

Homework 7

Due 18 April 2005

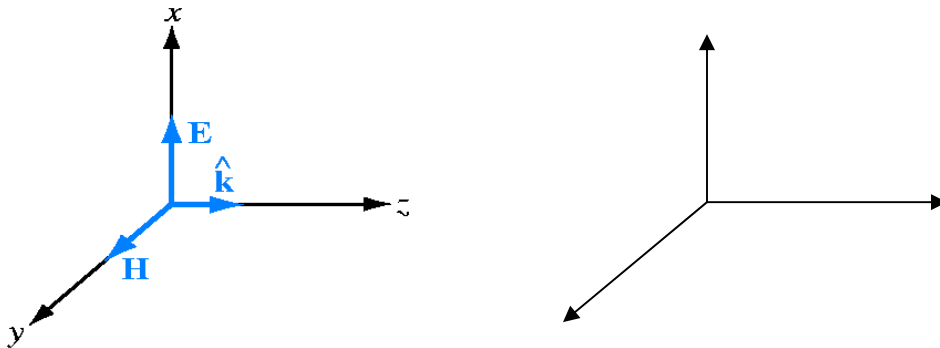
1. Plane Waves in Lossless Media

The magnetic field of a uniform plane wave in a lossless medium is given by

$$\vec{H}(z,t) = \hat{a}_x 10 \cos(2\pi 10^7 t + 0.08\pi z). \text{ Find:}$$

- The magnetic field phasor $\tilde{H}(z)$
- The direction of wave propagation
- The frequency of the wave, f , and its period, T .
- The phase velocity, u_p
- The wavelength in the material, λ , and the propagation constant of the wave, β .
- The relative permittivity of the material, ϵ_r , assuming the material is non-magnetic.

Since you now have the direction of the magnetic field phasor $\tilde{H}(z)$ and the direction of propagation, draw the diagram analogous to figure 7-4 of Ulaby showing the direction of the magnetic field phasor $\tilde{H}(z)$, the electric field phasor $\tilde{E}(z)$ and the direction of wave propagation. Figure 7-4 is copied below as a reference and a set of coordinate axes are also provided. Please thoroughly label your diagram.



We can now use the information obtained for the magnetic field to determine the electric field. Find:

- The electric field phasor, $\tilde{E}(z)$
- The electric field in time domain form, $\vec{E}(z,t)$

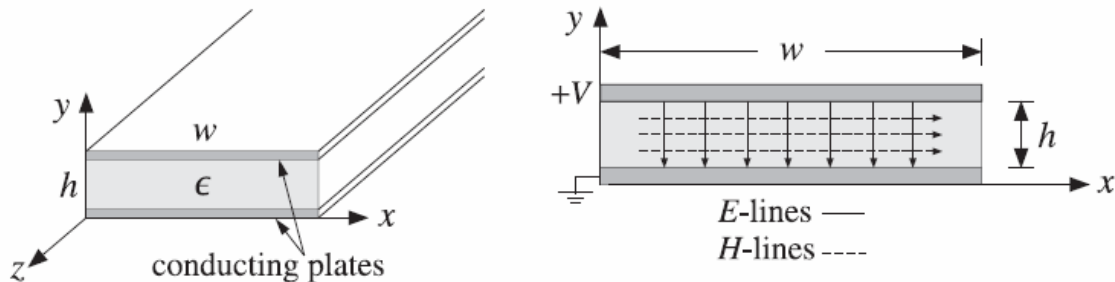
Let us now assume that the uniform electromagnetic wave is propagating in the insulating region between two conducting plates that form a parallel plate transmission line. (See figure below) We can use the information we have on the fields to find the voltage and current on the line. Find:

- The phasor voltage on the top plate $\hat{V}(z)$ by integrating the electric field phasor from the bottom plate to the top plate. Assume that the bottom plate is grounded (one of them has to be).
- The phasor current in either the top or bottom plate $\hat{I}(z)$ by applying Ampere's Law to a closed loop surrounding one of the plates. Note that the magnetic field is

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assumed to be zero in the regions outside of the transmission line (no fringing field assumption).

- k) The ratio of the voltage to the current. Compare your answer to the characteristic impedance of this transmission line. Indicate where you obtained the expression for the characteristic impedance.



Finally, using Maple, Matlab or some similar tool, plot the electric and magnetic fields of the waves as a function of position at three times, $t = 0$, $t = T/3$, $t = 2T/3$.

2. Power Absorption in a Lossy Material

A uniform plane wave ($f = 3\text{GHz}$) is propagating in a huge blob of beeswax. At $z = 0$, the electric field intensity of the wave is 100V/m . We wish to investigate the heating of the wax by the wave. A reference containing information on the electrical properties of a wide variety of materials:

http://www.rfcafe.com/references/electrical/dielectric_constants_strengths.htm

Note that the data provided gives the loss tangent, not the imaginary part of the permittivity. While it has nothing to do with this problem, there is another good reference on the electrical properties of human tissue provided by a laboratory in Florence, Italy:

<http://niremf.ifac.cnr.it/tissprop/htmlclie/htmlclie.htm> You can get a sense of the range of possible properties from these two sources.

Assume that the direction of wave propagation is $+z$. Determine the following:

- The complex permittivity $\epsilon_c = \epsilon' - j\epsilon''$.
- The basic wave parameters ω , α , β , λ , and η_c .
- The electric field phasor $\tilde{E}(z)$ and the magnetic field phasor $\tilde{H}(z)$.
- The phase velocity, u_p .
- The average power density (Poynting Vector) at $z = 0\text{m}$.
- The average power deposited in a square meter cross-sectional area of the material between $z = 0$ and $z \rightarrow \infty$ using the integral of $\frac{1}{2} \text{Re} \vec{J}(z) \cdot \vec{E}^*(z)$ as in lecture 21.

Compare your answers to parts e) and f).