1. a) $l = \frac{\mu_0}{2\pi} \ln \frac{b}{a} = \frac{4\pi \times 10^{-7}}{2\pi} \ln \frac{1.4}{.4} = 2.5 \times 10^{-7}$ H/m

$c = \frac{2\pi \varepsilon}{\ln b/a} = \frac{2\pi (2.25) \frac{1}{36\pi} \times 10^{-9}}{\ln \frac{1.4}{.4}} = 10^{-10}$ F/m

$v = \frac{1}{\sqrt{\mu_0 \varepsilon}} = \frac{2}{3} c = 2 \times 10^{8}$ m/s

$Z_0 = \sqrt{\frac{\mu}{\varepsilon}} \frac{\ln b/a}{2\pi} = 50\ \Omega$

b) $l = 800$ times larger $\Rightarrow l = 2 \times 10^{-4}$ H/m

$c = \frac{2\pi (4) \varepsilon_0}{\ln b/a} = 10^{-10} \frac{4}{2.3} = 1.7 \times 10^{-10}$ F/m

$v = \frac{1}{\sqrt{4 \cdot 800}} c = 5.3 \times 10^{6}$ m/s

$Z_0 = \sqrt{\frac{800 (2.25)}{4}}\ 50 \cong 1000\ \Omega$

c) Any reasonable reference accepted

2. Reluctance of the empty cylindrical hole

empty: $\mathcal{R}_1 = \dfrac{d}{\mu_0 \pi a^2}$

$\mu = \mu_r \mu_0$ in iron

Washer-like regions at the ends

$$\mathcal{R}_2 = \mathcal{R}_4 = \int_a^c \frac{dr}{\mu(2\pi r t)}$$

$$= \frac{1}{\mu 2\pi t} \int_a^c \frac{dr}{r}$$

$$= \frac{1}{\mu 2\pi t} \ln\frac{c}{a}$$

For the outer cylinder

$$\mathcal{R}_3 = \frac{d - 2t}{\mu\pi(-b^2+c^2)} \simeq \frac{d}{\pi\mu(c^2-b^2)}$$

$$\phi = \frac{NI}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4}$$

$$\Lambda = N\phi = LI$$

$$L = \frac{N^2}{\mathcal{R}_1+\mathcal{R}_2+\mathcal{R}_3+\mathcal{R}_4} \simeq \frac{N^2\mu_0\pi a^2}{d}$$

dominant term.

b. R_1 is now $\dfrac{d}{\mu \pi a^2}$ so all 4 terms are important

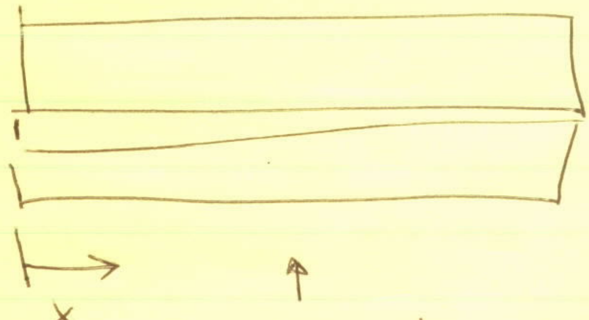
$$\phi = \frac{NI}{\sum R_i}$$

$$\Lambda = LI = \frac{N^2}{\sum R_i}$$

c. Energy $= \frac{1}{2} L I^2$

d. $$W_m = \frac{1}{2} I^2 \left(L_{empty} + \Delta L \frac{x}{d/2} \right)$$

$$F = -\nabla W_m = \frac{1}{2} I^2 \frac{\Delta L}{d/2}$$

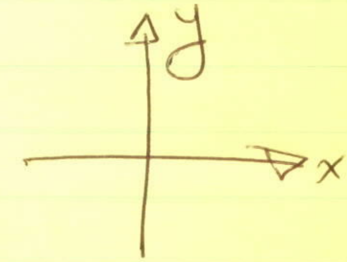


max when $x = \frac{d}{2}$

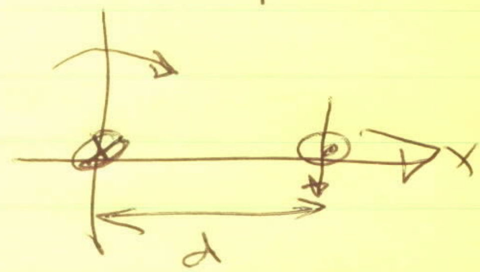
$$L_{max} = \Delta L + L_{empty}$$

e. any reasonable reference. Note that μ for iron is usually non linear so a typical value is OK

3. a. $\vec{B} = \dfrac{\mu_0 I}{2\pi r}\hat{\phi}$



$= -\dfrac{\mu_0 I}{2\pi d}\hat{y}$



b. $F = I L B \hat{x}$

$= I \dfrac{\mu_0 I}{2\pi d}\hat{x}$

$= \mu_0 \dfrac{I^2}{2\pi d}\hat{x}$

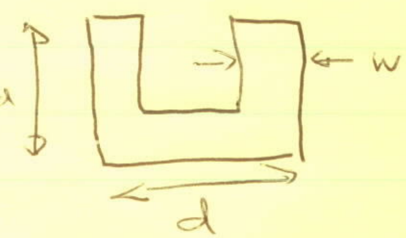
per unit length
$L = 1$

C. The force is repulsive

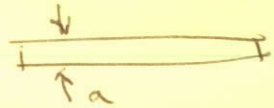
The force goes as $\vec{J} \times \vec{B}$

4. a. First find the reluctance

$R_{core} = \dfrac{3d}{\mu w^2}$



$R_{plate} = \dfrac{d}{\mu a w}$



$R_{gap} = \dfrac{g}{\mu_0 w^2}$

$R = \sum R_i = R_{core} + R_{plate} + 2 R_{gap}$

Now $\Lambda = N\phi = LI$

$$\phi = \frac{N_1 I}{R} \simeq \frac{N_1 I}{2R_{gap}}$$

$$\Lambda \cong \frac{N_1^2 I}{2R_{gap}} \cong \frac{N_1^2 I \mu_0 w^2}{2g}$$

$$\Rightarrow L_1 \simeq \frac{N_1^2 \mu_0 w^2}{2g}$$

b. ϕ done above $\quad \phi = \frac{N_1 I}{R}$

$$\simeq \frac{N_1 I}{2R_{gap}}$$

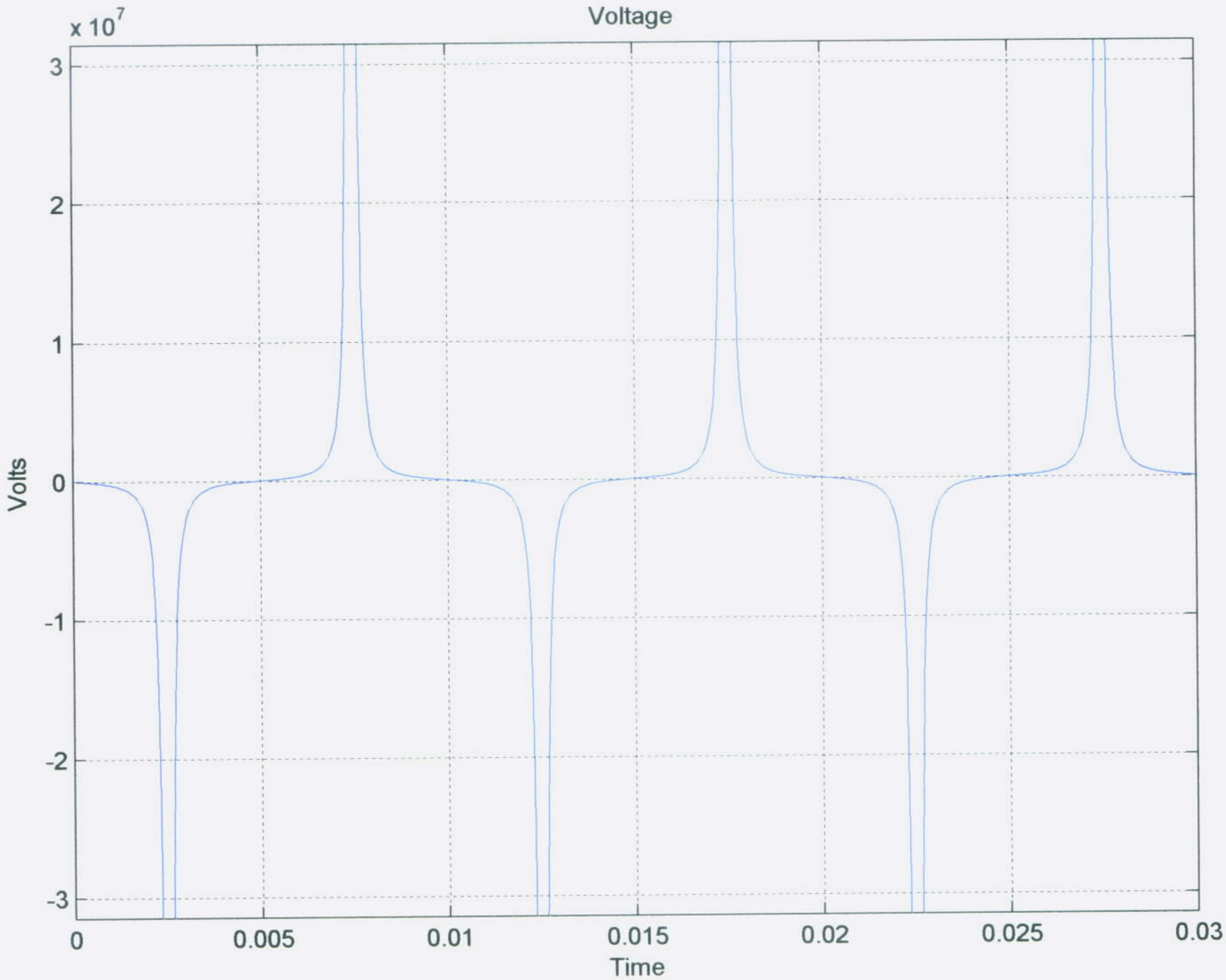
$$\Lambda_{21} \cong \frac{N_2 N_1 I}{2R_{gap}} \simeq \frac{N_1 N_2 I \mu_0 w^2}{2g}$$

c. $\Lambda_{21} \simeq \frac{N_1 N_2 I \mu_0 w^2}{2g(t)} \qquad g(t) = g_0 \cos wt$

d. $V = -\frac{d\Lambda_{21}}{dt} = -\frac{N_1 N_2 I \mu_0 w^2}{2} \frac{d}{dt} g^{-1}$

$$\frac{d}{dt} g^{-1} = g^{-2} \frac{dg}{dt} = \frac{-w g_0 \sin wt}{g_0^2 \cos^2 wt}$$

This is not a very practical measurement since the voltage can be very large (with $\cos^2 \omega t$ in the denominator) Practically, one can add an integrator circuit to the output and losses would keep the voltages finite.


Voltage
x 10^7
3
2
1
0
-1
-2
-3
Volts
0
0.005
0.01
0.015
0.02
0.025
0.03
Time