux and http://en.wikipedia.org/wiki/Flownet for Flownets. Also, from mostly Civil Engineering sources, since they use Flownets, we have http://doctorflood.rice.edu/envi518/Handouts/Ch02Flownets.ppt and http://uwp.edu/~li/geol460-00/chapter4.htm

Also from Wikipedia on grounding http://en.wikipedia.org/wiki/Ground_(electricity)

Facility and Equipment Grounding from IEEE http://www.ewh.ieee.org/r3/nashville/events/2007/2007.02.07_A.pdf

RF Earth System http://www.g4nsj.co.uk/earth.shtml

Curvilinear Squares and Flux Plotting: http://www.aa.washington.edu/courses/aa419/ch04_supp.pdf

From Google Books (http://books.google.com)

Electric Power Distribution by A. S. Pabla

Transmission Lines, Matching, and Crosstalk by Kenneth L. Kaiser

Practical Grounding, Bonding, Shielding and Surge Protection by G. Vijayaraghavan

Lightning: Physics and Effects by V. A. Rakov and M. A. Uman

You will also find some interesting information on grounding if you look up counterpoise.

If you find any other good references, let me know.

On the next page is a table of formulas for the resistance of a wide variety of grounding systems. We will look at only two of them (the first two) – the hemisphere and the ground rod. The latter, in effect, includes the former since the end of the rod can be modeled as a hemisphere.

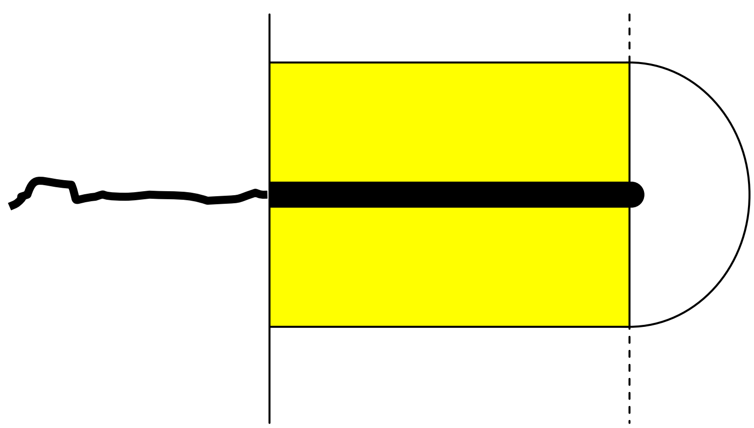
Table 4-5—Formulas for the calculation of resistances to ground

	Hemisphere radius a	$R = \frac{\rho}{2\pi a}$
•	One ground rod length L, radius a	$R = \frac{\rho}{2\pi L}\left(\ln\frac{4L}{a} - 1\right)$
• •	Two ground rods $s > L$; spacing s	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} - 1\right) + \frac{\rho}{4\pi s}\left(1 - \frac{L^2}{3s^2} + \frac{2L^4}{5s^4} \cdots\right)$
••	Two ground rods $s < L$; spacing s	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} + \ln\frac{4L}{s} - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4} \cdots\right)$
—	Buried horizontal wire length $2L$, depth $s/2$	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} + \ln\frac{4L}{s} - 2 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4} \cdots\right)$
L	Right-angle turn of wire length of arm L, depth $s/2$	$R = \frac{\rho}{4\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} - 0.2373 + 0.2146\frac{s}{L} + 0.1035\frac{s^2}{L^2} - 0.0424\frac{s^4}{L^4} \cdots\right)$
⅄	Three-point star length of arm L, depth $s/2$	$R = \frac{\rho}{6\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 1.071 - 0.209\frac{s}{L} + 0.238\frac{s^2}{L^2} - 0.054\frac{s^4}{L^4} \cdots\right)$
+	Four-point star length of arm L, depth $s/2$	$R = \frac{\rho}{8\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 2.912 - 1.071\frac{s}{L} + 0.645\frac{s^2}{L^2} - 0.145\frac{s^4}{L^4} \cdots\right)$
✳	Six-point star length of arm L, depth $s/2$	$R = \frac{\rho}{12\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 6.851 - 3.128\frac{s}{L} + 1.758\frac{s^2}{L^2} - 0.490\frac{s^4}{L^4} \cdots\right)$
✳	Eight-point star length of arm L, depth $s/2$	$R = \frac{\rho}{16\pi L}\left(\ln\frac{2L}{a} + \ln\frac{2L}{s} + 10.98 - 5.51\frac{s}{L} + 3.26\frac{s^2}{L^2} - 1.17\frac{s^4}{L^4} \cdots\right)$
○	Ring of wire diameter of ring D, diameter of wire d, depth $s/2$	$R = \frac{\rho}{2\pi^2 D}\left(\ln\frac{8D}{d} + \ln\frac{4D}{s}\right)$
▬	Buried horizontal strip length $2L$, section a by b, depth $s/2$, $b < a/8$	$R = \frac{\rho}{4\pi L}\left(\ln\frac{4L}{a} + \frac{a^2 - \pi ab}{2(a+b)^2} + \ln\frac{4L}{s} - 1 + \frac{s}{2L} - \frac{s^2}{16L^2} + \frac{s^4}{512L^4} \cdots\right)$
●	Buried horizontal round plate radius a, depth $s/2$	$R = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 - \frac{7}{12}\frac{a^2}{s^2} + \frac{33}{40}\frac{a^4}{s^4} \cdots\right)$
	Buried vertical round plate radius a, depth $s/2$	$R = \frac{\rho}{8a} + \frac{\rho}{4\pi s}\left(1 + \frac{7}{24}\frac{a^2}{s^2} + \frac{99}{320}\frac{a^4}{s^4} \cdots\right)$

NOTE—In Table 4-5, for 3 m (10 ft) rods of 12.7 mm, 15.88 mm, and 19.05 mm (1/2 in, 5/8 in, and 3/4 in) diameters, the grounding resistance may be quickly determined by dividing the soil resistivity ohm-cm, by 288, 298, and 307, respectively.[2]

1. First, to see that the hemisphere model makes sense, you should derive it from what you know about spherical capacitors. Begin by writing down the expression for the capacitance of a spherical structure consisting of two electrodes, one of radius *a* and one of radius *b*, where the region $b>r>a$ is filled with an insulator with dielectric constant ε. A hemispherical capacitor will be just half of this expression, since it is only half the size of a full sphere. Write this expression down and recall that two of these in parallel will make up a full sphere. Next, convert this expression to conductance using $\frac{G}{C} = \frac{\sigma}{\varepsilon}$. Then, invert the expression for the conductance to resistance by inverting it. $R = \frac{1}{G}$. We will come back to this expression in a minute, but first use it to find the expression for an isolated spherical electrode by setting $b = \infty$. You should find that this will now look like the first formula in the table on the previous page if you note that $\rho = \frac{1}{\sigma}$ is the resistivity of the surrounding material (soil). For this type of calculation, it is more typical to quote resistivity rather than conductivity but they are clearly equivalent. Usually in a Fields course, we avoid using resistivity since the use of the Greek letter ρ is reserved for charge density. Evaluate this expression for a sphere with diameter *16mm*. It turns out that the hemispherical ground will act as if it is in an infinite medium as long as it is reasonably well isolated from other objects. To see how isolated it has to be, return to the expression for the spherical capacitor with outer electrode at *b*. Given $a = 8mm$, find the value of *b* that produces *99.9%* of the resistance of an isolated hemisphere.

2. Next, consider a *16mm* diameter, *3m* long rod such as the one shown in figure 4-1 of the IEEE standard. Use the second formula in the table to find the resistance of this configuration in terms of an unspecified soil resistivity. Then, we want to see that this formula makes some sense by finding a simple approximation to the configuration. Rather than addressing the actual configuration, assume that it can be thought of as a combination of a coaxial cable with the outer conductor far away and a hemispherical end on the center conductor of the ground rod.



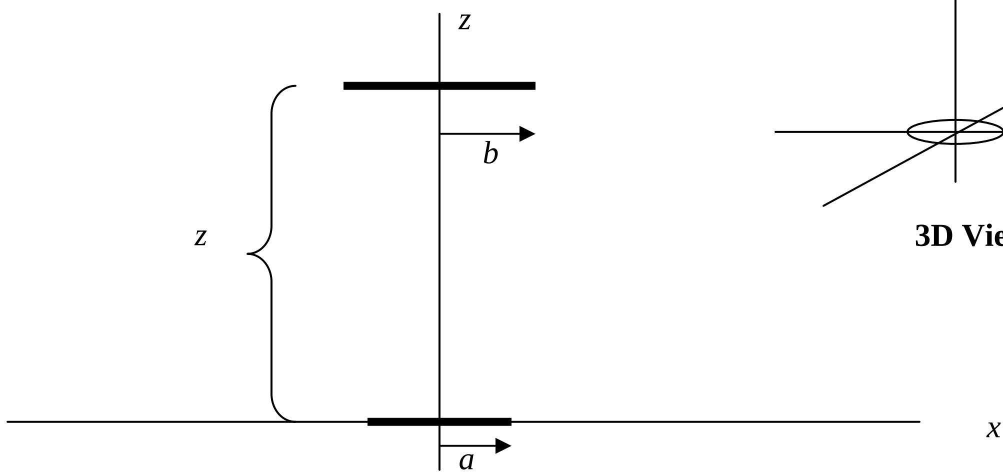
The figure above shows a wire connected to a ground rod (in black). The rod looks like a coax for its length *L* and then has a hemispherical end cap. One of our problems in modeling the coaxial cable is that we need an outer conductor or the resistance or capacitance of the structure will appear to be infinite if $b = \infty$. To keep things realistic, we will use the value for *b* found in the previous problem that produced *99.9%* of the resistance of an isolated hemisphere. We will discuss why this is a good model after we make our calculation. Again using the relationship $\frac{G'}{C'} = \frac{\sigma}{\varepsilon}$, where now *G'* and *C'* are per unit length values, find the conductance for a ground rod of length *L*. Invert this expression to get the resistance. Combine this resistance in parallel with the resistance of the hemispherical end cap (as you found in the first problem) to find the total resistance of the ground rod. Compare your answer with the resistance found using the second formula in the table. You should find that they are close but not identical, since our method is an approximation. The fact that they are reasonably close shows that the formula from the table is reasonable to use. Also, the fact that it is referenced by pretty much everyone in the world who does grounding should also give us some confidence that it is reasonably correct.

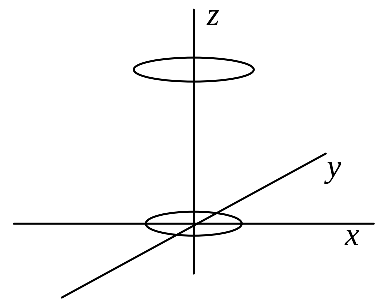
3. Now, assume that the grounding rod is located in soil with an average resistivity of 40,000 $\Omega - cm$ (see table 4-2 of the Standard document). Evaluate the expression for the resistance of the ground rod using this resistivity. Your value should be in the range listed in the table. Then, look through the IEEE Standard document and find the typical surge current for a lightning stroke. Using this current, find the voltage that will be produced at the feed end of the ground rod. You will see that it is quite an impressive value even for a typical stroke. For a high current stroke it is really big.

Magnetic Flux

4. Read over the second problem in HW 5 for Spring 2007, which involves the flux produced by a magnetic dipole. Such a field is produced by a current *I* in a circular loop of radius *a* located at the origin of our coordinate system. $\vec{B} \approx \frac{\mu_o a^2 I}{4r^3}\left(\hat{r}2\cos\theta + \hat{\theta}\sin\theta\right)$ where we have noted that this is an approximate expression. It is valid only for *r* >> *a*. From the latter part of Lecture 15 (see the slides), you will see that we can also represent the magnetic field using the vector potential $\vec{A}$. The link between the two expressions is $\vec{B} = \nabla \times \vec{A}$. From Stokes' Theorem, we also know that $\int \nabla \times \vec{A} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$ so that we can find the magnetic flux passing through some surface using either $\vec{A}$ or $\vec{B}$. The expression for flux is $\psi_m = \int \vec{B} \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$. Using the expression for $\vec{A}$ has a big advantage in the present problem, since we can evaluate the flux and never have to violate the requirement that the field expression we have is valid only for *r* >> *a*. Before

we do that, we need the dipole expression for $\vec{A}$. Looking it up online, one finds that all expressions look like $\vec{A} = K\dfrac{\sin\theta}{r^2}\hat{\phi}$ where K is a constant. Your first step then is to evaluate $\vec{B} = \nabla \times \vec{A}$, compare the result with our approximate expression for $\vec{B}$ and the identify the value of K. Next, find the flux passing through the coaxial loop of radius b whose center is located at z using the flux expression for the vector potential. *Hint: using* $\vec{A}$ *to find the flux, makes this task very simple.* (The following figure is copied from HW5 for Spring 2007)





3D View

Using the following parameters, find and plot the flux for z from *-50* to *50*. Assume that $I=100$, $a=1$, $b=5$. You can leave your expression in terms of μ_o since that keeps things simpler. Note that this can be done using Excel or Matlab, so use the one you know best. When one does the experiment of dropping a magnet through a conducting pipe, there will be a current and voltage induced in the pipe that is proportional to the time derivative of the flux. Since the dipole flux does not change, but rather the magnet or coil are moved, the time rate of change of the magnetic flux will be

$$\frac{d\psi_m}{dt} = \frac{d}{dt}\int \vec{B}\cdot d\vec{S} = \frac{d}{dt}\oint \vec{A}\cdot d\vec{l} = \frac{d\psi_m}{dz}\frac{dz}{dt} = \frac{d\psi_m}{dz}v$$

where v is the velocity of the magnet or coil. Thus, to see what the induced current looks like, we need to evaluate the z derivative of the flux. $\dfrac{d\psi_m}{dz} = ?$. This can either be done numerically or analytically (your choice). Either using the values for flux you evaluated above or the analytic expression for the derivative, find and plot the derivative of the flux with respect to z. Again, you can use either Excel or Matlab to plot your results.