Homework 7

Due 19 April 2005

1. Plane Waves in Lossless Media

The magnetic field of a uniform plane in a lossless medium is given by $\vec{H}(z,t) = \hat{a}_x 100\sin\left(6\pi 10^6 t + 0.04\pi z\right)$. Find:

a) The magnetic field phasor $\tilde{\vec{H}}(z)$

The phasor form of the field, when multiplied by $e^{j\omega t}$ *and finding the real part will give us back the original expression.* $\tilde{\vec{H}}(z) = \hat{a}_x 100 e^{-j\pi/2} e^{+j.04\pi z}$

$$\vec{H}(z,t) = \mathrm{Re}\left(\tilde{\vec{H}}(z)e^{j\omega t}\right) = \hat{a}_x 100\,\mathrm{Re}\left(e^{-j\pi/2}e^{+j.04\pi z}e^{j\omega t}\right) = -\hat{a}_x 100\,\mathrm{Re}\left(-j^2\sin(\beta z + \omega t)\right)$$

b) The direction of wave propagation

The wave travels in the –z direction

c) The frequency of the wave, f, and its period, T.

The frequency of the wave f is 3x10^6 *since* $\omega = 2\pi f = 6\pi 10^6$

d) The phase velocity, u_p *The phase velocity is* $u_p = \dfrac{\omega}{\beta} = \dfrac{6\pi 10^6}{0.04\pi} = 1.5x10^8$

e) The wavelength in the material, λ, and the propagation constant of the wave, β.

The propagation constant $\beta = 0.04\pi$

f) The relative permittivity of the material, ε_r,, assuming the material is non-magnetic. *The relative permittivity is* $\varepsilon_r = \left(\dfrac{c}{u_p}\right)^2 = 2^2 = 4$

g) The electric field phasor, $\tilde{\vec{E}}$ *To get this expression, we need the intrinsic impedance of he medium, which we an get from the dielectric constant.*

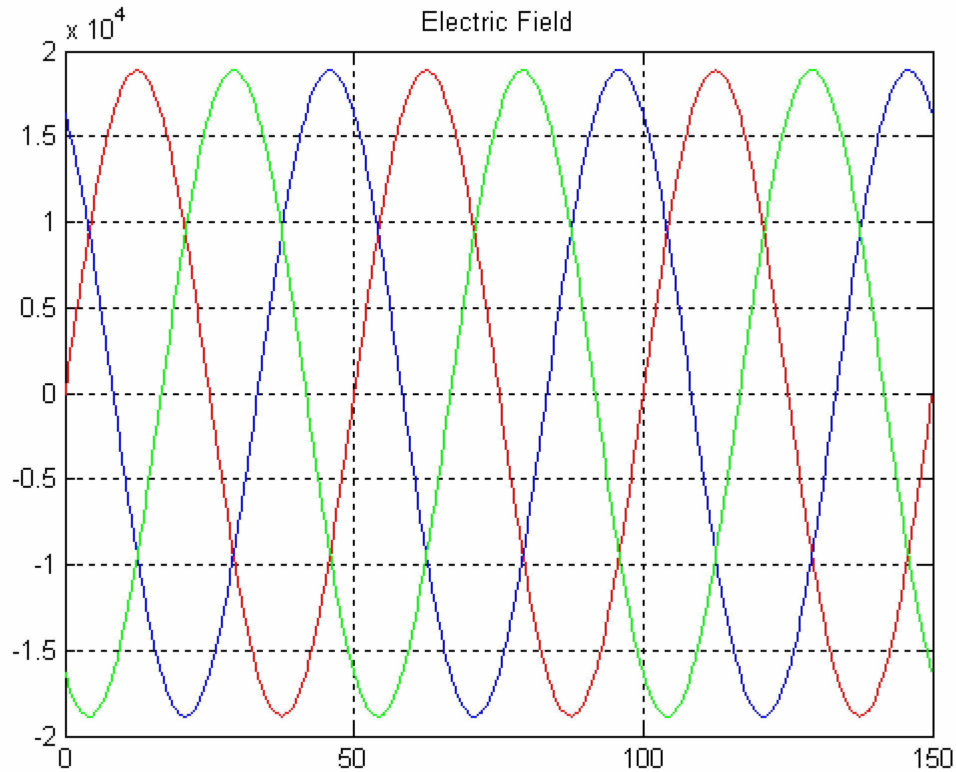
$\eta = \dfrac{120\pi}{\sqrt{\varepsilon_r}} = 60\pi$. *Then the phasor electric field must be*

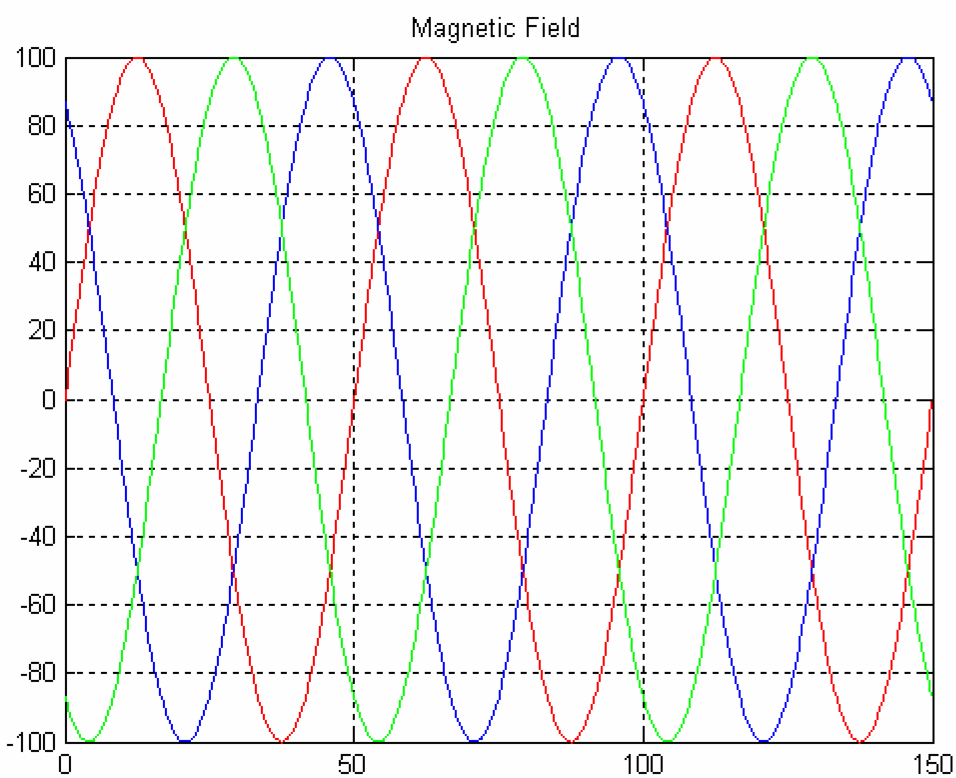
$$\tilde{\vec{E}}(z) = \hat{a}_y(\eta)100e^{-j\pi/2}e^{+j.04\pi z} = \hat{a}_y 6000\pi e^{-j\pi/2}e^{+j.04\pi z}$$

h) The electric field in time domain form, $\vec{E}(z,t)$ *The time domain form is*

$$\vec{E}(z,t) = \hat{a}_y 6000\pi\,\mathrm{Re}\left(e^{-j\pi/2}e^{+j.04\pi z}e^{j\omega t}\right) = \hat{a}_y 6000\pi\sin(6\pi 10^6 t + 0.04\pi z)$$

Finally, using Maple, Matlab or some similar tool, plot the electric and magnetic fields of the waves as a function of position at three times, *t = 0, t = T/3, t = 2T/3.*


x 10^4
Electric Field
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
50
100
150


Magnetic Field
100
80
60
40
20
0
-20
-40
-60
-80
-100
0
50
100
150

2. Power Absorption in a Lossy Material

A uniform plane wave (*f = 3GHz)* is propagating in a huge block of ice. At $z = 0$, the power density of the wave is $1kW/m^2$. We wish to investigate the heating of the ice by the wave. A reference containing information on the electrical properties of a wide variety of materials: http://www.rfcafe.com/references/electrical/dielectric_constants_strengths.htm Note that the data provided gives the loss tangent, not the imaginary part of the permittivity. While it has nothing to do with this problem, there is another good reference on the electrical properties of human tissue provided by a laboratory in Florence, Italy: http://niremf.ifac.cnr.it/tissprop/htmlclie/htmlclie.htm You can get a sense of the range of possible properties from these two sources.

Assume that the direction of wave propagation is +z. Determine the following:

a) The complex permittivity $\varepsilon_c = \varepsilon' - j\varepsilon''$. *From the data provided, the complex permittivity is* $\varepsilon_c = \varepsilon' - j\varepsilon'' = 3.2\varepsilon_o - j(.0009)3.2\varepsilon_o = 3.2\varepsilon_o - j.0029\varepsilon_o$ *Since the loss tangent is so small, this is a low loss dielectric, so we can use those approximations when we determine the wave parameters.*

b) The basic wave parameters ω, α, β, λ, and η_c.

$$\omega = 2\pi f = 2\pi 3x10^9 = 6\pi x10^9$$

$$\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{\left(6\pi 10^9\right)0.0029\left(\frac{1}{36\pi}10^{-9}\right)}{2}67.1\pi = \frac{(0.0029)(67.1\pi)}{12} = 0.051$$

$$\beta = \omega\sqrt{\mu\varepsilon'} = \frac{6\pi 10^9}{3x10^8}\sqrt{3.2} = 35.78\pi$$

$$\lambda = \frac{2\pi}{\beta} = 0.056m$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}} = \frac{120\pi}{\sqrt{3.2}} = 67.1\pi$$

c) The electric field phasor $\tilde{\vec{E}}(z)$ and the magnetic field phasor $\tilde{\vec{H}}(z)$.
The Poynting Vector magnitude at z=0 is $1kW/m^2$ *so that the magnitude of the electric field phasor is* $E_o = \left|\tilde{\vec{E}}(z)\right| = \sqrt{2\eta P} = \sqrt{2(67.1\pi)(1000)} = \sqrt{4.2x10^6} = 649$

Then, the electric field phasor is $\tilde{\vec{E}}(z) = \hat{a}_x E_o e^{-\alpha z} e^{-j\beta z}$ *and the magnetic field phasor is* $\tilde{\vec{H}}(z) = \hat{a}_y \frac{E_o}{\eta_c} e^{-\alpha z} e^{-j\beta z}$ *where we have assumed that the electric field is polarized in the x direction.*

d) The phase velocity, u_p. *The phase velocity is given by*

$$u_p = \frac{\omega}{\beta} = \frac{6\pi 10^9}{35.78\pi} = 1.68x10^8$$

e) The average power density (Poynting Vector) at *z = 1m*. *The average power density is given by the average Poynting Vector*

$$\vec{P}_{ave} = \hat{a}_z \frac{{E_o}^2}{2\eta_c} e^{-2\alpha z} = \hat{a}_z \frac{649^2}{2(67.1\pi)} e^{-2(0.051)1} = 1000(.903) = 903\frac{W}{m^2}$$

f) The power deposited in a cubic meter of the material between *z = 0* and *z = 1m* using the integral of $\vec{J}\cdot\vec{E}$ as in lecture 21. *The power deposition integral is given by*

$$\frac{1}{2}\mathrm{Re}\int_V \vec{J}\cdot\vec{E}^* dv = \frac{1}{2}\int_V \omega\varepsilon'' E^2 dv = \frac{1}{2}\omega\varepsilon'' {E_o}^2 \int_0^1 e^{-2\alpha z} dz =$$

$$\frac{1}{2}\omega\varepsilon'' {E_o}^2 \frac{1}{2\alpha}\left(1 - e^{-2\alpha}\right) = \frac{1}{2}\omega\varepsilon'' {E_o}^2 \frac{1}{\omega\varepsilon''}\frac{1}{\eta_c}(1-.903) = \frac{1}{2}\frac{{E_o}^2}{\eta_c}(0.097)$$

Compare your answers to parts e) and f). *Note that this is exactly the difference between the two power fluxes (one entering and one leaving the volume).*