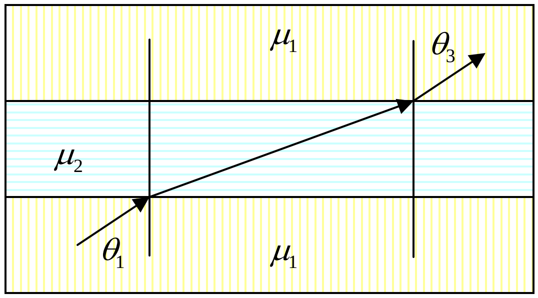
Homework 6

Due 5 April 2005

1. Boundary Conditions



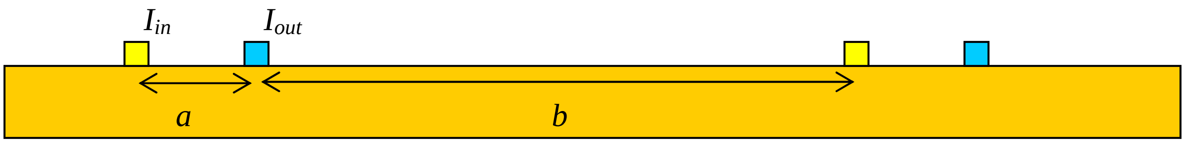
A magnetic field vector is obliquely incident on a slab of a material with a different permeability μ. From your knowledge of boundary conditions, determine the relationship between θ_1 and θ_3 where these angles are measured with respect to the normal.

From the BC we know that $B_{n1} = B_{n2}$ *and* $H_{t1} = H_{t2}$ *or* $\frac{B_{t1}}{\mu_1} = \frac{B_{t2}}{\mu_2}$ *at the first boundary.*

At the second boundary $B_{n2} = B_{n3}$ *and* $\frac{B_{t2}}{\mu_2} = \frac{B_{t3}}{\mu_1}$. *Combining these expressions we see that* $B_{n1} = B_{n3}$ *and* $\frac{B_{t1}}{\mu_1} = \frac{B_{t3}}{\mu_1}$. *Thus,* $\theta_1 = \theta_3$

2. Mutual Inductance

On a printed circuit board, there are two nearby wire traces carrying signals. We wish to determine the coupling between the traces. The board is made of an insulating material so it has no affect on the magnetic field produced by the currents.



Assume that there is current flowing in the pair on the left, with the current into the page in the left-most wire and returning in the right. The wire pairs are separated by a distance *a* and the two sets of wires are separated from one another by a distance *b*.

a. Determine the magnetic field produced by the currents in the left hand wire pair.

Each of the wires produces the magnetic field of a long straight wire centered on the wire: $\vec{B} = \hat{\phi}\frac{\mu_o I}{2\pi r}$. *To be rigorous, this expression should be written for each of the wires in rectangular coordinates and then added up. This is not necessary for the solution of this problem, as long as it is clear that both wires produce a field contribution. Here, we will show the entire solution, but only in the plane of the wires, since this is where we need the field to find the flux. If we choose the center of our coordinate system as the location of the left most wire, then the magnetic field on this plane is given by*

$$B_y = -\frac{\mu_o I}{2\pi x} + \frac{\mu_o I}{2\pi(x-a)}$$

b. Determine the flux linked by the second pair of wires. (Per unit length)

$$\psi_m = \int_{a+b}^{2a+b}\left(-\frac{\mu_o I}{2\pi x} + \frac{\mu_o I}{2\pi(x-a)}\right)dx = -\frac{\mu_o I}{2\pi}\int_{a+b}^{2a+b}\frac{dx}{x} + \frac{\mu_o I}{2\pi}\int_{b}^{a+b}\frac{dx}{x}$$

$$= \frac{\mu_o I}{2\pi}\left(\ln(a+b) - \ln(2a+b) + \ln(a+b) - \ln(b)\right) = \frac{\mu_o I}{2\pi}\ln\frac{(a+b)^2}{(2a+b)b}$$

If we assume that b >> a, then

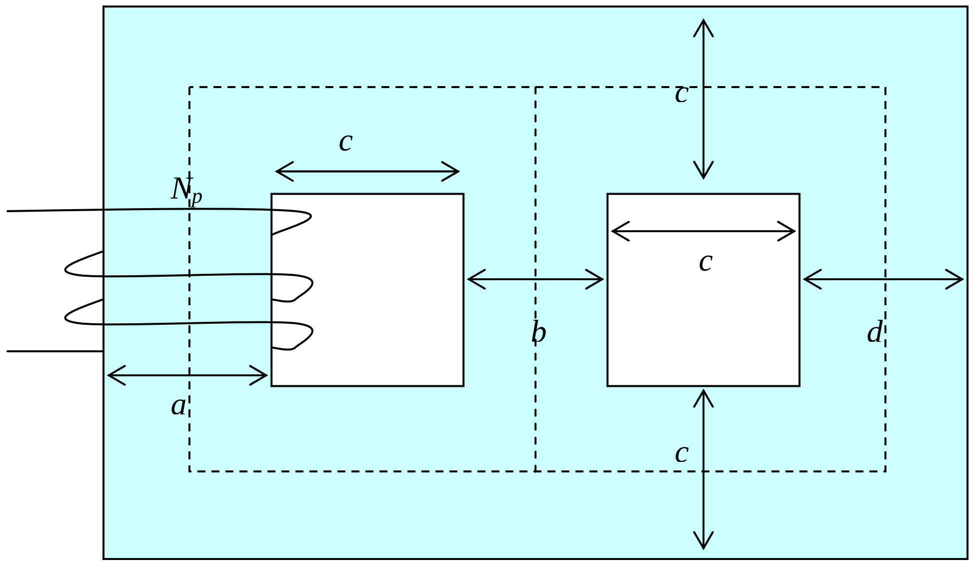
$$\psi_m = \frac{\mu_o I}{2\pi}\ln\frac{(a+b)^2}{(2a+b)b} = \frac{\mu_o I}{2\pi}\ln\frac{b^2\left(1+\frac{2a}{b}+\frac{a^2}{b^2}\right)}{b^2\left(1+\frac{2a}{b}\right)} \approx \frac{\mu_o I}{2\pi}\frac{a^2}{b^2}$$

but this information is only provided for completeness.

c. Determine the mutual inductance between the two sets of wires.

$$L_{12} = M = \frac{\mu_o}{2\pi}\ln\frac{(a+b)^2}{b(2a+b)}$$

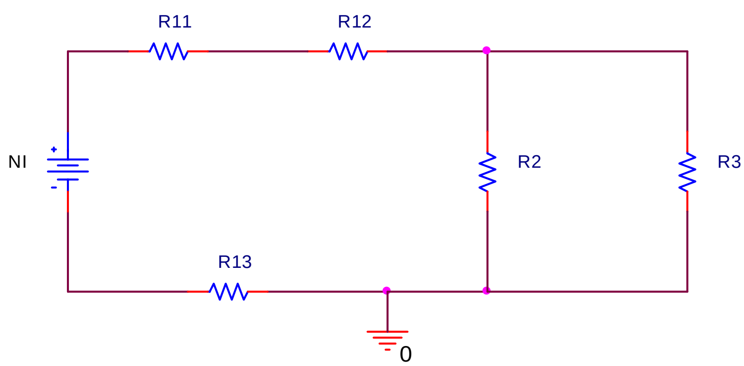
3. Magnetic Circuits



Note the dashed lines have been added to determine the length of each reluctance element.

a. A magnetic core with the geometry shown has N_p windings wrapped around its left post. The core has a rectangular cross section with depth w. The permeability of the core is μ. Using the magnetic circuit technique, find the inductance of this configuration and the total energy stored for a current I in the coil. Note that the total width of the core is $a+b+2c+d$ and the total height is $3c$.

The magnetic circuit looks like



where $R_{11} = \frac{2c}{\mu(aw)}$, $R_{12} = \frac{0.5a + c + 0.5b}{\mu(cw)}$, $R_{13} = R_{12}$, $R_2 = \frac{2c}{\mu(bw)}$, &

$R_3 = 2\frac{0.5b + 0.5d + c}{\mu(cw)} + \frac{2c}{\mu(dw)}$. *The total resistance is* $R = R_{11} + 2R_{12} + R_2 || R_3$

$\psi_m = \frac{N_p I}{R}$ *and* $L = \frac{N_p \psi_m}{I}$. *The total flux divides into the two parallel reluctance inversely with their reluctance values. Thus,* $\psi_{m2} = \psi_m \frac{R_3}{R_2 + R_3}$ *and* $\psi_{m3} = \psi_m \frac{R_2}{R_2 + R_3}$

b. If a second winding with N_s turns is wrapped around either the center post or the right post and if $b<d<c$, for which choice will the mutual inductance be larger? Evaluate the mutual inductance for both cases. Are there specific conditions (size of *a,b,c,* etc.) for one to be larger than the other?

The mutual inductance is given by $M_2 = \frac{N_s \psi_{m2}}{I}$ *or* $M_3 = \frac{N_s \psi_{m3}}{I}$ *Thus, the mutual inductance is larger when the flux linked is larger or, equivalently, when the reluctance is smaller.*

c. For completeness and simplicity, let *w = c = 2cm, a = 3cm, b = 1cm, d = 2cm.* What are the self and mutual inductances?

$$R_{11} = \frac{2c}{\mu(aw)} = \frac{2}{\mu 3},\ R_{12} = \frac{0.5a + c + 0.5b}{\mu(cw)} = \frac{1}{\mu},\ R_{13} = R_{12},\ R_2 = \frac{2c}{\mu(bw)} = \frac{2}{\mu},\ \&$$

$$R_3 = 2\frac{0.5b + c + 0.5d}{\mu(cw)} + \frac{2c}{\mu(dw)} = \frac{11}{4\mu}.\quad R = R_{11} + 2R_{12} + R_2 || R_3 = \frac{2}{3\mu} + \frac{2}{\mu} + \frac{22}{19\mu} = \frac{3.825}{\mu}.$$

Then, $\psi_m = \frac{N_p I}{R} = \frac{\mu N_p I}{3.825}$ *and* $L = \frac{\mu N_p^2}{3.825}$. *Finally, the two values for the mutual inductance are found from* $\psi_{m2} = \psi_m \frac{R_3}{R_2 + R_3} = \psi_m \frac{11}{19}$ *and* $\psi_{m3} = \psi_m \frac{R_2}{R_2 + R_3} = \psi_m \frac{8}{19}$

so $M_2 = \frac{N_s \psi_{m2}}{I} = \frac{11 \mu N_s N_p}{19(3.825)}$ *or* $M_3 = \frac{N_s \psi_{m3}}{I} = \frac{8 \mu N_s N_p}{19(3.825)}$.