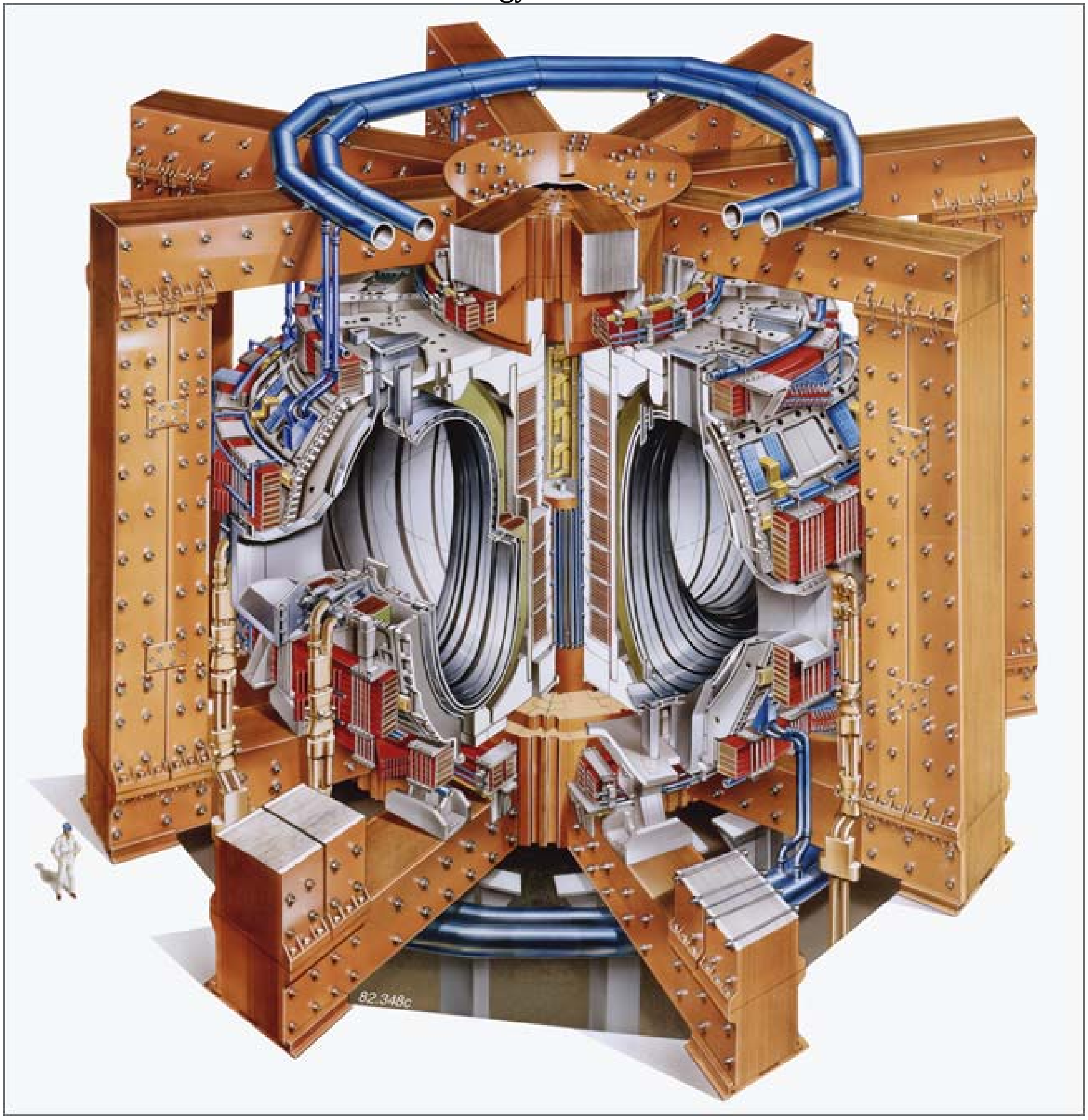
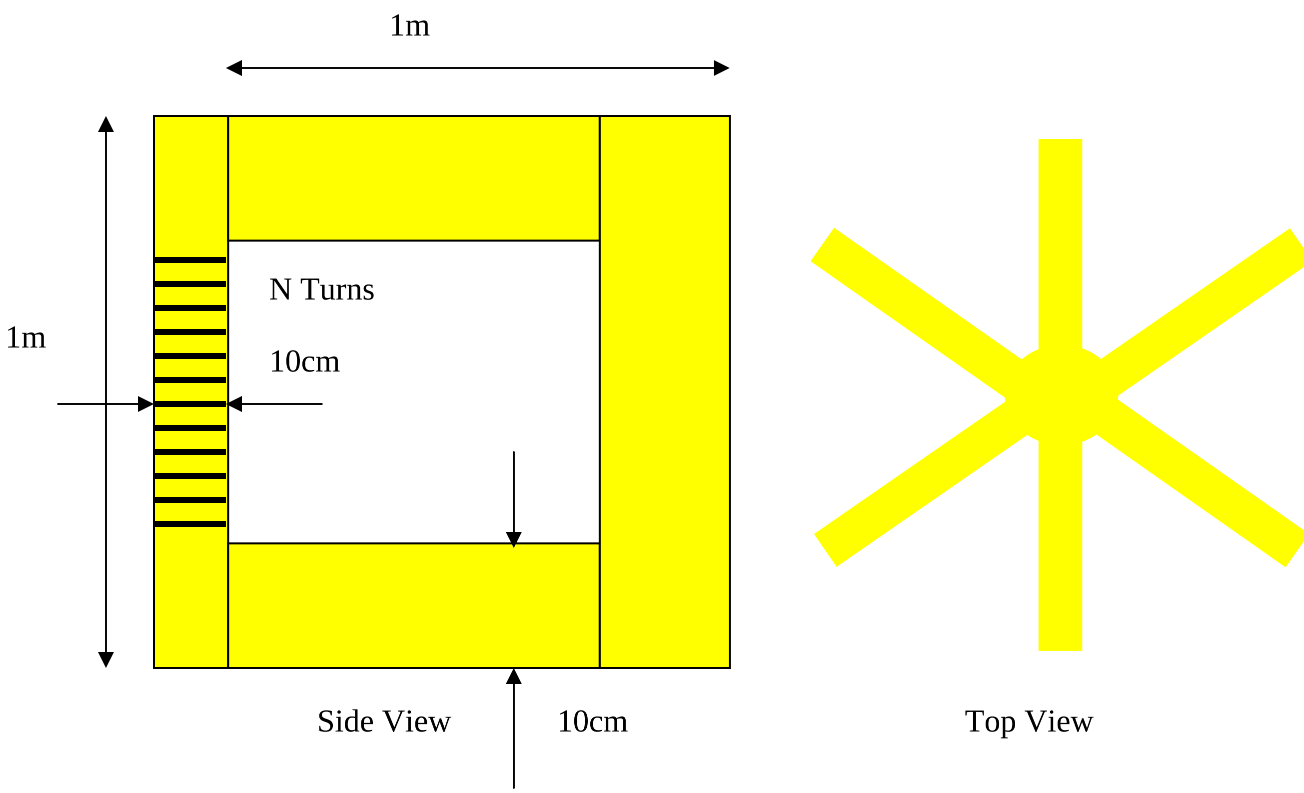
Due Wednesday 9 April 2008 at 6:00 PM (End of day on Wednesday)

1. Iron Core Inductance and Energy

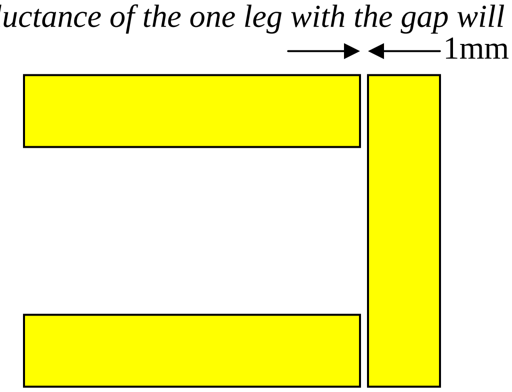


Shown above is a drawing of the Joint European Tokamak (JET) located near Culham in England. Part of the magnetic structure of this device involves an iron core with a cylindrical post in the center and 8 rectangular legs providing the path for the flux. Inspired by this configuration, we will consider the structure shown on the next page which also has a cylindrical central core, but six square cross section legs. The central core is 10cm in radius and the six outer legs are square and 10cm by 10cm in size. The length of each leg is shown in the diagram. The central core is wound with $N=1000$ turns of wire carrying $I=10000A$.



Determine the reluctance of each leg of this configuration, making any reasonable assumptions. Then draw the magnetic circuit diagram and solve for the magnetic flux. Determine also the magnetic flux density in each of the 7 legs of the structure. Find the inductance, the total energy and the energy density in each leg. Assume that $\mu = 5000\mu_o$ in the iron core material.

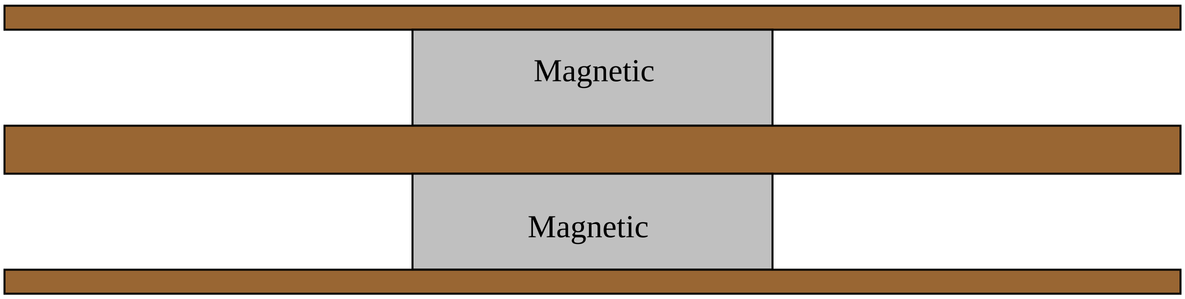
Now assume that a small gap of *1mm* appears at the connection between the vertical section of one of the six legs and the two horizontal sections. Repeat all calculations for this case. *Hint: Note that the reluctance of the one leg with the gap will now be different than the other five.*



The magnetic field values you will obtain in this problem (with and without the gaps) are not realistic. Can you explain why?

2. **Inductance of Coaxial Cable**

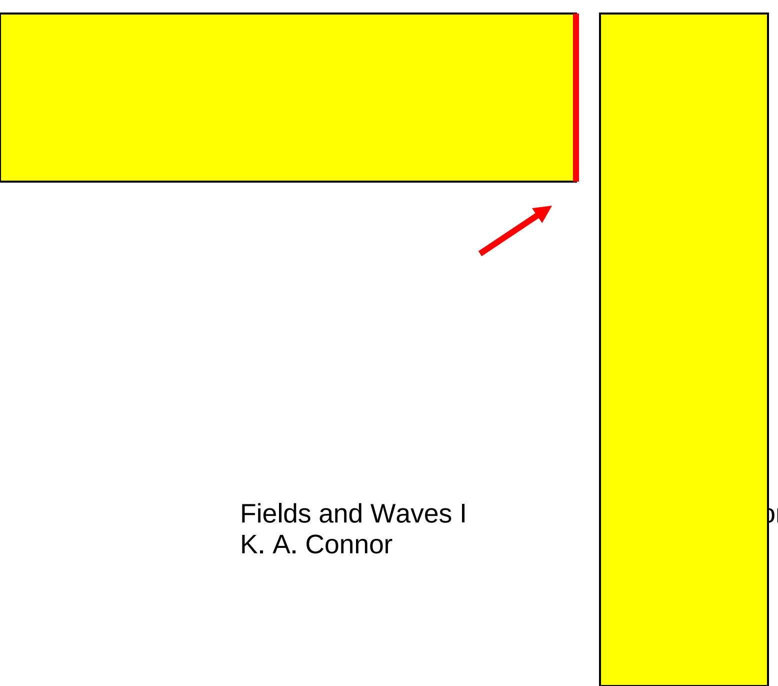
An engineer has an idea to change the inductance of a small section of a coaxial cable by placing a section of magnetic insulating material between two standard cables. Assume that $\mu = 100\mu_o$. In the remainder of the cable, the insulator is Teflon.



Assume the standard dimensions of an RG-142 cable with characteristic impedance of 50 Ohms. Assume also that this section of cable is 5cm in length. What is the inductance of this section? Assume also that the material is made by mixing a magnetic material in with an electric material. If the magnetic material makes up 20% of the volume, is it possible to find an electric material for the insulator that will keep the characteristic impedance of the short section the same? *Note that the cable type has been changed to a standard Teflon cable.*

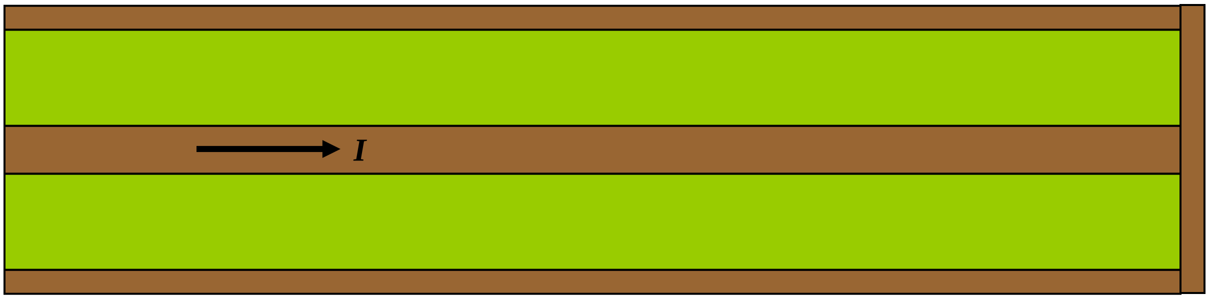
3. **Forces on Magnetic Materials**

In problem 1, you have determined the magnetic flux density in all regions of the structure with its several legs. If a gap appears, like the one you analyzed, there will be a force tending to either make it larger or smaller. Determine the value of the force and whether or not it will work to close or open the gap. *Hint: find the pressure difference on the two sides of the gap interface. From the pressure, find the force. The surface you should consider for each gap is indicated in red below.*



4. **Quasistatic Approximation**

A coaxial carries a DC current I, has an inner radius *a*, an outer radius *b*, and a length *d*. The insulating material is air. The right end of the cable is shorted by a circular plate.



Find the magnetic field intensity $\vec{H}$ in the insulating region. Now assume that the current is a sinusoidal function of time $I = I_o \cos\omega t$. Using the quasistatic approximation, find the electric field $\vec{E}$. You may use phasor notation or just use the time dependent form of Maxwell's equations. Either way, your final answer should be in time dependent form. *Hint: you should follow the method for the parallel plate transmission line in Unit 9 of the class notes posted on the WebCT site. Also, you can assume that the electric field has only a radial component. (Note that this problem has been solved to reduce the work required a bit on this assignment.)*

Answer: The magnetic field in the insulating region has been calculated many different places and is given by $\vec{B} = \frac{\mu_o I}{2\pi r}\hat{\phi}$. *If the current varies with time, this will produce an electric field through Faraday's Law* $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\frac{\mu_o}{2\pi r}\frac{\partial I}{\partial t}\hat{\phi} = \omega I_o \cos\omega t \frac{\mu_o}{2\pi r}\hat{\phi}$. *Simplifying since the electric field can only be in the radial direction and vary only with z, we have* $\frac{\partial E_r}{\partial z} = -\frac{\partial B}{\partial t} = \omega I_o \cos\omega t \frac{\mu_o}{2\pi r}$. *The electric field must be zero at the short circuited end, so we should integrate this expression from there to values of z elsewhere in the cable.* $E_r = \omega I_o \cos\omega t \frac{\mu_o}{2\pi r} z$ *where we have defined z=0 to be at the shorted end of the cable so all values of z in the cable are negative. Since we expect the voltage on the transmission line to be positive on the center conductor, at least in that part of the cycle where the current is positive, the electric field must point from the outer to the inner conductor, which it does.*

Extra Credit: Evaluate and compare the energy stored in the magnetic field and in the electric field for this length of cable.