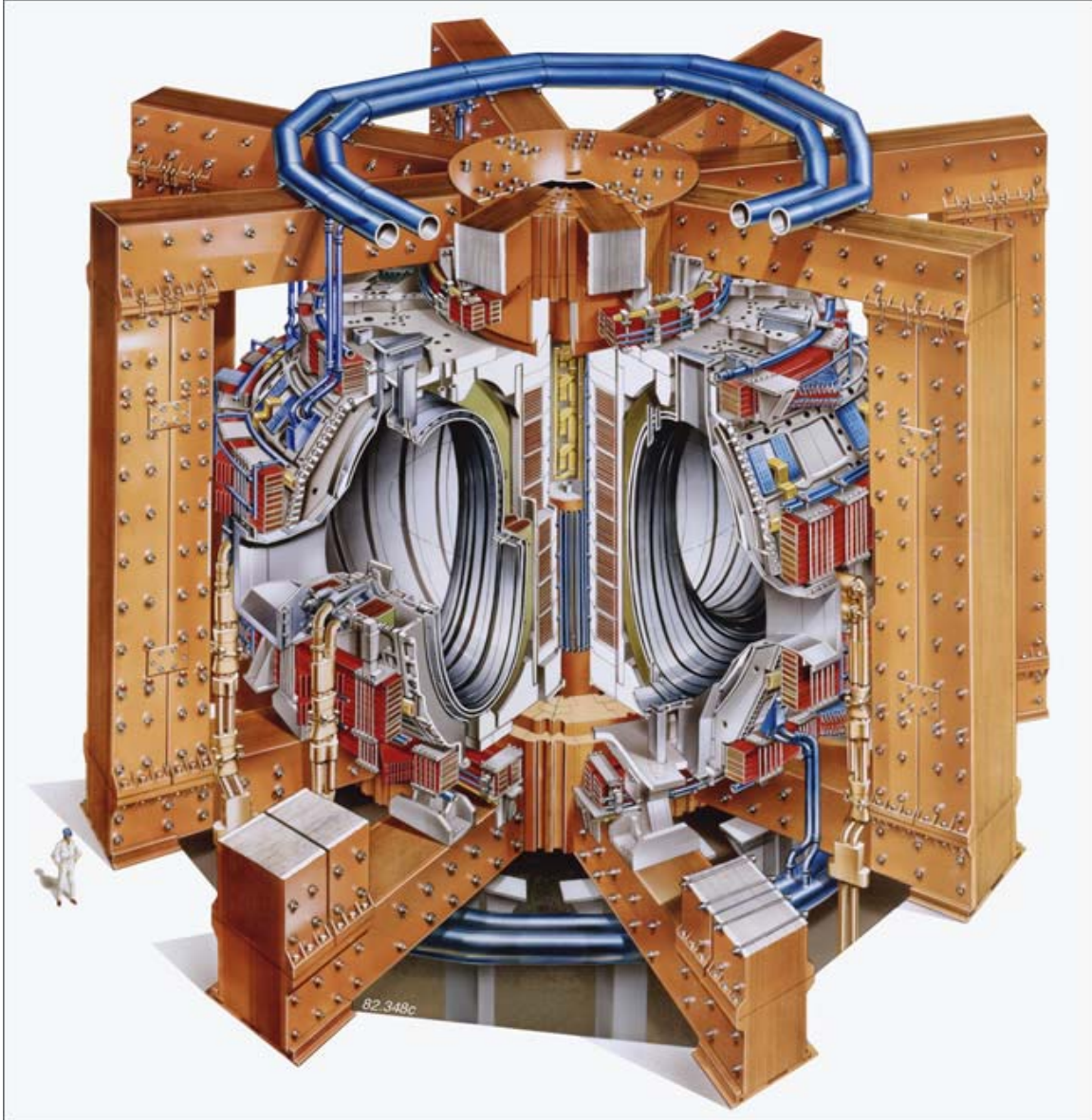
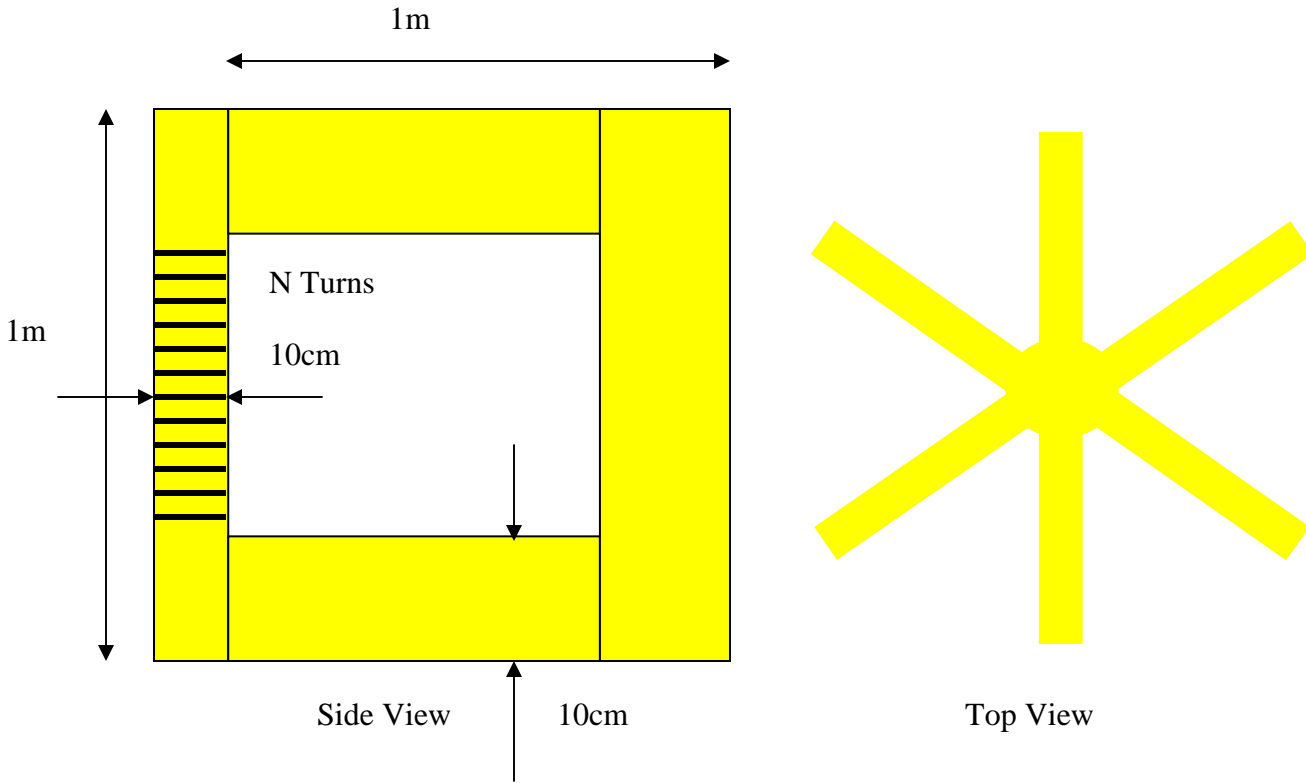




1. Iron Core Inductance and Energy

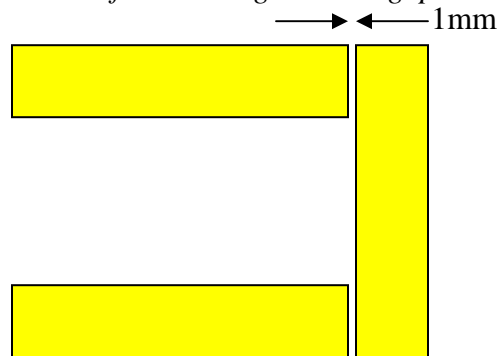


Shown above is a drawing of the Joint European Tokamak (JET) located near Culham in England. Part of the magnetic structure of this device involves an iron core with a cylindrical post in the center and 8 rectangular legs providing the path for the flux. Inspired by this configuration, we will consider the structure shown on the next page which also has a cylindrical central core, but six square cross section legs. The central core is 10cm in radius and the six outer legs are square and 10cm by 10cm in size. The length of each leg is shown in the diagram. The central core is wound with $N=1000$ turns of wire carrying $I=10000A$.



Determine the reluctance of each leg of this configuration, making any reasonable assumptions. Then draw the magnetic circuit diagram and solve for the magnetic flux. Determine also the magnetic flux density in each of the 7 legs of the structure. Find the inductance, the total energy and the energy density in each leg. Assume that $\mu = 5000\mu_0$ in the iron core material.

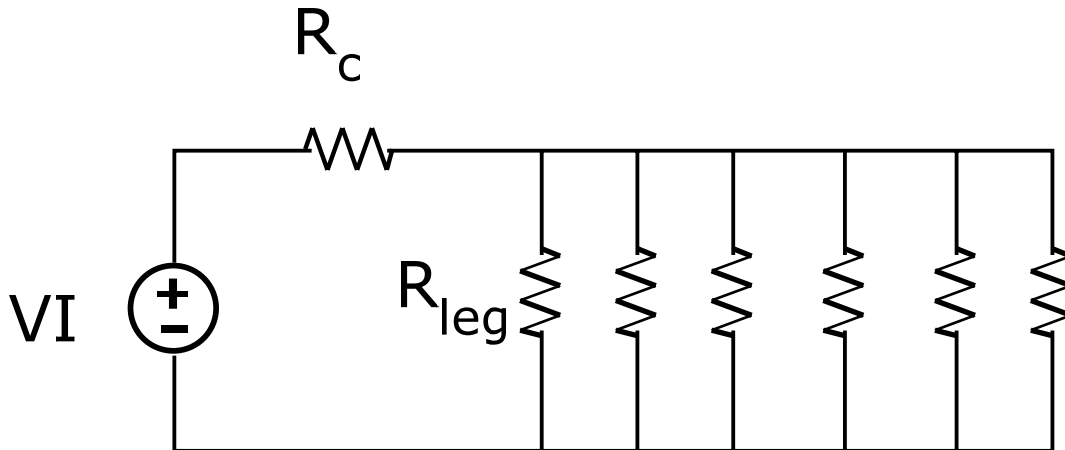
Now assume that a small gap of 1mm appears at the connection between the vertical section of one of the six legs and the two horizontal sections. Repeat all calculations for this case. *Hint: Note that the reluctance of the one leg with the gap will now be different than the other five.*



The magnetic field values you will obtain in this problem (with and without the gaps) are not realistic. Can you explain why?



The Matlab m-file below was used to find the parameters for this problem. $R = \frac{\text{length}}{\mu \text{Area}}$ for the reluctances.



The flux is found from $\Psi_m = \frac{NI}{R_{total}}$. The magnetic flux density and magnetic field intensity are $B = \frac{\Psi_m}{Area}$ and $H = \frac{B}{\mu}$. Note that one has to be careful to choose the right area for the flux path being considered. The inductance is $L = \frac{N\Psi_m}{I}$ and the energy is $\frac{1}{2} LI^2$. Then energy density in each leg is found from $w_m = \frac{1}{2} BH$. The inductance and total energy are found two ways in the file to check for math errors. It is always a good idea to find a solution using more than one technique. That way, one learns more and also has a way to check for errors.

Matlab m-file:

```
% Homework 6 Spring 2008
% Magnetic Circuit Calc Based on JET
% K. A. Connor, RPI

% Note that the Inductance and Total Energy are determined two ways to
% check for math errors.

% Core Parameters
mur=5000;muo=4*pi*1e-7;

% Core Dimensions
a=.1;w=.1;l=1;

% Areas
```



$Ac = \pi \cdot a^2$; $Aleg = w^2$;

% Number of turns and amount of current
 $N = 700$; $I = 5000$;

% Reluctance of Core
 $Rc = l / (\mu_r \cdot \mu_0 \cdot Ac)$;
% Reluctance of Legs
 $Rleg = 3 \cdot l / (\mu_r \cdot \mu_0 \cdot Aleg)$;
% Total Reluctance
 $R = Rc + Rleg / 6$;

% Flux
 $F = N \cdot I / R$;

% B (Flux Density)
 $Bc = F / Ac$; $Bleg = F / (6 \cdot Aleg)$;

% H
 $Hc = Bc / (\mu_r \cdot \mu_0)$; $Hleg = Bleg / (\mu_r \cdot \mu_0)$;

% Energy Density
 $wc = (Bc \cdot Hc) / 2$; $wleg = (Bleg \cdot Hleg) / 2$;

% Flux Linkage
 $Lam = F \cdot N$;

% Inductance
 $L = Lam / I$;

% Energy
 $W = 0.5 \cdot L \cdot I^2$;

% Alternate Inductance Calc
% Volume of Core and Legs
 $Vc = l \cdot Ac$; $Vleg = 3 \cdot l \cdot Aleg$;
% Energy
 $WW = (wc \cdot Vc + 6 \cdot wleg \cdot Vleg)$;
 $LL = 2 \cdot WW / (I^2)$;

% Displaying Numbers from Calcs
% L, W, F, Bc, Bleg, Hc, Hleg, wc, wleg
Params1 = [L = ' F = ' Bc = ' Bleg = ']
Ans1 = [L F Bc Bleg]
Params2 = [W = ' Hc = ' Hleg = ' wc = ' wleg = ']
Ans2 = [W Hc Hleg wc wleg]

% Gap Reluctance
% Gap Length
 $g = 0.001$;
 $Rg = g / (\mu_0 \cdot Aleg)$;



% Leg Plus 2 Gaps

$$R_{leggg} = R_{leg} + 2 * R_g;$$

% Total Reluctance

$$R_{gaps} = R_c + 1 / ((5 / R_{leg}) + (1 / R_{leggg}));$$

% New Flux with Gaps

$$F_{gaps} = N * I / R_{gaps};$$

$$\Lambda_{mgaps} = N * F_{gaps};$$

% Inductance with 2 Gaps

$$L_{gaps} = \Lambda_{mgaps} / I;$$

% Total Energy

$$W_{gaps} = 0.5 * L_{gaps} * I^2;$$

% Flux Density B with 2 Gaps

$$B_{cgaps} = F_{gaps} / A_c;$$

% Flux and B for Leg with 2 Gaps (Like Current Divider)

$$F_{leggg} = F_{gaps} * (R_{leg} / 5) / ((R_{leg} / 5) + R_{leggg});$$

$$B_{leggg} = F_{leggg} / A_{leg};$$

% Flux and B for 5 Legs with No Gaps (Like Current Divider)

$$F_{legg} = (F_{gaps} - F_{leggg}) / 5;$$

$$B_{legg} = F_{legg} / A_{leg};$$

% H

$$H_{cgaps} = B_{cgaps} / (\mu_r * \mu_0);$$

$$H_{legg} = B_{legg} / (\mu_r * \mu_0);$$

$$H_{leggg} = B_{leggg} / (\mu_r * \mu_0);$$

$$H_{leggap} = B_{leggg} / \mu_0;$$

% Energy Density

$$w_{cgaps} = 0.5 * B_{cgaps} * H_{cgaps};$$

$$w_{legg} = 0.5 * B_{legg} * H_{legg};$$

$$w_{leggg} = 0.5 * B_{leggg} * H_{leggg};$$

$$w_{leggap} = 0.5 * B_{leggg} * H_{leggap};$$

% Total Energy Calc from Energy Density

$$V_{gap} = g * A_{leg};$$

$$W_{Wgaps} = w_{cgaps} * V_c + 5 * w_{legg} * V_{leg} + w_{leggg} * V_{leg} + 2 * w_{leggap} * V_{gap};$$

% Inductance

$$L_{Lgaps} = 2 * W_{Wgaps} / (I^2);$$

% Determining the Force trying to close the gaps

% Pressure is energy density

% Net pressure on the gap is the difference in the energy densities on

% either side of the gap face

$$P_{gaps} = w_{leggap} - w_{leggg};$$

% Force is pressure times area

$$Force = 2 * P_{gaps} * A_{leg};$$

% Displaying Numbers from Calcs

% L_{gaps} , W_{gaps} , F_{gaps} , B_{cgaps} , B_{legg} , B_{leggg} , H_{cgaps} , H_{legg} , H_{leggg} , H_{leggap}



```

% wcgaps, wlegg, wleggg, wleggap
Params3=[ Lgaps= ' Fgaps= ' Bcgaps= ' Blegg= ' Bleggg= ' ]
Ans3=[Lgaps Fgaps Bcgaps Blegg Bleggg]
Params4=[ Wgaps= ' Hcgaps= ' Hlegg= ' Hleggap= ' wcgaps= ' wlegg= ' wleggg= '
'wleggap/100= ' ]
Ans4=[Wgaps Hcgaps Hlegg Hleggap wcgaps wlegg wleggg wleggap/100]

```

```

% Displaying Reluctances
Params5=[ R= ' Rc= ' Rg= ' Rgaps= ' Rleg= ' Rleggg= ' ]
Ans5=[R Rc Rg Rgaps Rleg Rleggg]

```

% Flux density really must be kept below 2.2T so the numbers in this
% problem a not realistic

Results:

Params1 =

L= F= Bc= Bleg=

Ans1 =

1.0e+003 *
0.0376 0.2687 8.5542 4.4790

Params2 =

W= Hc= Hleg= wc= wleg=

Ans2 =

1.0e+009 *
0.4703 0.0014 0.0007 5.8231 1.5964

Params3 =

Lgaps= Fgaps= Bcgaps= Blegg= Bleggg=

Ans3 =



1.0e+003 *

0.0345 0.2466 7.8489 4.7141 1.0879

Params4 =

Wgaps= Hcgaps= Hlegg= Hleggap= wcgaps= wlegg= wleggg= wleggap/100=

Ans4 =

1.0e+009 *

0.4315 0.0012 0.0008 0.8657 4.9024 1.7684 0.0942 4.7088

Params5 =

R= Rc= Rg= Rgaps= Rleg= Rleggg=

Ans5 =

1.0e+005 *

0.1302 0.0507 0.7958 0.1419 0.4775 2.0690

From the Matlab results:

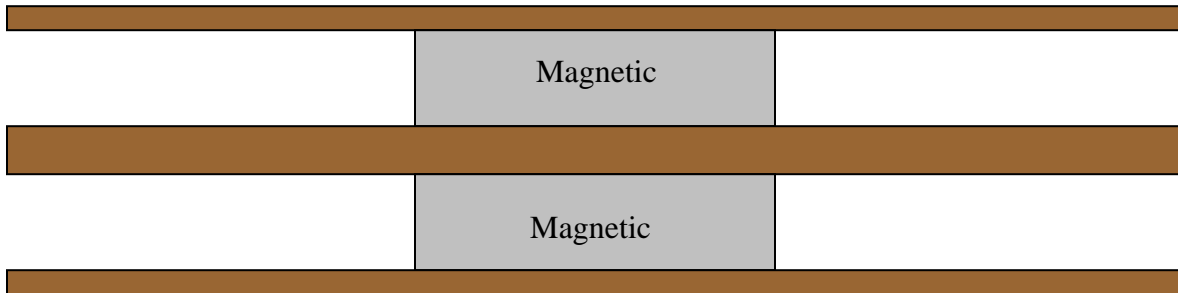
No Gap Case: Inductance is 37.6Henries, the reluctance of the central core is 5070, the reluctance of each of the six outer legs is 47750. The flux is 269. The magnetic flux density in the core is 8554T while the magnetic flux density in the outer six legs is 4479T. The total energy is 4.7e8, the energy density in the central core is 4.8e9 and the outer leg energy density is 1.6e9. Note that these numbers cannot be real since most magnetic materials saturate at around 2-3T.

Gap Case: Inductance is 34.5Henries, the reluctance of the central core is the same as is the reluctance of five of the six outer legs. The reluctance of the gap is 79580. The flux is 247. The magnetic flux density in the core is 7849T while the magnetic flux density in five of the outer six legs is 4714T and 1088T in the leg with the gaps. The total energy is 4.3e8, the energy density in the central core is 4.9e9 and in five of the outer legs energy density is 1.8e9. The energy density in the core material in the leg with the gaps is 9.4e7 and in the gaps is 4.7e11.



2. Inductance of Coaxial Cable

An engineer has an idea to change the inductance of a small section of a coaxial cable by placing a section of magnetic insulating material between two standard cables. Assume that $\mu = 100\mu_o$. In the remainder of the cable, the insulator is Teflon.



Assume the standard dimensions of an RG-142 cable with characteristic impedance of 50 Ohms. Assume also that this section of cable is 5cm in length. What is the inductance of this section? Assume also that the material is made by mixing a magnetic material in with an electric material. If the magnetic material makes up 20% of the volume, is it possible to find an electric material to replace Teflon as the insulator that will keep the characteristic impedance of the short section the same?

Answer: The inductance per unit length of coaxial cable is $L = \frac{\mu}{2\pi} \ln \frac{b}{a}$. For a standard RG142 cable, the dimensions are given by $a=.47\text{mm}$, $b=1.52\text{mm}$ and $\epsilon_r = 2$ (we will need this value shortly). The inductance of the 5cm section is then

$L = 100(0.235\mu\text{H} / \text{m})(.05\text{m}) = 1.18\mu\text{H}$ where the μ is for microHenries, not for permeability. The inductance per unit length is 100 times larger so we require a capacitance per unit length that is also 100 times larger if the characteristic impedance is to remain the same. Note that, because the magnetic material only makes up 20% of the volume, the permeability of the material must be 500 times that of free space. The insulating material makes up 80% of the volume, so we need something that has a dielectric constant of 250, to achieve the value of 200 that we need to have the capacitance be 100 times bigger. This is unrealistic, although there are some exotic materials that achieve values this large. (e.g. CaTiO_3)



To obtain the parameters of the coaxial cable from what is published, the following m-file was written.

```
% Coaxial Cable Property Calculator
% for Cables Specs Given the Standard Way
% Given are Zo, velocity factor, center conductor diameter, and nominal
% capacitance per unit length.
% K. A. Connor, RPI, Spring 2008

muo=4*pi*1e-7;epso=(1/(36*pi))*1e-9; % Vacuum constants for reference
a=0.037/2; % Center conductor size is usually given as a diameter in inches
a=a*.0254; % Converting inches to meters
c=3e8;% Speed of light in vacuum
vel=.709; % Velocity Factor
epsr=1/(vel^2); % Assumes no magnetic properties
vel=vel*c; %Converts to actual velocity
Zo=50; % Characteristic Impedance
cap=32e-12; % Usually given in pF per foot
cap=cap*3.28; % Convert to pF per meter, this is just used to check the calc since it is a nominal
number

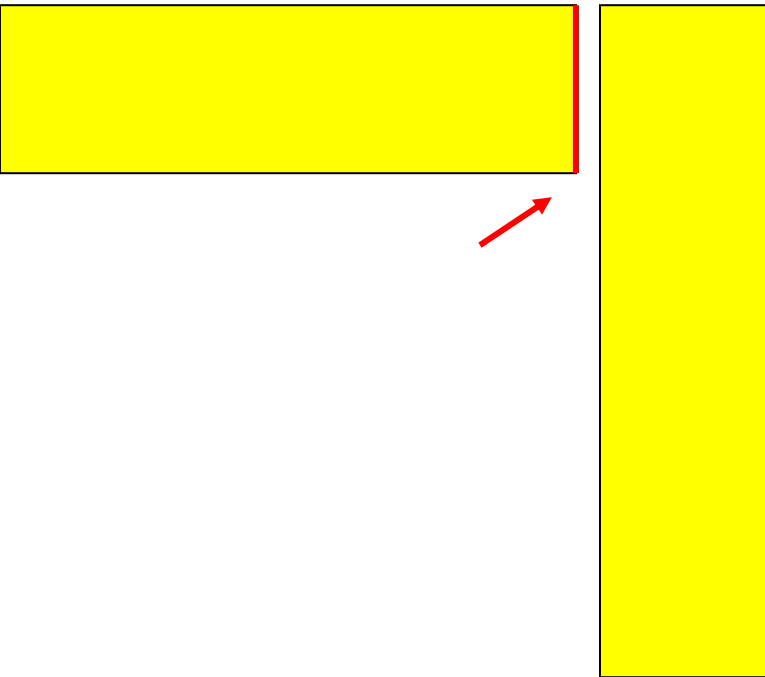
capcalc=1/(Zo*vel); % Calculated from the given specs
indcalc=Zo/vel; % Again calculated from the given specs
eta=120*pi/sqrt(epsr); % Intrinsic impedance of the material
geomterm=2*pi*Zo/eta; % This is log(b/a) from specs
b=exp(geomterm)*a;

% Printing the parameters
epsr, capcalc, indcalc, a, b
```



3. Forces on Magnetic Materials

In problem 1, you have determined the magnetic flux density in all regions of the structure with its several legs. If a gap appears, like the one you analyzed, there will be a force tending to either make it larger or smaller. Determine the value of the force and whether or not it will work to close or open the gap. *Hint: find the pressure difference on the two sides of the gap interface. From the pressure, find the force. The surface you should consider for each gap is indicated in red below.*



From the Matlab File (see problem 1), the pressure difference and force are given by (note that the gap force appears twice since there are two gaps).

$P_{\text{gaps}} =$

4.7078e+011

Force =

9.4157e+009

which is again a very large, unrealistic force.



4. Quasistatic Approximation

A coaxial carries a DC current I , has an inner radius a , an outer radius b , and a length d . The insulating material is air. The right end of the cable is shorted by a circular plate.



Find the magnetic field intensity \vec{H} in the insulating region. Now assume that the current is a sinusoidal function of time $I = I_o \cos \omega t$. Using the quasistatic approximation, find the electric field \vec{E} . You may use phasor notation or just use the time dependent form of Maxwell's equations. Either way, your final answer should be in time dependent form. *Hint: you should follow the method for the parallel plate transmission line in Unit 9 of the class notes posted on the WebCT site. Also, you can assume that the electric field has only a radial component.*

Answer: The magnetic field in the insulating region has been calculated many different places and is given by $\vec{B} = \frac{\mu_o I}{2\pi r} \hat{\phi}$. If the current varies with time, this will produce an

electric field through Faraday's Law $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\mu_o}{2\pi r} \frac{\partial I}{\partial t} \hat{\phi} = \omega I_o \cos \omega t \frac{\mu_o}{2\pi r} \hat{\phi}$.

Simplifying since the electric field can only be in the radial direction and vary only with z ,

we have $\frac{\partial E_r}{\partial z} = -\frac{\partial B}{\partial t} = \omega I_o \cos \omega t \frac{\mu_o}{2\pi r}$. The electric field must be zero at the short

circuited end, so we should integrate this expression from there to values of z elsewhere

in the cable. $E_r = \omega I_o \cos \omega t \frac{\mu_o}{2\pi r} z$ where we have defined $z=0$ to be at the shorted end of

the cable so all values of z in the cable are negative. Since we expect the voltage on the transmission line to be positive on the center conductor, at least in that part of the cycle where the current is positive, the electric field must point from the outer to the inner conductor, which it does.

Extra Credit: Evaluate and compare the energy stored in the magnetic field and in the electric field for this length of cable.