Due Wednesday 16 April 2008 at 6:00 PM (End of day on Wednesday)

1. Laser Wave Propagation

A low noise diode pumped blue laser (457nm) available from Information Unlimited (http://www.amazing1.com/blue_lasers.htm) produces 5mW of continuous output power in a beam that is 3mm in diameter (at least when first emitted from the source). Find the average Poynting vector, electrical field and magnetic field of this beam and write them all in Phasor form. (e.g. for the electric field, find $E_i = E_{io}e^{-j\beta z}$)

Answer: The average Poynting vector is $S_{avei} = \frac{P}{Area} = \frac{5e-3}{7.07e-6} = 707.4\frac{W}{m^2}$. *From this we can determine the magnitudes of the electric and magnetic fields.*

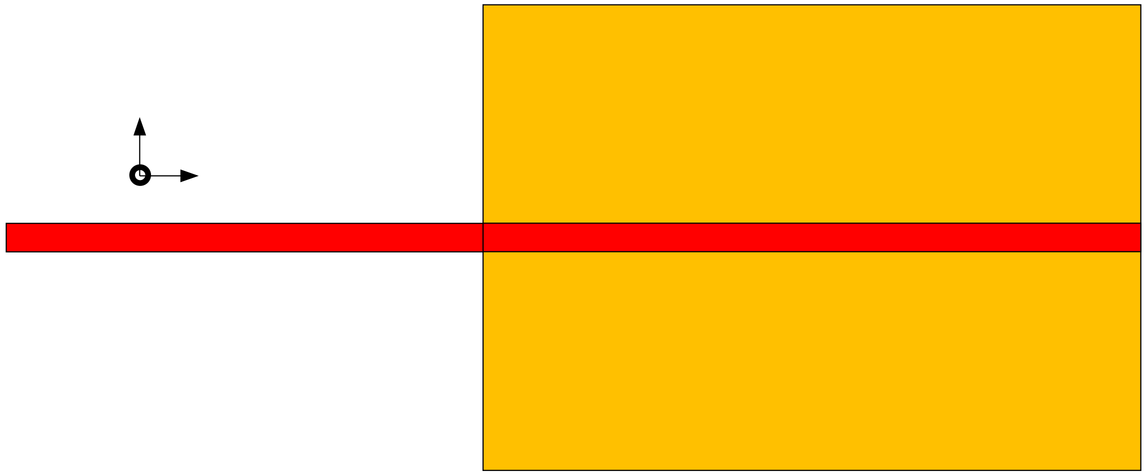
$E_{io} = \sqrt{2\eta_o S} = 730.3\frac{V}{m}$ *and* $H_{io} = \frac{E_{io}}{\eta_o} = 1.94\frac{A}{m}$. *The phase term* $\beta_o = \frac{2\pi}{\lambda} = 1.37e7m^{-1}$.

Then $\vec{E}_i = \hat{x}E_{io}e^{-j\beta_o z}$ *and* $\vec{H}_i = \hat{y}\frac{E_{io}}{\eta_o}e^{-j\beta_o z}$ *and the Poynting vector is* $\vec{S}_i = \hat{z}S_{avei}e^{-j2\beta_o z}$

Note that the Matlab m-file used to find the answers to all questions and to plot results is reproduced at the end of this document.

Plane Wave at Normal Incidence

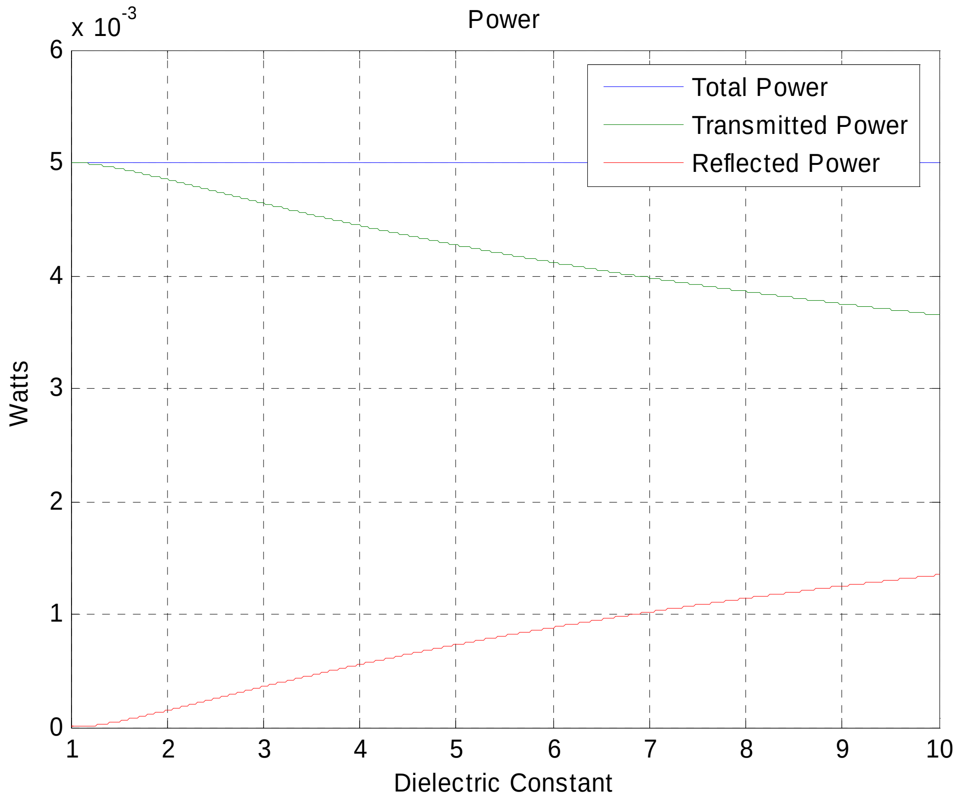
Given the plane wave of problem 1 is normally incident on the boundary of a dielectric medium with $\varepsilon = \varepsilon_r \varepsilon_o$, find the general form for the reflected and transmitted average power in terms of the incident power. Note that this question asks for power, not power density. You may assume that the beam diameter remains constant, even thought this is not exactly the case. Also, assume that the boundary is at *z = 0*.



Plot the two ratios for $1 \le \varepsilon_r \le 10$ which is the range of values we find for most practical materials.

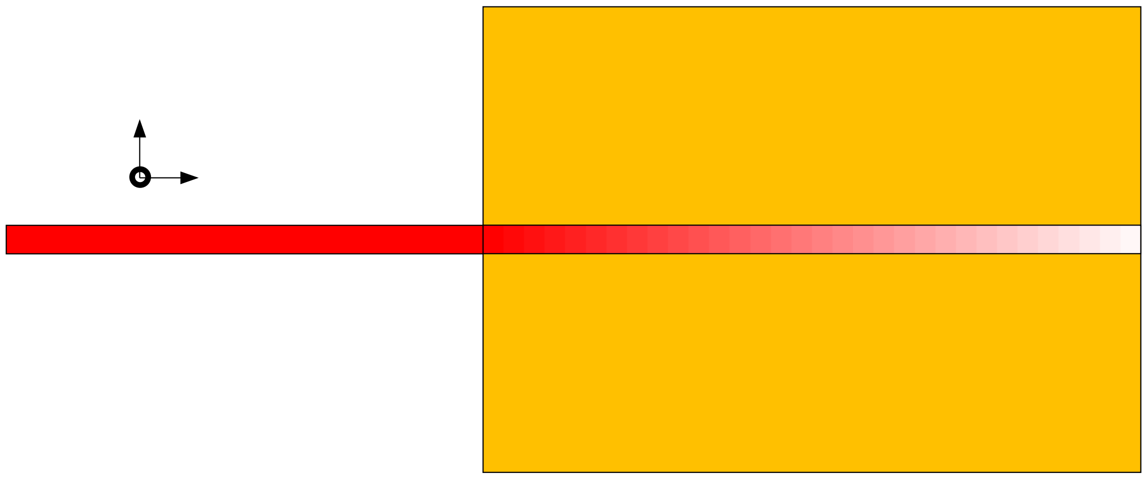
Answer: The reflection coefficient is $\Gamma = \dfrac{\eta - \eta_o}{\eta + \eta_o} = \dfrac{1-\sqrt{\varepsilon_r}}{1+\sqrt{\varepsilon_r}}$ *while the transmission coefficient is* $\tau = \dfrac{2\eta}{\eta + \eta_o} = \dfrac{2}{1+\sqrt{\varepsilon_r}}$. *The reflected electric field magnitude is* $E_{ro} = \Gamma E_{io}$ *while the transmitted electric field magnitude is* $E_{to} = \tau E_{io}$. *Then* $\vec{E}_r = \hat{x} E_{ro} e^{j\beta_o z}$, $\vec{H}_r = -\hat{y}\dfrac{E_{ro}}{\eta_o} e^{j\beta_o z}$, $\vec{E}_t = \hat{x} E_{to} e^{-j\beta z}$ *and* $\vec{H}_t = \hat{y}\dfrac{E_{to}}{\eta} e^{-j\beta z}$. *The reflected and transmitted average power densities are* $\vec{S}_r = -\hat{z} S_{aver} e^{j2\beta_o z}$ *and* $\vec{S}_t = \hat{z} S_{avet} e^{-j2\beta z}$ *where* $S_{aver} = \dfrac{E_{ro}^{\ 2}}{2\eta_o} = \Gamma^2 S_{avei}$ *and* $S_{avet} = \dfrac{E_{to}^{\ 2}}{2\eta} = \sqrt{\varepsilon_r}\,\tau^2 S_{avei}$. *For power, we must multiply these expressions by the area of the beam.* $P_{aver} = -A\Gamma^2 S_{avei}$ *and* $P_{avet} = A\sqrt{\varepsilon_r}\,\tau^2 S_{avei}$

Then the power as a function of the dielectric constant is plotted (using Matlab) as:



2. **Plane Waves in Lossy Media**

Now let the frequency be reduced to 10GHz, but use the same electric field as in the previous problems. Also, region 2 is now seawater. Assume, for simplicity, that the conductivity of seawater is $\sigma = 5\frac{S}{m}$ and the dielectric constant is $\varepsilon_r = 80$.



Find the reflected and transmitted power densities. *Reminder: the electric field magnitude is 730V/m from problem 1.*

Answer: First, we need to determine the general wave properties in seawater at 10GHz. The complex permittivity is $\varepsilon_c = \varepsilon_r \varepsilon_o - j\frac{\sigma}{\omega}$. *First, check to see if this is a low loss dielectric or a good conductor.* $\frac{\sigma}{\omega\varepsilon} = 0.11$ *so seawater can be treated as a low loss dielectric. In the solution that follows, we will generally use both the full solution and the approximate solution to see how accurate things are. The complex intrinsic impedance of seawater is* $\eta_c = \sqrt{\frac{\mu_o}{\varepsilon_c}} \approx \sqrt{\frac{\mu_o}{\varepsilon}}\left(1 + j\frac{\sigma}{2\omega\varepsilon}\right) = 42 + j2.4$ *for both expressions. The propagation constant is*

$\gamma = j\omega\sqrt{\mu_o \varepsilon_c} \approx j\omega\sqrt{\mu_o \varepsilon}\left(1 - j\frac{\sigma}{2\omega\varepsilon}\right) = \alpha + j\beta = 105.2 + j1876.2$ *with the two expressions very close. The reflection coefficient is now complex as is the transmission coefficient.* $\Gamma_c = \frac{\eta_c - \eta_o}{\eta_c + \eta_o}$ $\Gamma_c = -0.8 + j0.01$ *and* $\tau_c = \frac{2\eta_c}{\eta_c + \eta_o}$ $\tau_c = 0.2 + j0.01$. *Since the imaginary parts of both the reflection and transmission coefficients are so small, we can neglect them. Then the expressions for the reflected and transmitted power densities look,* *in general, like those for problem 2* $S_{aver} = \text{Re}\left(\frac{E_{ro}^{\ 2}}{2\eta_o}\right) = \text{Re}\left(\Gamma^2 S_{avei}\right) = 452.4\frac{W}{m^2}$ *and*

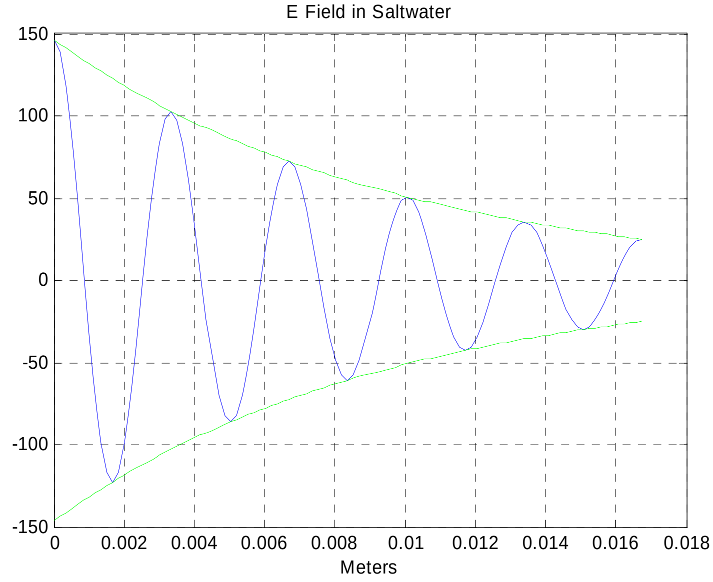
$S_{avet} = \text{Re}\left(\frac{E_{to}^{\ 2}}{2\eta}\right) = \text{Re}\left(\sqrt{\varepsilon_r}\,\tau^2 S_{avei}\right) = 255\frac{W}{m^2}$ *where we have to be careful for other cases to be sure that we use the full expressions (when the imaginary part is larger). The reflected and transmitted power densities are the Poynting vectors, so we have to write them in their full form.* $\vec{S}_{aver} = -\hat{z}S_{aver}$ *and* $\vec{S}_{avet} = \hat{z}S_{avet}e^{-2\alpha z}$.

3. **Penetration Depth in the Lossy Medium**

From the last problem, we see that some of the incident power will be transmitted beyond the boundary. Of the power that does enter the lossy material, 90% will be absorbed by the medium after the wave propagates some distance into the medium. Determine this distance.

Answer: 90% of the power will be gone when $e^{-2\alpha z} = 0.1$ *or*

$z = \frac{\ln(0.1)}{-2\alpha} = 0.0109m = 3.27\lambda$. *Thus, we can see that the wave decays quite fast. The following is not asked for, but it is instructive in understanding how the wave decays. Plotting the E field in the saltwater, we see that it decays rapidly. The power density decays twice as fast because it goes as the square of the E field.*



The m-file for the entire assignment follows. Each problem is addressed in turn.

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% Calculations for HW7 Spring 2008
% Uniform Plane Waves
% K.A. Connor, 13 April 2008

% Problem 1

epso=(1/(36*pi))*1e-9; muo=4*pi*1e-7; % Basic parameters

P=5e-3; % Laser beam power
lam=457e-9; % Blue wavelength
d=.003; % Beam diameter

A=pi*(d/2)^2; % Beam area
S=P/A; % Average Poynting Vector Magnitude

etao=120*pi; % Impedance of free space
c=3e8; % Speed of light
f=c/lam; % Frequency of blue laser light
w=2*pi*f; % Angular frequency

Eio=sqrt(2*etao*S); % E field magnitude
Hio=Eio/etao; % H field magnitude

Sio=Eio*Hio/2; % Checking math

betao=2*pi/lam; % Wave nuumber in free space for blue light

% Problem 2

epsr=[1:.01:10]; % Range of relative permittivity
eta=etao./sqrt(epsr); % Range of intrinsic impedances
gam=(eta-etao)./(eta+etao); % Range of reflection coefficients
tau=2.*eta./(eta+etao); % Range of transmission coefficients

Sro=Sio.*gam.^2; % Reflected average power density
Sto=Sio.*(sqrt(epsr)).*tau.^2; % Transmitted average power density
Pro=A.*Sro; % Reflected average power
Pto=A.*Sto; % Transmitted average power

plot(epsr,Pto+Pro,epsr,Pto,epsr,Pro); %Plotting the Power vs Dielectric Constant
grid;
legend('Total Power','Transmitted Power','Reflected Power');
title('Power');
xlabel('Dielectric Constant');
ylabel('Watts');

% Problem 3
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```
sig=5;epsrw=80; % Properties of seawater
fw=1e10; % Microwave frequency
wf=2*pi*fw; % Angular frequency
epsc=epsrw*epso-j*sig/wf; % Complex permittivity
KK=sig/(wf*epsrw*epso); % If KK << 1, then lossy dielectric, if >>1 then conductor

etac=sqrt(muo/epsc); % Complex permittivity
etacll=(1+j*sig/(2*epsrw*epso*wf))*120*pi/sqrt(epsrw); % Approx complex permittivity for low loss

gw=j*wf*sqrt(muo*epsc); % Complex propagation constant
alf=real(gw); % Decay constant
bw=imag(gw); % Real prop constant
bwll=wf*sqrt(muo*epsrw*epso); % Low loss expression for beta
alfll=bwll*sig/(2*wf*epsrw*epso); % Low loss expression for alpha
lamw=2*pi/bw; % Wavelength
sd=1/alf; % Skin depth
sdlam=sd/lamw; % Skin depth in wavelengths

gamw=(etac-etao)/(etac+etao); % Reflection coefficient for wave propagating toward water from
air
tauw=2*etac/(etac+etao); % Transmission coefficient for same conditions
% Can check to be sure that the imaginary parts can be neglected
gamw2=gamw*conj(gamw); % This is the square of the reflection coeff
tauw2=tauw*conj(tauw); % The square of the trans coeff

Srw=real(gamw2*Sio); % Reflected power density magnitude
Stw=real(sqrt(epsc/epso)*tauw2*Sio); % Transmitted power density magnitude

% Problem 4
z90=log(0.1)/(-2*alf); % Distance over which power density decays by 90%
z90lam=z90/lamw; % Same distance in wavelengths

% Not required, but informative is to plot the electric field in the saltwater
Eowt=tauw*Eio; % Magnitude
zw=[0:.01:1].*5.*lamw; % We will plot the E field over 5 wavelengths

Etw=Eowt*exp(-j.*bw.*zw).*exp(-alf.*zw); % Phasor E field in the saltwater
figure; plot(zw,real(Etw),zw,real(Eowt).*exp(-alf.*zw),'g',zw,-real(Eowt).*exp(-alf.*zw),'g');
grid;xlabel('Meters');
title('E Field in Saltwater');
```