Problem Solution # 6

Problem 1

$B_{1n} = B_{2n}$

$\int \vec{H} \cdot d\vec{l} = NI \qquad \vec{H} = \frac{\vec{B}}{\mu} \qquad \frac{B}{\mu} l + \frac{B}{\mu_0} g = NI$

as $\mu \to \infty$, $\quad B = \frac{NI\mu_0}{g}$

$$\Phi = \int \vec{B} \cdot d\vec{s} = \frac{NI\mu_0}{g} A = \frac{10 \times 6 \times 4\pi \times 10^{-7} \times 2 \times \cancel{10^{-3}}}{2.5 \times \cancel{10^{-3}}} \approx 6 \times 10^{-5} (Wb)$$

$$\Lambda = N\Phi \qquad L = \frac{\Lambda}{I} = \frac{N\Phi}{I} = \frac{10 \times 6 \times 10^{-5}}{6} = 10^{-4} (H)$$

as $\mu_r = 4000$, $\quad \mu = \mu_r \mu_0 \qquad B = NI\mu_0 \Big/ \left(\frac{l}{2000} + g \right)$

$$\Phi = \int \vec{B} \cdot d\vec{s} = NI\mu_0 A \Big/ \left(\frac{l}{4000} + g \right) = \frac{10 \times 6 \times 4\pi \times 10^{-7} \times 2 \times 10^{-3}}{\left(\frac{0.95}{4000} + 2.5 \times 10^{-3} \right)} \approx 5.5 \times 10^{-5} (Wb)$$

$$L = \frac{\Lambda}{I} = \frac{N\Phi}{I} = \frac{10 \times 5.5 \times 10^{-5}}{6} \approx 9.17 \times 10^{-5} (H)$$

Problem 2

$\Phi = \int \vec{B} \cdot d\vec{s} = BA$

$V = -N \frac{d\Phi}{dt} = -NA \frac{dB}{dt} = -100 \times 0.15 \times 1.2 \times 377 \cos(377t) = -6786 \cos(377t) (V)$

From the plot we can see H=250A/m as B=1.2T.

$\int \vec{H} \cdot d\vec{l} = Hl = 250 \times 0.25 = NI = 100I \Rightarrow I = 0.625 (A)$

Problem 3

$I = \frac{V}{R} \qquad F = BIl = \frac{BVl}{R}$

When $V_{induced} = V$, $\quad v = v_{\max}$. $V_{induced} = Blv \quad \Rightarrow \quad v_{\max} = \frac{V}{Bl}$

$F_{mech} = F = BI'l \Rightarrow I' = \frac{F_{mech}}{Bl}$

$$F_{mech} = \frac{B\left(V - V_{induced}\right)l}{R} = \frac{Bl}{R}\left(V - Blv'\right) \Rightarrow v' = \frac{V}{Bl} - \frac{RF_{mech}}{(Bl)^2}$$

power balance: $VI' - I'^2R = V\left(\frac{V - Blv'}{R}\right) - \left(\frac{V - Blv'}{R}\right)^2 R = \left(\frac{V - Blv'}{R}\right)Blv' = F_{mech}v'$

Problem 4

a)

$$B = \frac{\mu_0 I}{2\pi r} \qquad V_{induced} = \left(\frac{\mu_0 I}{2\pi r}\Big|_{r=y_0} - \frac{\mu_0 I}{2\pi r}\Big|_{r=y_0+0.1}\right)lv = 2\times10^{-6}\left(\frac{1}{y_0} - \frac{1}{y_0+0.1}\right)\times 0.2\times 5$$

$$I = \frac{V_{induced}}{R} = 10^{-7}\times\left(\frac{1}{y_0} - \frac{1}{y_0+0.1}\right)A$$

b)

$$\Phi = \int \vec{B}\cdot d\vec{s} = \frac{\mu_0 i}{2\pi}\int_{y_0}^{y_0+0.1}\left(\frac{1}{y}\right)dy\times 0.2 = \frac{\mu_0 i}{10\pi}\ln\left(\frac{y_0+0.1}{y_0}\right) = \frac{\mu_0}{\pi}\ln\left(\frac{y_0+0.1}{y_0}\right)\sin(1000t)$$

$$V = -\frac{d\Phi}{dt} = -1000\frac{\mu_0}{\pi}\ln\left(\frac{y_0+0.1}{y_0}\right)\cos(1000t) = -4\times10^{-4}\ln\left(\frac{y_0+0.1}{y_0}\right)\cos(1000t)$$

$$I = \frac{V}{R} = -2\times10^{-5}\ln\left(\frac{y_0+0.1}{y_0}\right)\cos(1000t)$$