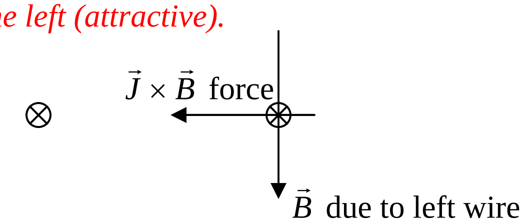
Homework 8

1. Short Questions

a. An air insulated parallel plate capacitor consists of two plates each with an area equal to A_o separated by a distance d. What is the capacitance of this configuration? (5 pts) *This is one of the simplest and most important expressions to remember from this course.* $C = \frac{\varepsilon_o A_o}{d}$ *Farads*

b. Two long straight wires carrying a current I run parallel to one another. Will these wires experience an attractive force or a repulsive force? (5 pts) *This is also a very fundamental phenomenon. Forces between two parallel conductors are such that like currents attract. The diagram below shows this. Assume the currents are going into the page. Then the field from the current on the left points vertically down at the right hand wire. The* $\vec{J} \times \vec{B}$ *force points to the left (attractive).*



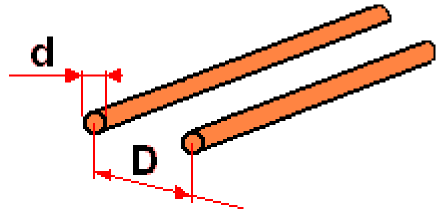
c. What is the input impedance of a transmission line with an open circuited load if the length of the line is a half wavelength ($\lambda/2$)? (5 pts) *For any half wavelength long transmission line, the input impedance equals the load impedance. Thus, the input impedance is infinity (open circuit).*

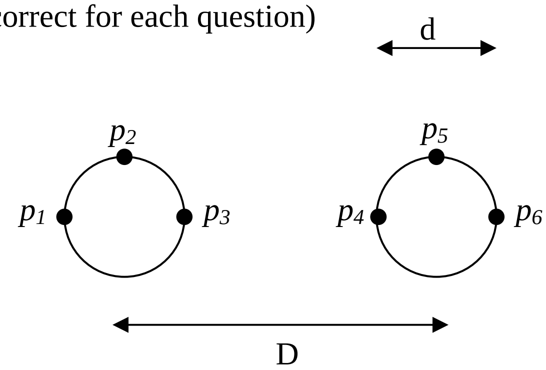
d. Will sunlight reflecting off of a body of water be polarized predominantly in the horizontal or vertical direction? (5 pts) *Light reflecting off of any dielectric will be primarily perpendicularly polarized. That means it will be primarily horizontally polarized since the water surface is horizontal.*

e. A uniform plane wave propagates in free space. It carries an average power density of 100 Watts per square meter. What is the magnitude of the electric field of this wave? (5 pts) *The average Poynting Vector is given by*

$\vec{S}_{ave} = \frac{1}{2}\mathrm{Re}\left(\vec{E} \times \vec{H}^*\right) = \frac{E_m^{\,2}}{2\eta_o}\hat{a}_S$ *in free space. Thus,* $E_m = \sqrt{2\eta_o 100} = 275\ V/m$ *(approximately)*

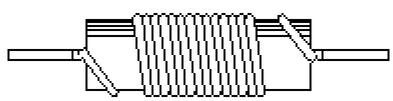
2. Multiple Choice (circle all answers that are correct for each question)





a. For the two-wire transmission line shown above, assume that there is a positive voltage V_o on the right hand wire and a negative voltage $-V_o$ on the left hand wire. As with all capacitors, there will be a surface charge density on each of the wires. Which of the following statements are true? (5 pts)

   i. The surface charge density is the most negative at point p_1 and most positive at the point p_6 and increases in the order $p_1, p_2, p_3, p_4, p_5, p_6$
   ii. The surface charge density is the most negative at point p_3 and most positive at the point p_4 and increases in the order $p_3, p_2, p_1, p_6, p_5, p_4$
   iii. The magnitude of the surface charge density is the same at the following points: p_1 & p_2, p_3 & p_4, p_5 & p_6
   iv. The magnitude of the surface charge density is the same at the following points: p_1 & p_6, p_3 & p_4, p_5 & p_2



b. The inductance of the solenoid shown above can be made larger by (5 pts)

   i. Winding it on a core with a larger μ
   ii. Winding more turns
   iii. Winding it on a core with the same length but larger area
   iv. Winding it on a core with the same area but larger length
   v. Winding it with thicker wire but all other parameters the same

   For all five cases, assume all other parameters remain exactly the same.

c. Relative to free space, waves propagating in a lossless dielectric medium at a given frequency will have which of the following characteristics? (5 pts)

   i. Higher propagation speed
   ii. Lower propagation speed
   iii. Higher intrinsic impedance
   iv. Lower intrinsic impedance
   v. Larger wavelength
   vi. Shorter wavelength

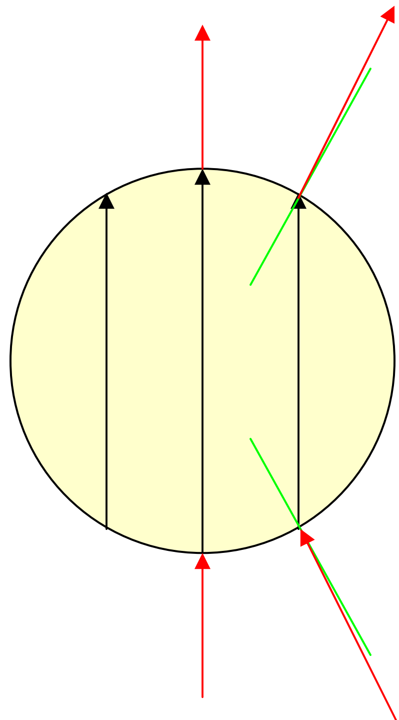
d. Two transmission lines are identical except for the insulator used. One uses Teflon with $\varepsilon = 2.1\varepsilon_o$ while the other uses Polystyrene with $\varepsilon = 2.6\varepsilon_o$. Otherwise, all dimensions and conductors are exactly the same. Comparing these two lines, which of the following are true? (5 pts)
   i. The capacitance per unit length of the Teflon cable is larger than the capacitance per unit length of the Polystyrene cable.
   ii. The time it takes a pulse to propagate from one end of a 100 meter cable is larger for the Teflon cable than the Polystyrene cable.
   iii. The characteristic impedance is larger for the Teflon cable than the Polystyrene cable.
   iv. The two cables have the same inductance per unit length.

e. Which of the following applications of Fields and Waves were mentioned in lecture? (5 pts)
   i. Polarizing sunglasses
   ii. Antennas for satellite communication
   iii. Transformers
   iv. Motors & generators
   v. Microwave ovens
   vi. Diodes (actually PN junctions)
   vii. Cable TV transmission lines

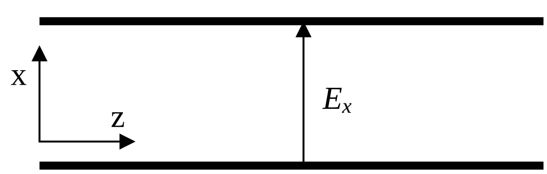
f. Standing wave patterns for both transmission lines and plane waves are characterized by which of the following? (5 pts)
   i. Maxima (voltage, current, electric field, magnetic field) are separated by one wavelength λ.
   ii. Minima (voltage, current, electric field, magnetic field) are separated by one half wavelength $\lambda/2$.
   iii. The standing wave ratio can take values between zero and *1*.
   iv. The reflection coefficient can take values between zero and *1*.

3. Boundary Conditions

a. The static magnetic field inside of a spherical magnetic object is known to be uniform and oriented vertically upward, as shown below. If this object has a high permeability $\mu >> \mu_o$, sketch the field lines in air just outside of the sphere. You only need the show the direction of the field lines near the sphere. Be sure to draw carefully. (10 pts) *First draw the normals, then trivial. Only showing on the right for clarity*



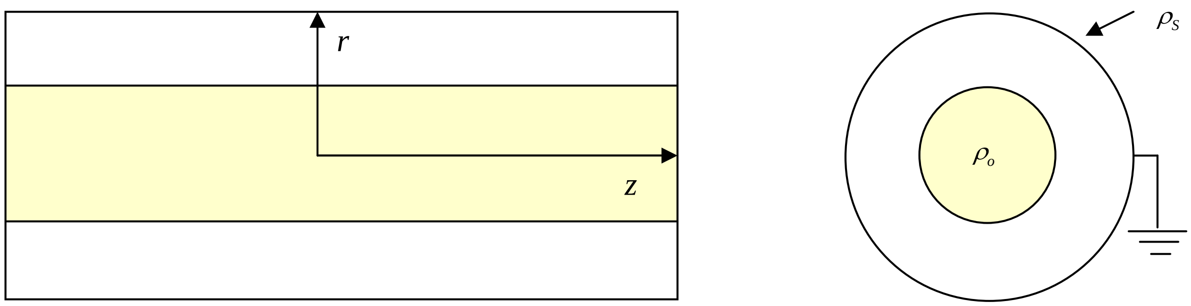
b. The following electric field is thought to exist in the region (air) between two conducting parallel plates $\vec{E} = \hat{x}10\cos\left(2\pi 10^6 t - \frac{\pi}{100}z\right)$, as shown. Determine the surface charge density on each of the plates (top and bottom). (10 pts) *One only needs to evaluate the boundary condition for normal D to get the surface charge density.* $\rho_S = D_n = \varepsilon_o E_x = \varepsilon_o 10\cos\left(2\pi 10^6 t - \frac{\pi}{100}z\right)$ *where the sign is negative for the top plate and positive for the bottom. The velocity of propagation is also wrong since it is not equal to the speed of light in vacuum.*



While part a only requires that you show a reasonable approximation of the field direction, a specific answer is required for part b.

4. Gauss' Law

A uniformly charged beam (volume charge density equals ρ_o) exists in the region *0<r<a*. This beam is surrounded by a thin conducting shell at $r = b$. The outer conductor is grounded and, thus, must have an equal and opposite total charge per unit length as is found in the beam.



a. Using Gauss's Law, solve for the electric field for the region from *0<r<a*. (5 pts)
The Gaussian surface for this case is a cylinder (the end caps contribute nothing because the field must be in the radial direction. $\oint \vec{D}\cdot d\vec{S} = D_r 2\pi r l = Q_{encl} = \rho_o \pi r^2 l$ *thus*

$$E_r = \frac{\rho_o \pi r^2}{\varepsilon_o 2\pi r} = \frac{\rho_o r}{\varepsilon_o 2}$$

b. Again, using Gauss' Law, solve for the electric field for the region from *a<r<b*. (5 pts)
The same Gaussian surface $\oint \vec{D}\cdot d\vec{S} = D_r 2\pi r l = Q_{encl} = \rho_o \pi a^2 l$ *so that*

$$E_r = \frac{\rho_o \pi a^2}{\varepsilon_o 2\pi r} = \frac{\rho_o a^2}{\varepsilon_o 2r}$$

c. Finally, what is the electric field for the region *r>b*? (5 pts)
$E_r = 0$ *because no net charge is enclosed.*

d. Verify your solutions to parts a, b, and c by applying the differential form of Gauss' Law. (5 pts) *Neither expression has a curl since they vary and are directed only in the r direction. The divergence must also be zero.*

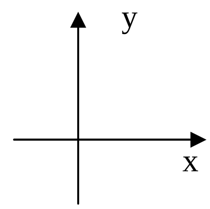
$$\nabla\cdot E_r\hat{r} = \frac{1}{r}\frac{\partial}{\partial r} r\frac{\rho_o r}{\varepsilon_o 2} = \frac{1}{r}\frac{\partial}{\partial r}\frac{\rho_o r^2}{\varepsilon_o 2} = \frac{1}{r}\frac{\rho_o 2r}{\varepsilon_o 2} = \frac{\rho_o}{\varepsilon_o}$$ *as it should while*

$$\nabla\cdot E_r\hat{r} = \frac{1}{r}\frac{\partial}{\partial r} r\frac{\rho_o a^2}{\varepsilon_o 2r} = \frac{1}{r}\frac{\partial}{\partial r}\frac{\rho_o a^2}{\varepsilon_o 2} = 0$$ *again as it should.*

5. Laplace's Equation

The spreadsheet, finite difference method was used to determine the voltage values in some region of space. In the table below, the voltages in the shaded cells on the outside are fixed. The actual voltages in the nine interior cells have been rounded up or down to display no decimal digits. The cell separation is 1mm.

110	111	113	111	110
100	100	100	100	100
90	89	88	89	90
80	77	75	77	80
70	65	55	65	70



Identify one cell which has the correct relationship to its neighbors. (circle it and justify your answer) (5 pts) *For the upper left cell, 100=(100+100+111+89)/4 so it works as it should.*
Identify one cell which has an incorrect relationship to its neighbors. (circle it and justify your answer) (5 pts) *For the upper center cell, (113+88+100+100)/4 does not equal 100, so it does not work.*

Using the given voltages, estimate the electric field vector for the middle cell (88 Volts), giving both direction and magnitude. Note the given coordinate system. (5 pts) *For the center cell, there is no variation in the horizontal direction and the configuration has a line of symmetry vertically through the center cell. Thus, the electric field must point vertically downward through this cell. Its magnitude is given by the average of* $\frac{100-88}{0.001} = 12000$ *and* $\frac{88-75}{0.001} = 13000$ *Thus, the electric field is given by* $\vec{E} = -\hat{y}12500$

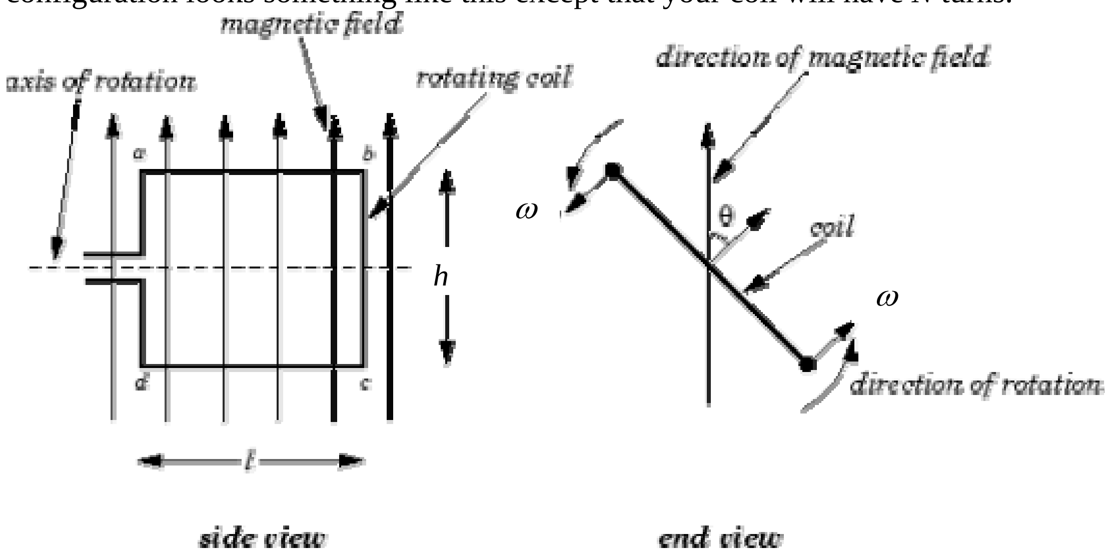
Five of the cell voltages are actually integers and, thus, are completely correct as shown. Can you identify them? (5 pts) *Just make an educated guess. Any answer will be accepted.*
They are the ones in the four corners and the center. The only good way to tell this is to iterate a few times on the six cells that have unique values. The final result is shown below. In a quiz, one cannot use Excel. Thus, one has to do this by hand. (see below)

110	111	113	111	110
100	100	100.25	100	100
90	88.75	88	88.75	90
80	77	74.25	77	80
70	65	55	65	70

Upper left is already OK, upper center new value =(113+88+100+100)/4=100.25
Center left new value = (100+88+77+90)/4=88.75
Bottom center new value = (88+77+77+55)/4=74.25
Then center and bottom center work out as is. This is just good guess work, largely or brute force for several iterations.

6. Faraday's Law

Last weekend, LITEC students tried to fly their blimps in the new Biotech center. This building has a very large atrium that should be ideal for this purpose since it is several stories high and at least 25 feet wide, more than large enough for the blimps to pivot in. Unfortunately, the blimps could not fly because the large amounts of steel in the building made their compasses useless. Because the magnetic fields in the building were significantly modified from the usual value and direction for the earth's field, some of the people with labs there became concerned that their sensitive and expensive lab equipment might also be affected. To allay their fears, assume that you are given the job to quickly measure the field to see if you can, at least, demonstrate that it is largely a direction problem and the field magnitude is not large enough to be a problem for anything. To do this, you wind a coil like the one we used for the Beakman's motor. You spin the coil on its axis and measure the voltage induced. Assume that you were clever enough to wind a square coil (can be done on a square frame) to keep the analysis simple. The resulting configuration looks something like this except that your coil will have *N* turns.



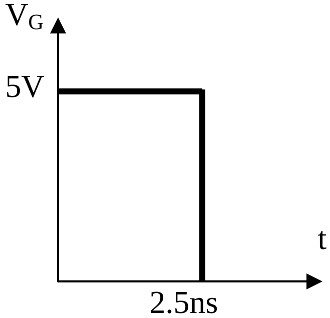
The coil has a height *h* and a length *l* and turns at a frequency ω.

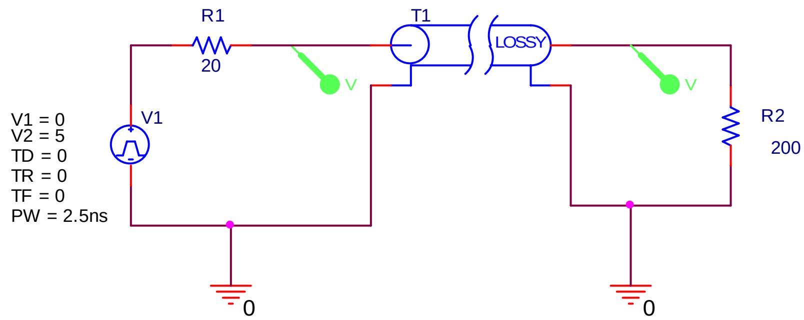
First find a general expression for the voltage induced across the terminals of the coil. Assume that the magnitude of the magnetic field is B_o. (10 pts) *The induced voltage is given by the time derivative of the flux passing through the coil. The flux is given by the product of the magnetic field and the loop area. The total flux is N times for N turns. Thus* $V = NB_o\omega hl\sin(\omega t)$ *where* $Area = hl\cos(\omega t)$ *is the area aligned with the magnetic field.*

Next, assume that *w = h = 2cm*, B_o*=1mT*, the rotation frequency *f=10Hz,* and then number of turns is *N = 1000.* Determine the numerical magnitude of the induced voltage. As long as the real voltage measured is less than this number, the actual field magnitude will be very small. (10 pts)

$V = NB_o\omega hl\sin(\omega t) = 1000(0.001)(2\pi 10)(.02)(.02)\sin(20\pi t) = 0.025\sin(20\pi t)$ *so that, if the field is larger than this, it can be seen, since 25mV is a reasonable signal level.*

7. Pulses on Transmission Lines





The 5V pulse is launched on the 20 cm long transmission line shown. For this line, the capacitance per unit length is $1.7x10^{-10}$ F/m and the inductance per unit length is $1.1x10^{-6}$ H/m. The propagation speed on this line was designed to be quite slow.

a. Find the characteristic impedance of the line. (5) *The characteristic impedance is given by* $Z_o = \sqrt{\frac{l}{c}} = \sqrt{\frac{1.1x10^{-6}}{1.7x10^{-10}}} = 80.44$ *for the purposes of this problem, we can approximate this as 80 to keep the math simpler. For the next part, we need the reflection coefficients at each end.* $\Gamma_L = \frac{200-80}{200+80} = .429$ *(actually .426) and* $\Gamma_S = \frac{20-80}{20+80} = -.6$ *while the voltage divider relationship is* $V_{IN} = 5\frac{80}{80+20} = 4$

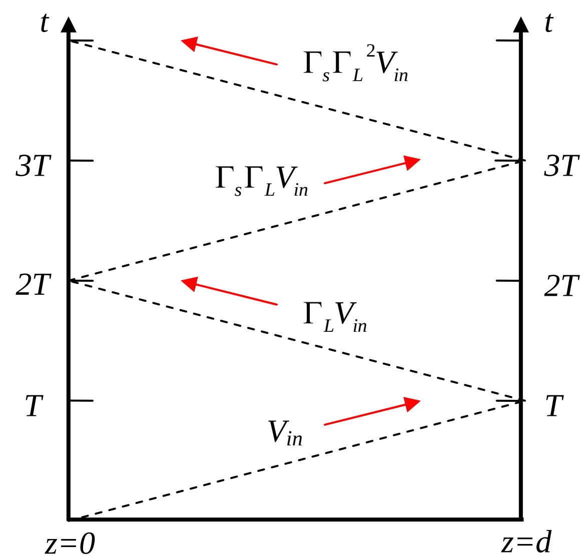
b. Generate the bounce diagram for this configuration. Include all numbers. Continue the diagram until the voltages observed at **both** the input and output ends of the lines are less than 1 Volt. Draw carefully. (10)

$V_{IN} = 4$

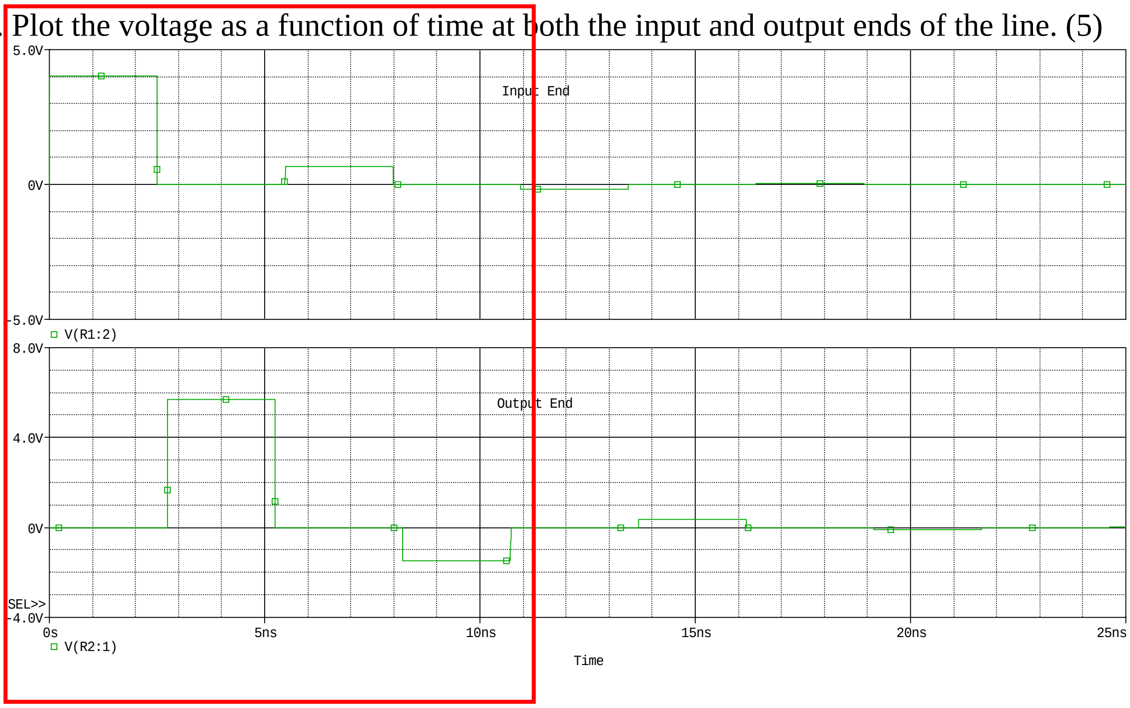
$\Gamma_L V_{IN} = (0.429)4 = 1.7$

$\Gamma_S \Gamma_L V_{IN} = (-0.6)1.7 = -1.03$

$\Gamma_S \Gamma_L{}^2 V_{IN} = (0.429)(-1.03) = -0.44$

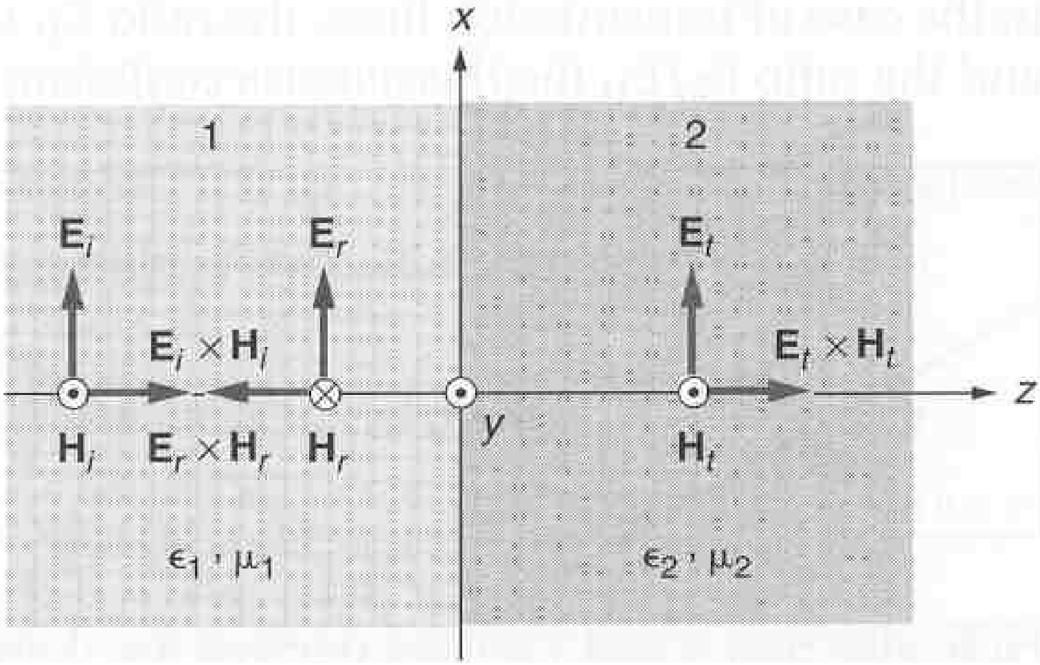


c. Plot the voltage as a function of time at both the input and output ends of the line. (5)



The section in the box shows the only pulses with amplitudes greater than 1 V. The first pulse at the load should have a voltage of (1.429)4=5.7 which it does. The second pulse at the source end should have a voltage of (1-0.6)1.7=.68 which it does. The second pulse at the load end should have a voltage of (1.429)(-1.023)=-1.462 which it does. The rest of the pulses are all smaller than 1V.

8. Reflection and Transmission



a. A uniform plane wave is incident in air ($\varepsilon_1 = \varepsilon_o$ & $\mu_1 = \mu_o$) on a lossless dielectric medium. ($\varepsilon_2 = \varepsilon_r \varepsilon_o$ & $\mu_2 = \mu_o$). The incident power density of the wave is 100 Watts per square meter. Assume that the frequency of the wave, ω, is given. Write the incident electric field in phasor form. (4) *The incident electric field is given by* $\vec{E}_i = \hat{x} E_m^{\,+} e^{-j\beta_o z}$ *where* $\beta_0 = \omega\sqrt{\mu_o \varepsilon_o}$ *and* $E_m^{\,+} = \sqrt{2\eta_o 100} = 275$

b. Assume that 20% of the incident power is reflected. Write the reflected electric field in phasor form. (4) *The reflected electric field is given by* $\vec{E}_r = -\hat{x} E_m^{\,-} e^{+j\beta_o z}$ *where* $E_m^{\,-} = \sqrt{2\eta_o 20} = 123$*. The reflected field must have a negative magnitude since the impedance of the dielectric must be less than that of air. Note that we could also get this by dividing the incident amplitude by the square root of 5 since the E field magnitude must reduce by the square root of the reduction in the power density.*

c. From your answers to parts a and b, determine the dielectric constant ε_r for the second medium. (4) *The reflection coefficient is given by*

$$\Gamma = -\frac{E_m^{\,-}}{E_m^{\,+}} = -\frac{123}{275} = -0.45 = \frac{\sqrt{\varepsilon_1} - \sqrt{\varepsilon_2}}{\sqrt{\varepsilon_1} + \sqrt{\varepsilon_2}}$$ *which we can rearrange to solve for* ε_r.

$$\varepsilon_r = \left(\frac{1-\Gamma}{1+\Gamma}\right)^2 = 7$$

d. Write the transmitted electric field in phasor form. (4) *The transmitted electric field is given by* $\vec{E}_t = \hat{x}E_{m2}^{\ +}e^{-j\beta z}$ *where* $\beta = \omega\sqrt{\mu_o\varepsilon_r\varepsilon_o} = \sqrt{7}\beta$*,* $\eta = \sqrt{\frac{\mu_o}{\varepsilon_r\varepsilon_o}} = \frac{\eta_o}{\sqrt{7}} = 143$ *and*

$E_{m2}^{\ +} = \sqrt{2\eta 80} = 152$

e. Using your answer to part d, evaluate the average Poynting vector in the second medium to demonstrate that all power is fully accounted for. That is, the incident power equals the sum of the reflected and transmitted power. (4) *The average transmitted is given by* $\frac{1}{2}\frac{E_{m2}^{\ +}}{\eta} = \frac{(152)^2}{2(143)} = 80$ *as expected (or at least very close).*

9. Comprehensive Question on Plane Waves and Transmission Lines.

Given a uniform plane wave with $\vec{E} = \hat{x}100\cos(6\pi 10^8 t - 3\pi z)$ is propagating in a lossless dielectric medium.

a. What are the direction of propagation, the wavelength λ, and the frequency f of this wave? (3) *The wave propagates in the +z direction, has a frequency of 3x10*^{8}*Hz and a wavelength given by* $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{3\pi} = 0.667$ *We can also check this by looking at the speed of propagation* $u = \frac{\omega}{\beta} = \frac{6\pi 10^8}{3\pi} = 2x10^8$ *The speed will be smaller than c by the index of refraction* $n = \frac{3}{2}$. *The wavelength must be* $\lambda = \frac{c}{nf} = \frac{3x10^8}{\left(\frac{3}{2}\right)3x10^8} = \frac{2}{3}$ *as above.*

b. Write this electric field wave in phasor form. (3) $\vec{E} = \hat{x}100e^{-j3\pi z}$

c. From what you know of the wave characteristics, what is the permittivity $\varepsilon = \varepsilon_r \varepsilon_o$? (3) *The propagation speed is 2/3 that of light in vacuum, thus* $n = \frac{3}{2}$ *and* $\varepsilon_r = (1.5)^2 = 2.25$

d. What is the intrinsic impedance of the medium η? (3) $\eta = \frac{\eta_o}{n} = \frac{120\pi}{1.5} = 80\pi$

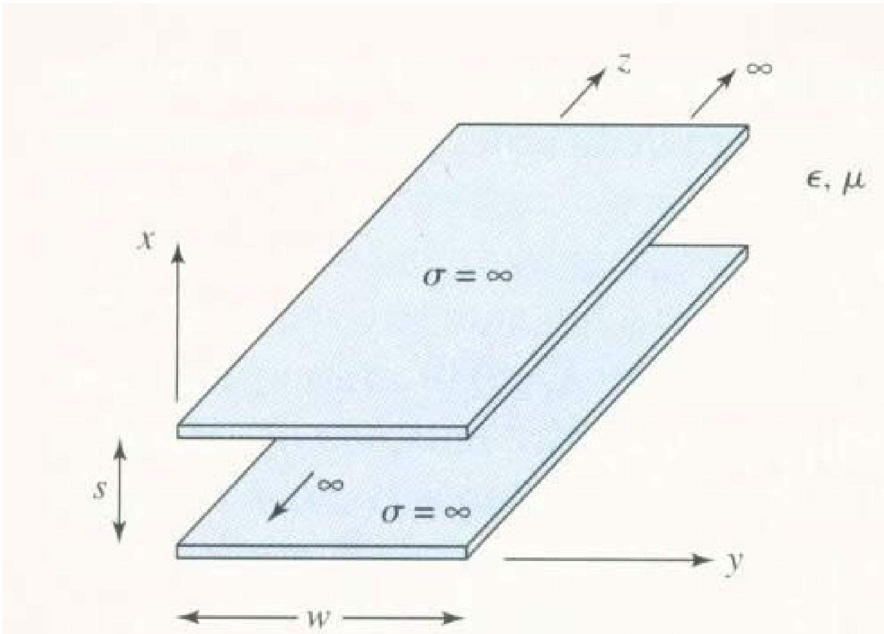
e. Write the phasor form of the magnetic field vector $\vec{H}$? (3)

$$\vec{H} = \hat{y}\frac{100}{80\pi}e^{-j3\pi z} = \hat{y}\frac{5}{4\pi}e^{-j3\pi z}$$

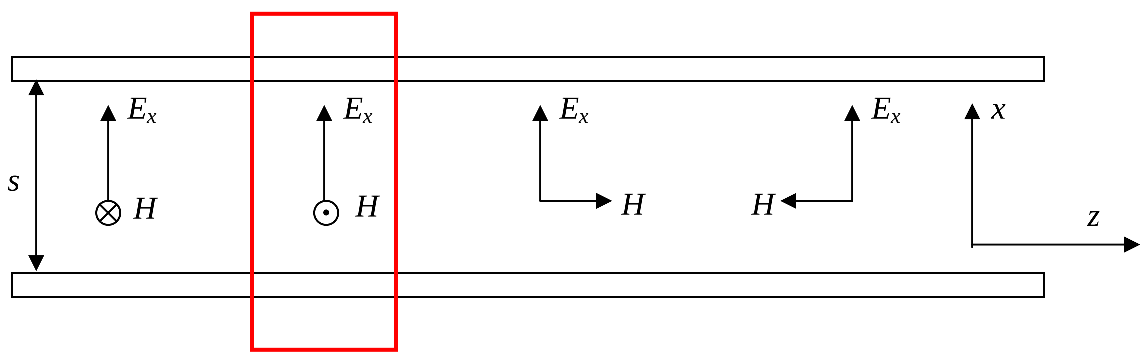
f. What is the average power density (Poynting Vector) carried by this wave? (4)

$$\vec{S} = \frac{1}{2}\mathrm{Re}\,\vec{E}\times\vec{H} = \hat{z}\frac{100^2}{2(80\pi)} = \hat{z}\frac{62.5}{\pi}$$

Now we want to demonstrate that this wave can propagate in the space between two perfectly conducting plates that form a parallel plate transmission line (also known as a stripline) as shown below. From the given information on the electric field, we know that the electric field vector is as shown.



g. On the diagram below (a side view of the transmission line, also can be described as looking in the negative y-direction), pick out the correct direction for the magnetic field using the information you have given above, by circling the correct configuration. The symbol ⊗ designates into the page while the symbol ⊙ designates out of the page. (2)



h. From the information given on the uniform plane wave above, determine either the phasor form of the voltage *v(z)* or the current *i(z)* on the transmission line. (4)

$v(z) = |\vec{E}|s = 100se^{-j3\pi z}$ *while for the current,* $i(z) = |\vec{H}|w = w\dfrac{5}{4\pi}e^{-j3\pi z}$ *we can check this result by looking at* $Z_o = \dfrac{v(z)}{i(z)} = \dfrac{(4\pi)100se^{-j3\pi z}}{w5e^{-j3\pi z}} = \dfrac{s80\pi}{w} = \eta\dfrac{s}{w}$ *which is the same result obtained in lecture 25.*