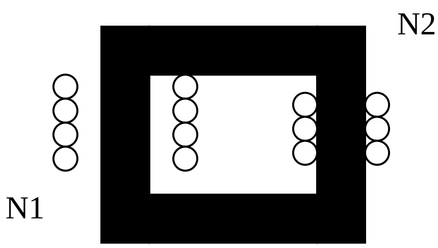
Fields and Waves I
Fall 2007
Homework 6

1) For the core below we have voltage source connected to the primary winding (N1 turns) producing a sinusoidal voltage. The secondary winding (N2 turns) is open circuited. The cross- sectional area is $A = 0.1\ m^2$. The core length is $\ell = 0.25\ m$. The flux density in the core is $B = 1.2\sin(377t)\ T$. There are N1 = 100 turns in the primary coil and N2 =300 turns in the secondary. Ignore the resistance in the windings. .

A) Find the applied voltage to the primary and the open circuit voltage on the secondary..

B) We now connect a load to the secondary (N2) of 50 Ohms resistance. Find the current in the primary and secondary windings.



Solution :

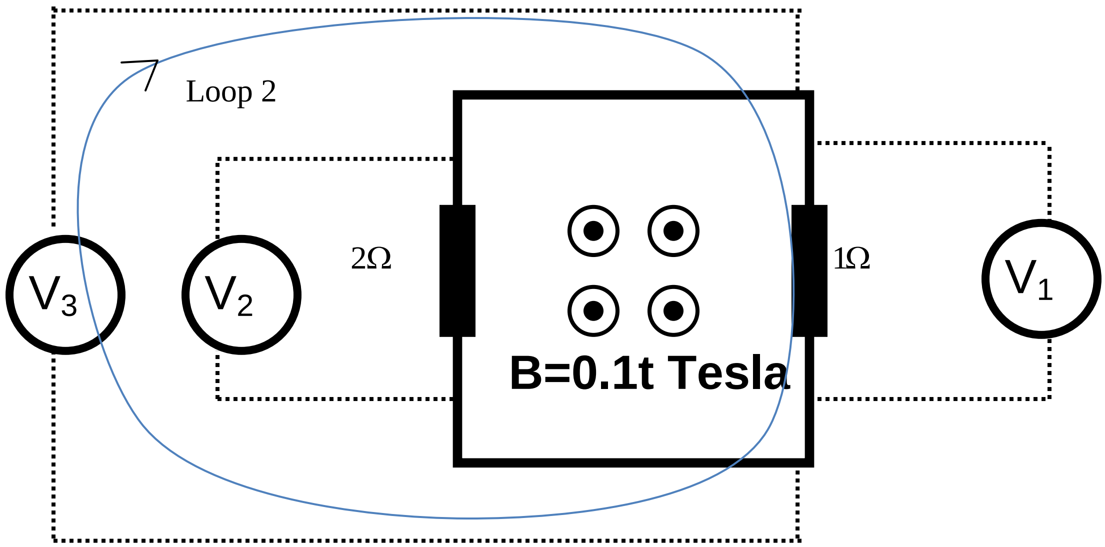
A)

$$B = 1.2\sin(\omega t)$$
$$\psi = B\ A = 0.12\sin(377\ t)$$
$$V_1 = -\frac{d\psi}{dt} = 100 \times 0.12 \times 377\cos(377\ t) = 4524\cos(377\ t)$$
$$V_2 = \frac{N_2}{N_1}V_1 = 3 \times 4534\cos(377\ t) = 13572\cos(377\ t)$$

B)

$$I_2 = \frac{V_2}{R_2} = \frac{13752}{50}\cos(377\ t) = 271.44\cos(377\ t)$$
$$I_1 = \frac{N_2}{N_1}I_2 = 814.32\cos(377\ t)$$

2. A Flux density is increasing as B = 0.1t Tesla through a loop of area 1 square meter. The loop has 2 resistors as shown below. There are 3 volt meters attached. What is the reading of each volt meter?



Solution :

$$E = -\frac{d\psi}{dt} = -\frac{d}{dt}(0.1.t.1m^2) = 0.1\,V$$

$$I = \frac{E}{2\Omega + 1\,\Omega} = 0.033\,mA$$

$$V_1 = I.1\Omega = 0.033\,V$$

$$V_2 = I.2\Omega = 0.067\,V$$

For V_3 consider Loop 2 shown in the figure.

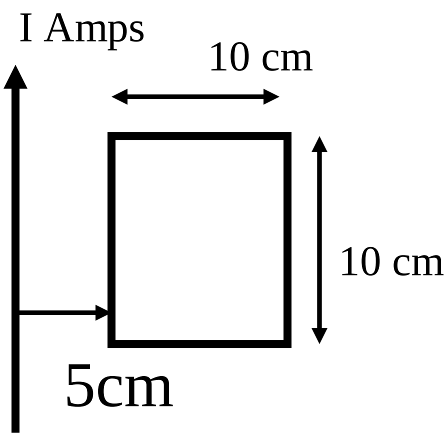
$$E_{loop\,2} = -\frac{d\psi}{dt} = -\frac{d}{dt}(0.1.t.1m^2) = 0.1\,V$$

There is also a resistive drop, V_1.

Therefore,

$$V_3 = E_{loop\ 2} - V_1 = 0.067\,V$$

3. A rectangular loop 10 cm by 10 cm is coplanar with a long wire with current I = $2.5\cos(2\pi * 10^4 t)$ Amps. The resistance of the loop is 20 Ohms and we can ignore the inductance. Find the current circulating in the loop and the direction.



Solution :

$$B = \frac{\mu_0 I}{2\pi r}$$

$$\psi = \oint_s B.ds = \int_{0.05}^{0.15} \frac{\mu_0 I.l}{2\pi r} dr = \frac{\mu_0.I.l}{2\pi} \ln\left(\frac{0.15}{0.05}\right) = 5.5 \times 10^{-8} \cos(2\,\pi \times 10^4 t)$$

$$E = -\frac{d\psi}{dt} = 3.45 \times 10^{-3} \sin(2\,\pi \times 10^4 t)$$

$$I = \frac{E}{R} = 1.72 \times 10^{-4} \sin(2\,\pi \times 10^4 t)\ (Clockwise)$$

Direction : For small t, B is decreasing and the direction is into the plane of the paper. According to Faraday's law, the flux generated by the current should oppose the rate of change of magnetic flux. i.e current should be clockwise when it is positive.