Homework 7

Due 18 April 2006

1. Plane Waves in Lossless Media

The magnetic field of a uniform plane in a lossless medium is given by $\vec{H}(z,t)=\hat{a}_x 10\cos\left(2\pi 10^7 t+0.08\pi z\right)$. Find:

a) The magnetic field phasor $\tilde{\vec{H}}(z)$

The phasor form of the field, when multiplied by $e^{j\omega t}$ *and finding the real part will give us back the original expression.* $\tilde{\vec{H}}(z)=\hat{a}_x 10e^{+j.08\pi z}$

$$\vec{H}(z,t)=\mathrm{Re}\left(\tilde{\vec{H}}(z)e^{j\omega t}\right)=\hat{a}_x 10\,\mathrm{Re}\left(e^{+j.08\pi z}e^{j\omega t}\right)=\hat{a}_x 10\left(\cos(\beta z+\omega t)\right)$$

b) The direction of wave propagation

The wave travels in the –z direction

c) The frequency of the wave, *f*, and its period, *T*.

The frequency of the wave f is 10^7 *since* $\omega=2\pi f=2\pi 10^7$

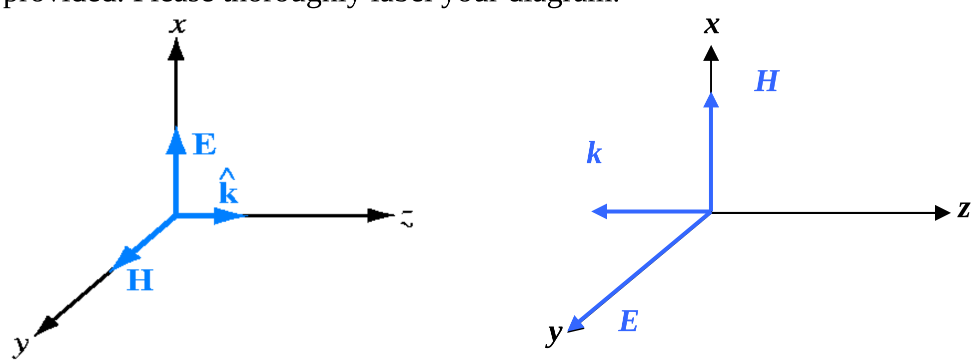
d) The phase velocity, u_p *The phase velocity is* $u_p=\frac{\omega}{\beta}=\frac{2\pi 10^7}{0.08\pi}=2.5x10^8$

e) The wavelength in the material, λ, and the propagation constant of the wave, β.

The propagation constant $\beta=0.08\pi$. *The wavelength* $\lambda=\frac{2\pi}{\beta}=\frac{2\pi}{0.08\pi}=25\,meters$

f) The relative permittivity of the material, ε_r, assuming the material is non-magnetic. *The relative permittivity is* $\varepsilon_r=\left(\frac{c}{u_p}\right)^2=\left(\frac{3}{2.5}\right)^2=1.44$

Since you now have the direction of the magnetic field phasor $\tilde{\vec{H}}(z)$ and the direction of propagation, draw the diagram analogous to figure 7-4 of Ulaby showing the direction of the magnetic field phasor $\tilde{\vec{H}}(z)$, the electric field phasor $\tilde{\vec{E}}(z)$ and the direction of wave propagation. Figure 7-4 is copied below as a reference and a set of coordinate axes are also provided. Please thoroughly label your diagram.



We can now use the information obtained for the magnetic field to determine the electric field. Find:

g) The electric field phasor, $\tilde{\vec{E}}$ *To get this expression, we need the intrinsic impedance of he medium, which we an get from the dielectric constant.*
$\eta = \frac{120\pi}{\sqrt{\varepsilon_r}} = \frac{120\pi}{1.2} = 100\pi$. *Then the phasor electric field must be*
$\tilde{\vec{E}}(z) = \hat{a}_y(\eta)10e^{+j.08\pi z} = \hat{a}_y 1000\pi e^{+j.08\pi z}$

h) The electric field in time domain form, $\vec{E}(z,t)$ *The time domain form is*
$\vec{E}(z,t) = \hat{a}_y 1000\pi \,\mathrm{Re}\left(e^{+j.08\pi z}e^{j\omega t}\right) = \hat{a}_y 1000\pi \cos(2\pi 10^7 t + 0.08\pi z)$

Let us now assume that the uniform electromagnetic wave is propagating in the insulating region between two conducting plates that forma parallel plate transmission line. (See figure below) We can use the information we have on the fields to find the voltage and current on the line. Find:

i) The phasor voltage on the top plate $\hat{V}(z)$ by integrating the electric field phasor from the bottom plate to the top plate. Assume that the bottom plate is grounded (one of them has to be).

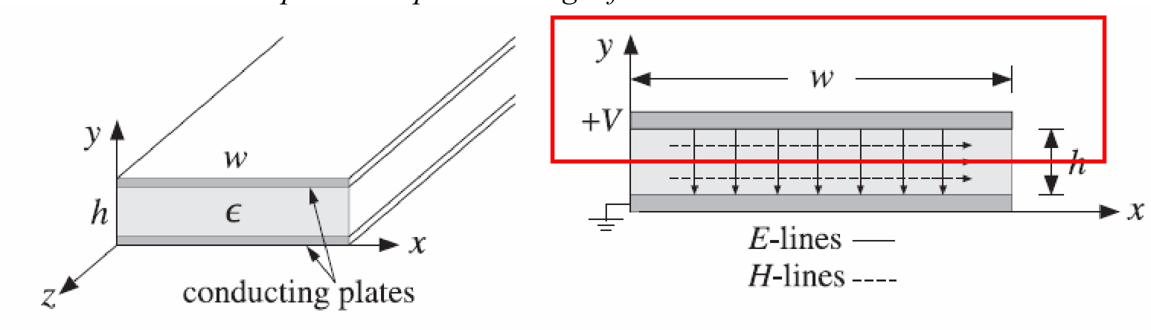
The relationship between the voltage and the electric field is given by
$\hat{V}(z) = -\int_0^h \tilde{\vec{E}}(z)\cdot\hat{y}dy = h1000\pi e^{+j.08\pi z}$

j) The phasor current in either the top or bottom plate $\hat{I}(z)$ by applying Ampere's Law to a closed loop surrounding one of the plates. Note that the magnetic field is assumed to be zero in the regions outside of the transmission line (no fringing field assumption).

For clarity, draw the integration path (in red) on the diagram below. Then we see that $I_{enclosed} = \oint \tilde{\vec{H}}(z)\cdot d\vec{l} = w10e^{+j.08\pi z}$

k) The ratio of the voltage to the current. Compare your answer to the characteristic impedance of this transmission line. Indicate where you obtained the expression for the characteristic impedance.

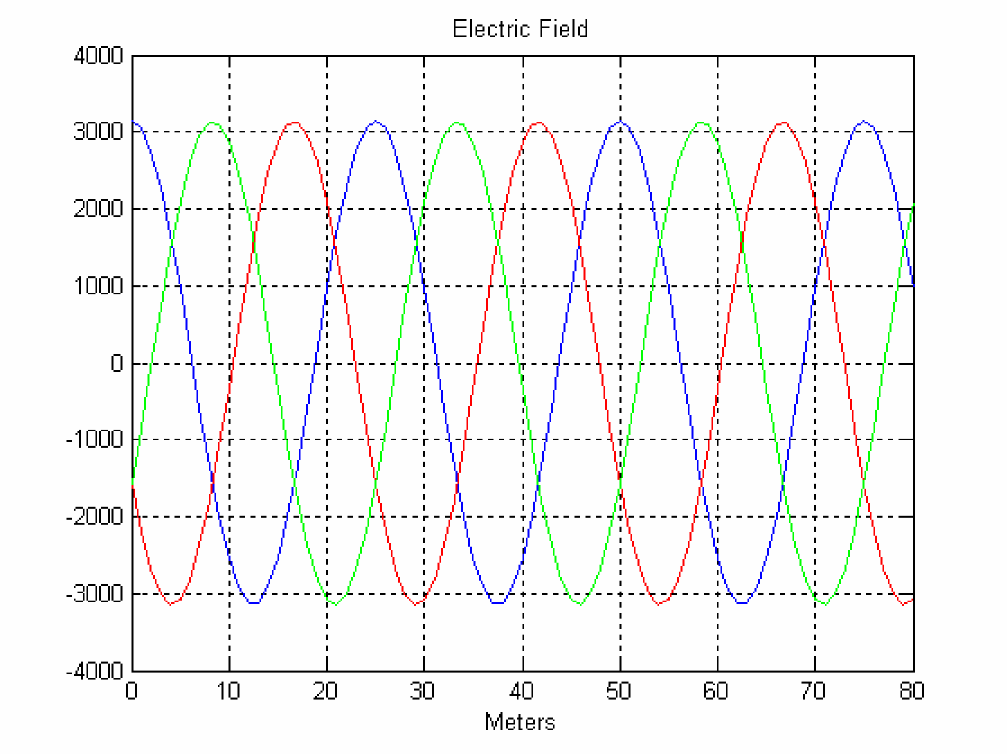
The ratio of the voltage to the current should be
$Z_o = \frac{V(z)}{I(z)} = \frac{h1000\pi e^{+j0.08\pi z}}{w10e^{+j0.08\pi z}} = \frac{h}{w}\eta = \frac{h}{w}\sqrt{\frac{\mu_o}{\varepsilon}} = \sqrt{\frac{l}{c}} = \frac{100\pi h}{w}$ *where l and c are the inductance and capacitance per unit length for such a transmission line.*



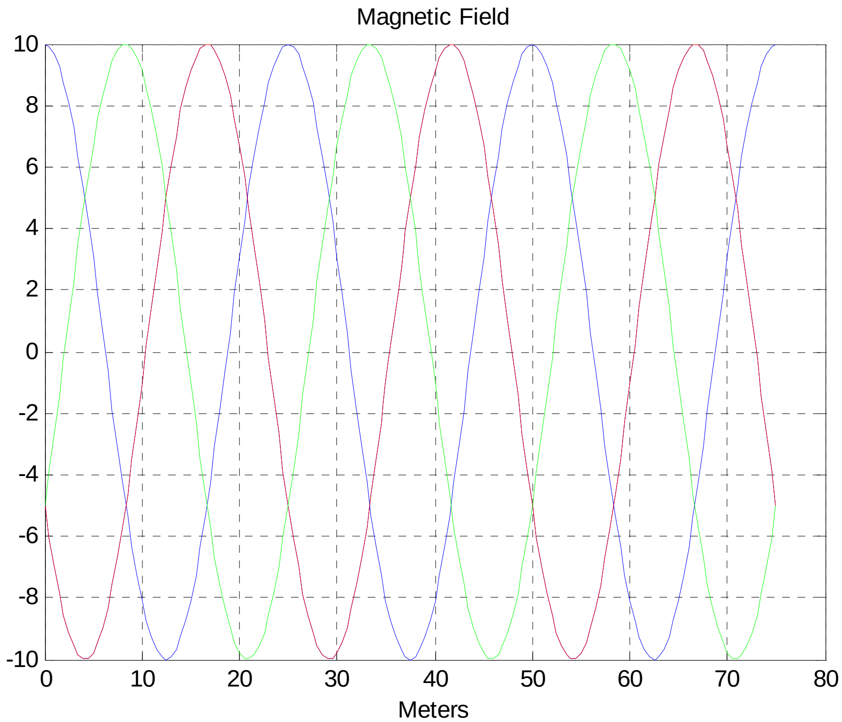
From Ulaby, page 41, the inductance and capacitance per unit length for a parallel plate transmission line are $l = \frac{\mu_o h}{w}$ *and* $c = \frac{\varepsilon w}{h}$.

Finally, using Maple, Matlab or some similar tool, plot the electric and magnetic fields of the waves as a function of position at three times, *t = 0*, *t = T/3*, *t = 2T/3*. *Note that the colors are chosen so that the time goes from blue to red to green. In the plots below, we can see that the wave propagates from right to left.*

The electric field:

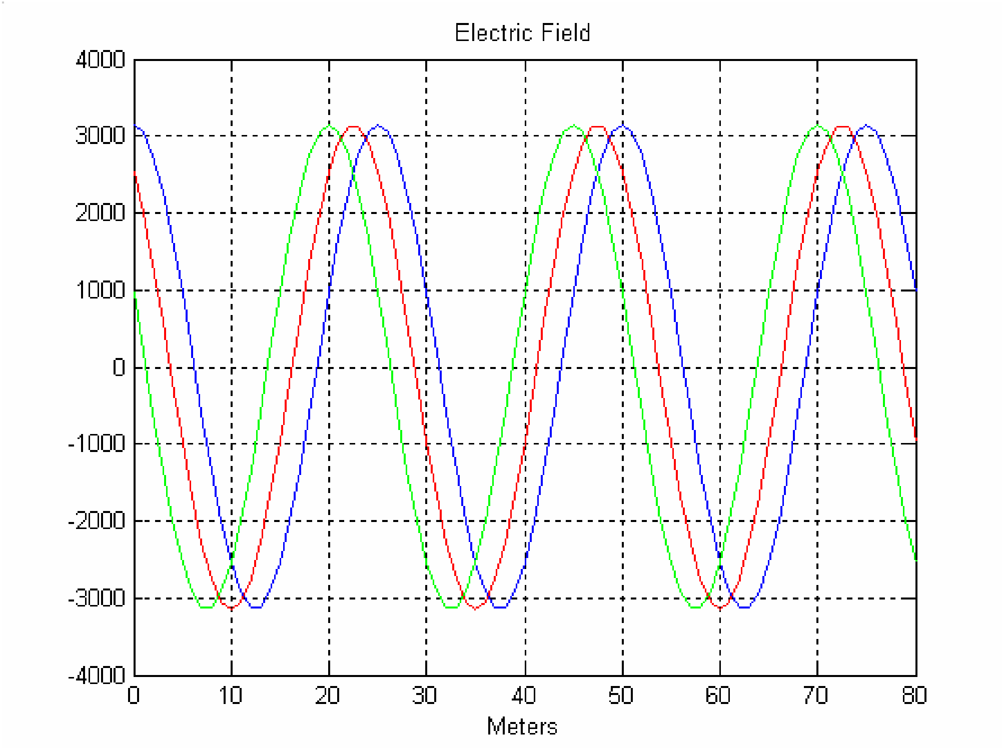


The magnetic field:

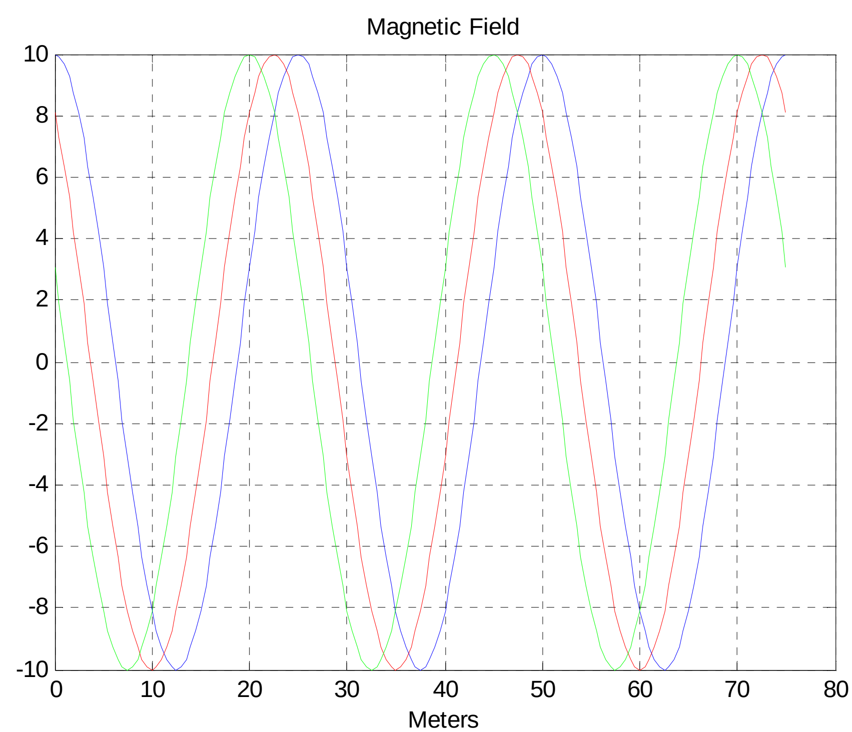


These plots do not show the direction of propagation as clearly as is possible with other choices of times (not asked for here). If we plot smaller times, we can see the wave direction much better. $t = 0$, $t = T/10$, $t = T/5$

Electric Field:



The Magnetic Fields:



2. Power Absorption in a Lossy Material

A uniform plane wave (*f = 3GHz)* is propagating in a huge blob of beeswax. At *z = 0*, the electric field intensity of the wave is *100V/m*. We wish to investigate the heating of the wax by the wave. A reference containing information on the electrical properties of a wide variety of materials:
http://www.rfcafe.com/references/electrical/dielectric_constants_strengths.htm
Note that the data provided gives the loss tangent, not the imaginary part of the permittivity. While it has nothing to do with this problem, there is another good reference on the electrical properties of human tissue provided by a laboratory in Florence, Italy: http://niremf.ifac.cnr.it/tissprop/htmlclie/htmlclie.htm You can get a sense of the range of possible properties from these two sources.

Assume that the direction of wave propagation is +z. Determine the following:

a) The complex permittivity $\varepsilon_c = \varepsilon' - j\varepsilon''$. *From the data provided, the complex permittivity is* $\varepsilon_c = \varepsilon' - j\varepsilon'' = 2.39\varepsilon_o - j(.0075)2.39\varepsilon_o = 2.39\varepsilon_o - j0.018\varepsilon_o$ *Since the loss tangent is so small (less than 0.01), this is a low loss dielectric, so we can use those approximations when we determine the wave parameters.*

b) The basic wave parameters ω, α, β, λ, and η_c.

$$\omega = 2\pi f = 2\pi 3x10^9 = 6\pi x10^9$$

$$\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{\left(6\pi 10^9\right)(0.018)\left(\frac{1}{36\pi}10^{-9}\right)}{2}77.6\pi = 0.3647$$

$$\beta = \omega\sqrt{\mu\varepsilon'} = \frac{6\pi 10^9}{3x10^8}\sqrt{2.39} = 30.92\pi = 97.138$$

$$\lambda = \frac{2\pi}{\beta} = 0.065m$$

$$\eta_c = \sqrt{\frac{\mu}{\varepsilon'}} = \frac{120\pi}{\sqrt{2.39}} = 77.6\pi = 243.79$$

c) The electric field phasor $\tilde{\tilde{E}}(z)$ and the magnetic field phasor $\tilde{\tilde{H}}(z)$.
The magnitude of the electric field phasor is $E_o = 100$ *Then, the electric field phasor is* $\tilde{\tilde{E}}(z) = \hat{a}_x E_o e^{-\alpha z} e^{-j\beta z}$ *and the magnetic field phasor is* $\tilde{\tilde{H}}(z) = \hat{a}_y \frac{E_o}{\eta_c} e^{-\alpha z} e^{-j\beta z}$ *where we have assumed that the electric field is polarized in the x direction.*

d) The phase velocity, u_p. *The phase velocity is given by*

$$u_p = \frac{\omega}{\beta} = \frac{6\pi 10^9}{30.92\pi} = 1.94x10^8 = \frac{3x10^8}{\sqrt{2.39}}$$

e) The average power density (Poynting Vector) at $z = 0m$. *The average power density is given by the average Poynting Vector*

$$\vec{P}_{ave} = \hat{a}_z \frac{{E_o}^2}{2\eta_c} = \hat{a}_z \frac{100^2}{2(77.6\pi)} = \hat{a}_z 20.5 \frac{W}{m^2}$$

f) The average power deposited in a square meter cross-sectional area of the material between $z = 0$ and $z \to \infty$ using the integral of $\frac{1}{2}\mathrm{Re}\,\vec{J}(z)\cdot\vec{E}^*(z)$ as in lecture 21. *The power deposition integral is given by*

$$\frac{1}{2}\mathrm{Re}\int_V \vec{J}\cdot\vec{E}^*dv = \frac{1}{2}\int_V \omega\varepsilon'' E^2 dv = \frac{1}{2}\omega\varepsilon'' {E_o}^2 \int_0^\infty e^{-2\alpha z}dz =$$

$$\frac{1}{2}\omega\varepsilon'' {E_o}^2 \frac{1}{2\alpha} = \frac{1}{2}\omega\varepsilon'' {E_o}^2 \frac{1}{\omega\varepsilon''}\frac{1}{\eta_c} = \frac{1}{2}\frac{{E_o}^2}{\eta_c}$$

Compare your answers to parts e) and f). *Note that the two expressions are identical.*