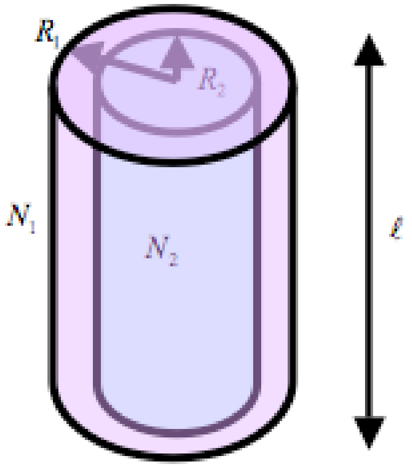
Homework 5
Fields and Waves I
Fall 2007

1. We have 2 solenoid coils. The outer one has N1 turns and a radius of R1 and the inner one has N2 turns and a radius of R2. They are parallel to each other and both are the same length l, We can assume that l >> R1 or R2. The inner cylinder is filled with a magnetic material of relative permeability of 100. Find the self inductance of each coil and the mutual inductance.



Inner cylinder:

$$H_2 = \frac{N_2 I_2}{l}$$

$$B_2 = \mu H_2 = \mu_0 \mu_r \frac{N_2 I_2}{l}$$

$$\Phi_2 = \int_0^{r_2} B_2 \cdot dS = \pi r_2^2 \mu_0 \mu_r \frac{N_2 I_2}{l}$$

$$\Lambda_2 = N_2 \Phi_2 = \pi r_2^2 \mu_0 \mu_r \frac{N_2^2 I_2}{l}$$

$$L_2 = \frac{\Lambda_2}{I_2} = \pi r_2^2 \mu_0 \mu_r \frac{N_2^2}{l}$$

Outer cylinder:

$$H_1 = \frac{N_1 I_1}{l}$$

$$B_1 = \mu H_1 = \mu_0 \frac{N_1 I_1}{l}$$

$$\Phi_1 = \int_0^{r_2} \mu_0 \mu_r H_1 \cdot dS + \int_{r_2}^{r_1} \mu_0 H_1 \cdot dS = \pi r_2^2 \mu_0 \mu_r \frac{N_1 I_1}{l} + \pi \mu_0 \frac{N_1 I_1}{l}\left(r_1^2 - r_2^2\right)$$

$$\Lambda_1 = N_1 \Phi_1 = \pi r_2^2 \mu_0 \mu_r \frac{N_1^2 I_1}{l} + \pi \mu_0 \frac{N_1^2 I_1}{l}\left(r_1^2 - r_2^2\right)$$

$$L_1 = \frac{\Lambda_1}{I_1} = \pi \mu_0 \frac{N_1^2}{l}\left(r_2^2 \mu_r + \left(r_1^2 - r_2^2\right)\right)$$

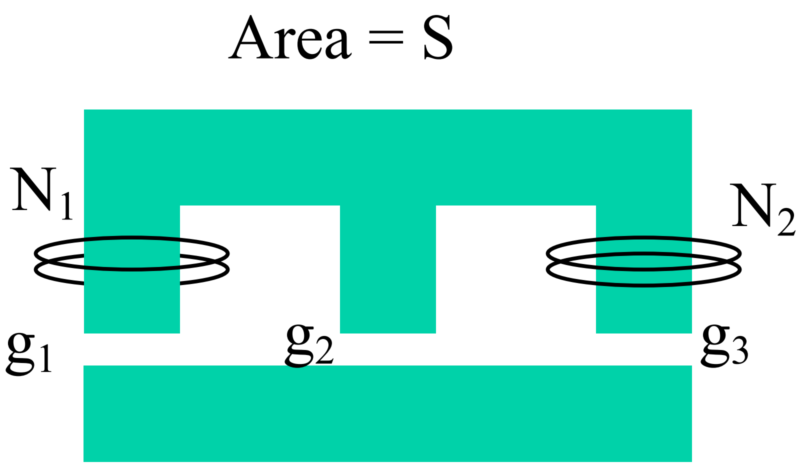
Mutual inductance:

$$\Phi_{12} = \int_0^{r_2} B_1 \cdot dS = \pi r_2^2 \frac{\mu_0 \mu_r N_1 I_1}{l}$$

$$\Lambda_{12} = N_2 \Phi_{12} = \pi r_2^2 \frac{\mu_0 \mu_r N_1 N_2 I_1}{l}$$

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \pi r_2^2 \frac{\mu_0 \mu_r N_1 N_2}{l}$$

2. A magnetic circuit is shown below. Assume that $\mu = \infty$ for the iron parts. Draw the equivalent circuit and evaluate the reluctances. Find the flux density in the three air gaps. Find the self and mutual inductances of the coils.



$$R_1 = \frac{g_1}{\mu_0 S} \quad R_2 = \frac{g_2}{\mu_0 S} \quad R_3 = \frac{g_3}{\mu_0 S}$$

$$N_1 I_1 = \Phi_1 R_1 + (\Phi_1 + \Phi_2) R_2$$

$$N_2 I_2 = \Phi_2 R_3 + (\Phi_1 + \Phi_2) R_2$$

$$\Phi_1 = \frac{N_1 I_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}, \quad \Lambda_1 = \frac{N_1^2 I_1}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$\Phi_2 = \frac{N_2 I_2}{R_3 + \frac{R_2 R_1}{R_2 + R_1}}, \quad \Lambda_2 = \frac{N_2^2 I_2}{R_3 + \frac{R_2 R_1}{R_2 + R_1}}$$

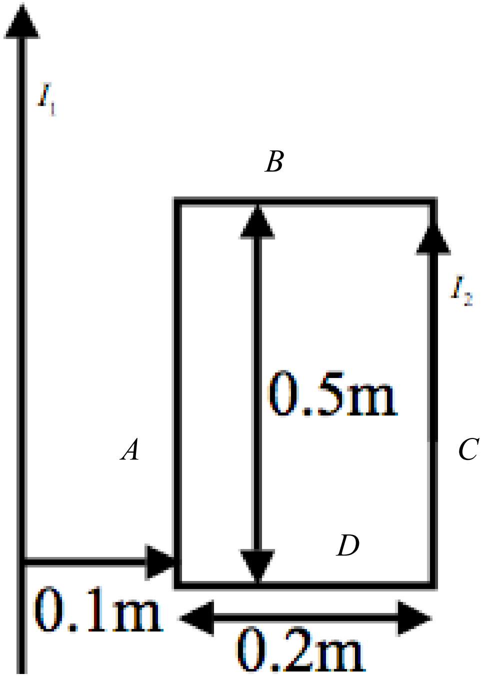
$$L_1 = \frac{\Lambda_1}{I_1} = \frac{N_1^2}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$L_2 = \frac{\Lambda_2}{I_2} = \frac{N_2^2}{R_3 + \frac{R_2 R_1}{R_2 + R_1}}$$

$$\Phi_{12} = I_1 \left(\frac{R_2}{R_2 + R_3} \right), \quad \Lambda_{12} = N_2 \Phi_{12} = \frac{N_1 N_2 I_1 \left(\frac{R_2}{R_2 + R_3} \right)}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

$$L_{12} = \frac{\Lambda_{12}}{I_1} = \frac{N_1 N_2 \left(\frac{R_2}{R_2 + R_3} \right)}{R_1 + \frac{R_2 R_3}{R_2 + R_3}}$$

3. In the figure below we have a current of I_1 = 10 Amperes in the y direction. There is a loop with current I_2 = 15 Amperes. Find the force on the loop.



No force on sides B and D.

On side A:

$$F = il \times B$$

$$F = (15\text{A})(0.5\text{m})\left(\frac{\mu_0(10\text{A})}{2\pi(0.1\text{m})}\right) = 150\mu\text{N}\ \hat{a}_x$$

On side C:

$$F = (15\text{A})(0.5\text{m})\left(\frac{\mu_0(10\text{A})}{2\pi(0.3\text{m})}\right) = 50\mu\text{N}\ -\hat{a}_x$$

Total force:

$$F = (15\text{A})(0.5\text{m})\left(\frac{\mu_0(10\text{A})}{2\pi}\right)\left(\frac{1}{0.1\text{m}} - \frac{1}{0.3\text{m}}\right) = 100\mu\text{N}\ \hat{a}_x$$

4. In a cylindrical coordinate system the magnetic field in a conducting region is $H = \frac{4}{r}\left(1-(1+2r)e^{-2r}\right)\hat{a}_\phi$. Find the current density.

$$J = \nabla \times H$$

$$J = \frac{1}{r}\left(\frac{\delta}{\delta r}(rH)\right)\hat{a}_z$$

$$J = \frac{1}{r}\left(\frac{\delta}{\delta r}\left(r\left(\frac{4}{r}\left(1-(1+2r)e^{-2r}\right)\right)\right)\right)\hat{a}_z$$

$$J = \frac{1}{r}\left(8^{-2r}(1+2r) - 8e^{-2r}\right)\hat{a}_z$$

$$J = 16e^{-2r}\hat{a}_z$$

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