Homework 8

Due 2 May 2005

1. Short Questions

a. An air insulated parallel plate capacitor consists of two plates each with an area equal to A_o separated by a distance d. What is the capacitance of this configuration? (5 pts)

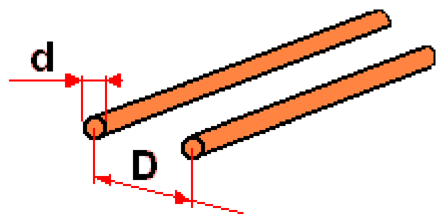
b. Two long straight wires carrying a current I run parallel to one another. Will these wires experience an attractive force or a repulsive force? (5 pts)

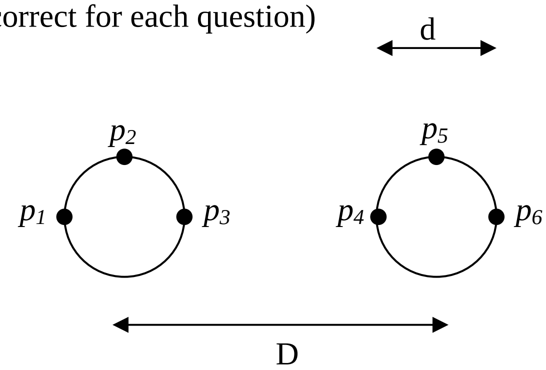
c. What is the input impedance of a transmission line with an open circuited load if the length of the line is a half wavelength ($\lambda/2$)? (5 pts)

d. Will sunlight reflecting off of a body of water be polarized predominantly in the horizontal or vertical direction? (5 pts)

e. A uniform plane wave propagates in free space. It carries an average power density of 100 Watts per square meter. What is the magnitude of the electric field of this wave? (5 pts)

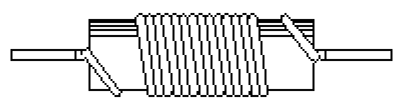
2. Multiple Choice (circle all answers that are correct for each question)





a. For the two-wire transmission line shown above, assume that there is a positive voltage V_o on the right hand wire and a negative voltage $-V_o$ on the left hand wire. As with all capacitors, there will be a surface charge density on each of the wires. Which of the following statements are true? (5 pts)

  i. The surface charge density is the most negative at point p_1 and most positive at the point p_6 and increases in the order $p_1, p_2, p_3, p_4, p_5, p_6$
  ii. The surface charge density is the most negative at point p_3 and most positive at the point p_4 and increases in the order $p_3, p_2, p_1, p_6, p_5, p_4$
  iii. The magnitude of the surface charge density is the same at the following points: p_1 & p_2, p_3 & p_4, p_5 & p_6
  iv. The magnitude of the surface charge density is the same at the following points: p_1 & p_6, p_3 & p_4, p_5 & p_2



b. The inductance of the solenoid shown above can be made larger by (5 pts)

  i. Winding it on a core with a larger μ
  ii. Winding more turns
  iii. Winding it on a core with the same length but larger area
  iv. Winding it on a core with the same area but larger length
  v. Winding it with thicker wire but all other parameters the same

For all five cases, assume all other parameters remain exactly the same.

c. Relative to free space, waves propagating in a lossless dielectric medium at a given frequency will have which of the following characteristics? (5 pts)

  i. Higher propagation speed
  ii. Lower propagation speed
  iii. Higher intrinsic impedance
  iv. Lower intrinsic impedance
  v. Larger wavelength
  vi. Shorter wavelength

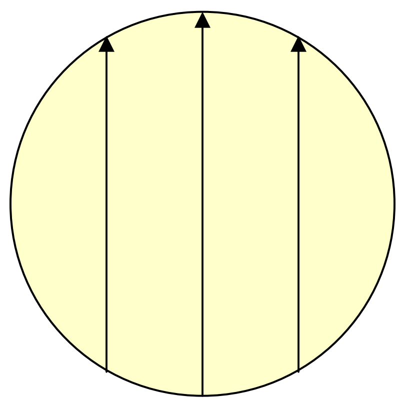
d. Two transmission lines are identical except for the insulator used. One uses Teflon with $\varepsilon = 2.1\varepsilon_o$ while the other uses Polystyrene with $\varepsilon = 2.6\varepsilon_o$. Otherwise, all dimensions and conductors are exactly the same. Comparing these two lines, which of the following are true? (5 pts)
   i. The capacitance per unit length of the Teflon cable is larger than the capacitance per unit length of the Polystyrene cable.
   ii. The time it takes a pulse to propagate from one end of a 100 meter cable is larger for the Teflon cable than the Polystyrene cable.
   iii. The characteristic impedance is larger for the Teflon cable than the Polystyrene cable.
   iv. The two cables have the same inductance per unit length.

e. Which of the following applications of Fields and Waves were mentioned in lecture? (5 pts)
   i. Polarizing sunglasses
   ii. Antennas for satellite communication
   iii. Transformers
   iv. Motors & generators
   v. Microwave ovens
   vi. Diodes (actually PN junctions)
   vii. Cable TV transmission lines

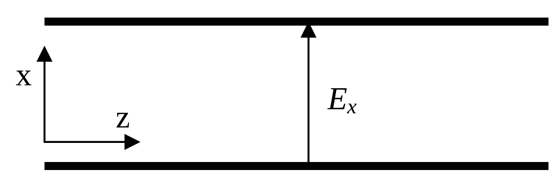
f. Standing wave patterns for both transmission lines and plane waves are characterized by which of the following? (5 pts)
   i. Maxima (voltage, current, electric field, magnetic field) are separated by one wavelength λ.
   ii. Minima (voltage, current, electric field, magnetic field) are separated by one half wavelength $\lambda/2$.
   iii. The standing wave ratio can take values between zero and *1*.
   iv. The reflection coefficient can take values between zero and *1*.

3. Boundary Conditions

a. The static magnetic field inside of a spherical magnetic object is known to be uniform and oriented vertically upward, as shown below. If this object has a high permeability $\mu >> \mu_o$, sketch the field lines in air just outside of the sphere. You only need the show the direction of the field lines near the sphere. *Be sure to draw carefully and remember what happens at a planar boundary.* (10 pts)

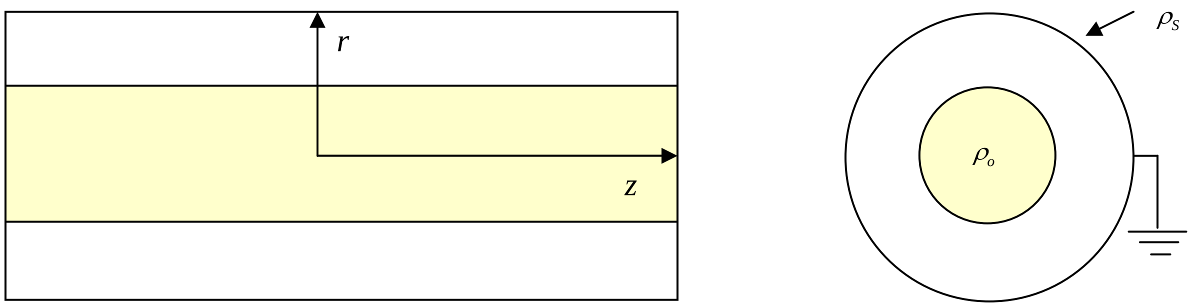


b. What is wrong with the answer to this question? The following electric field is thought to exist in the region (air) between two conducting parallel plates $\vec{E} = \hat{x}10\cos\left(2\pi 10^6 t - \frac{\pi}{100}z\right)$, as shown. Determine the surface charge density on each of the plates (top and bottom). (10 pts) Ans: *One only needs to evaluate the boundary condition for normal D to get the surface charge density.* $\rho_S = D_n = \varepsilon_o E_x = \varepsilon_o 10\cos\left(2\pi 10^6 t - \frac{\pi}{100}z\right)$ *where the sign is positive for the top plate and negative for the bottom.*



4. Gauss' Law

A uniformly charged beam (volume charge density equals ρ_o) exists in the region $0<r<a$. This beam is surrounded by a thin conducting shell at $r = b$. The outer conductor is grounded and, thus, must have an equal and opposite total charge per unit length as is found in the beam.



a. Using Gauss's Law, solve for the electric field for the region from $0<r<a$. (5 pts)

b. Again, using Gauss' Law, solve for the electric field for the region from $a<r<b$. (5 pts)

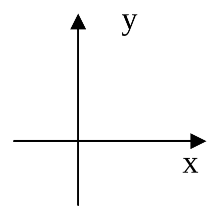
c. Finally, what is the electric field for the region $r>b$? (5 pts)

d. Verify your solutions to parts a, b, and c by applying the differential form of Gauss' Law. (5 pts)

5. Laplace's Equation

The spreadsheet, finite difference method was used to determine the voltage values in some region of space. In the table below, the voltages in the shaded cells on the outside are fixed. The actual voltages in the nine interior cells have been rounded up or down to display no decimal digits. The cell separation is 1mm.

110	111	113	111	110
100	100	100	100	100
90	89	88	89	90
80	77	75	77	80
70	65	55	65	70



Identify one cell which has the correct relationship to its neighbors. (circle it and justify your answer) (5 pts)

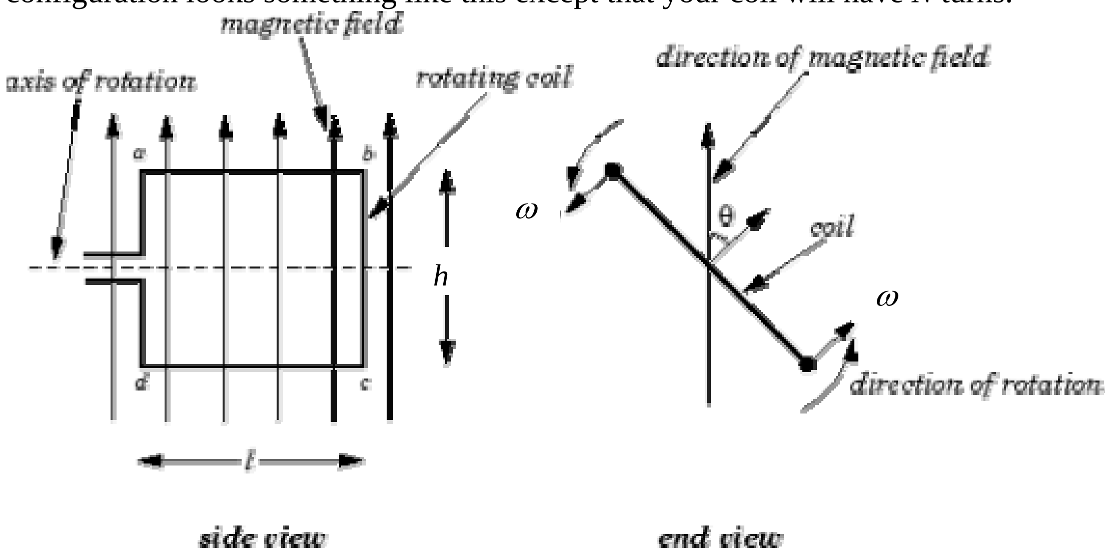
Identify one cell which has an incorrect relationship to its neighbors. (circle it and justify your answer) (5 pts)

Using the given voltages, estimate the electric field vector for the middle cell (88 Volts), giving both direction and magnitude. Note the given coordinate system. (5 pts)

Five of the cell voltages are actually integers and, thus, are completely correct as shown. Can you identify them? (5 pts) *Just make an educated guess. Any answer will be accepted.*

6. Faraday's Law

Last weekend, LITEC students tried to fly their blimps in the new Biotech center. This building has a very large atrium that should be ideal for this purpose since it is several stories high and at least 25 feet wide, more than large enough for the blimps to pivot in. Unfortunately, the blimps could not fly because the large amounts of steel in the building made their compasses useless. Because the magnetic fields in the building were significantly modified from the usual value and direction for the earth's field, some of the people with labs there became concerned that their sensitive and expensive lab equipment might also be affected. To allay their fears, assume that you are given the job to quickly measure the field to see if you can, at least, demonstrate that it is largely a direction problem and the field magnitude is not large enough to be a problem for anything. To do this, you wind a coil like the one we used for the Beakman's motor. You spin the coil on its axis and measure the voltage induced. Assume that you were clever enough to wind a square coil (can be done on a square frame) to keep the analysis simple. The resulting configuration looks something like this except that your coil will have *N* turns.

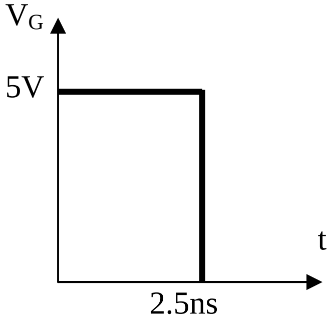


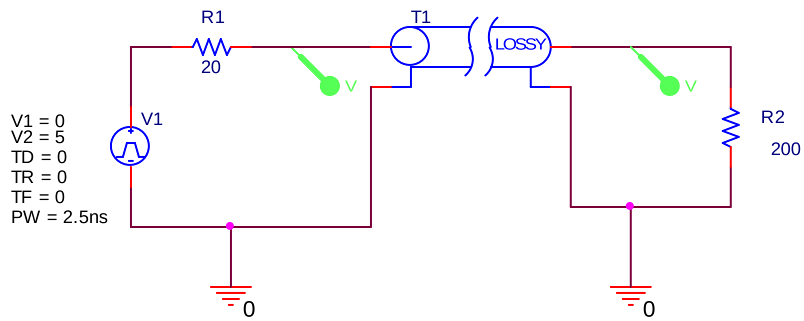
The coil has a height *h* and a length *l* and turns at a frequency ω.

First find a general expression for the voltage induced across the terminals of the coil. Assume that the magnitude of the magnetic field is B_o. (10 pts)

Next, assume that $w = h = 2cm$, $B_o=1mT$, the rotation frequency $f=10Hz$, and then number of turns is $N = 1000$. Determine the numerical magnitude of the induced voltage. As long as the real voltage measured is less than this number, the actual field magnitude will be very small. (10 pts)

7. Pulses on Transmission Lines





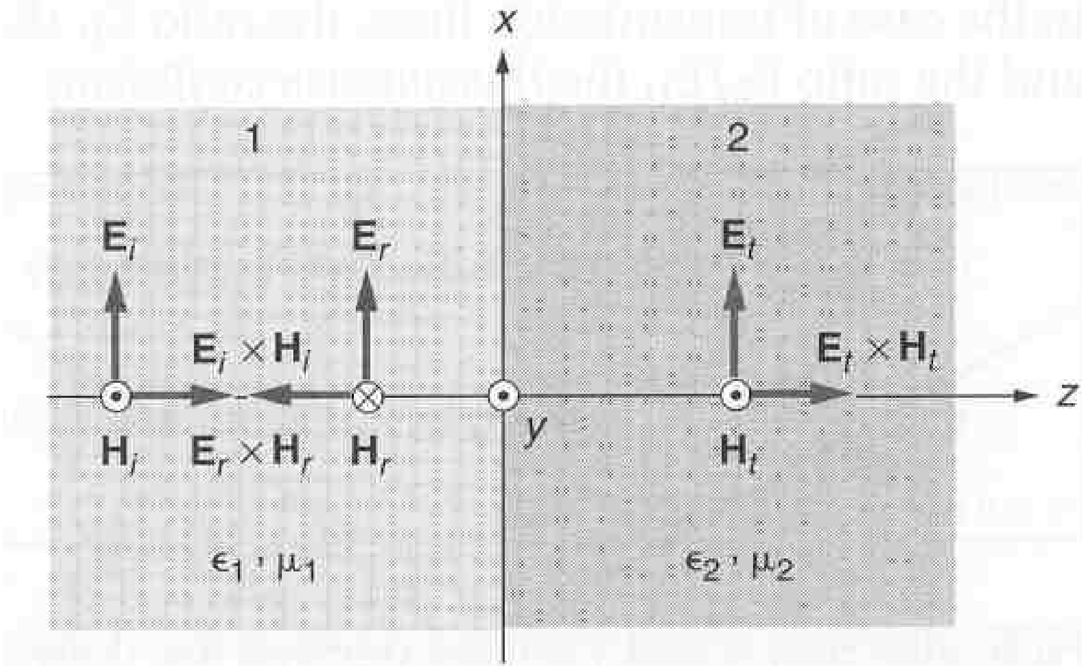
The 5V pulse is launched on the 20 cm long transmission line shown. For this line, the capacitance per unit length is 1.7×10^{-10} F/m and the inductance per unit length is 1.1×10^{-6} H/m. The propagation speed on this line was designed to be quite slow.

a. Find the characteristic impedance of the line. (5)

b. Generate the bounce diagram for this configuration. Include all numbers. Continue the diagram until the voltages observed at **both** the input and output ends of the lines are less than 1 Volt. Draw carefully. (10)

c. Plot the voltage as a function of time at both the input and output ends of the line. (5)

8. Reflection and Transmission



a. A uniform plane wave is incident in air ($\varepsilon_1 = \varepsilon_o$ & $\mu_1 = \mu_o$) on a lossless dielectric medium. ($\varepsilon_2 = \varepsilon_r \varepsilon_o$ & $\mu_2 = \mu_o$). The incident power density of the wave is 100 Watts per square meter. Assume that the frequency of the wave, ω , is given. Write the incident electric field in phasor form. (4)

b. Assume that 20% of the incident power is reflected. Write the reflected electric field in phasor form. (4)

c. From your answers to parts a and b, determine the dielectric constant ε_r for the second medium. (4)

d. Write the transmitted electric field in phasor form. (4)

e. Using your answer to part d, evaluate the average Poynting vector in the second medium to demonstrate that all power is fully accounted for. That is, the incident power equals the sum of the reflected and transmitted power. (4)

9. Comprehensive Question on Plane Waves and Transmission Lines.

Given a uniform plane wave with $\vec{E} = \hat{x}100\cos(6\pi 10^8 t - 3\pi z)$ is propagating in a lossless dielectric medium.

a. What are the direction of propagation, the wavelength λ, and the frequency f of this wave? (3)

b. Write this electric field wave in phasor form. (3)

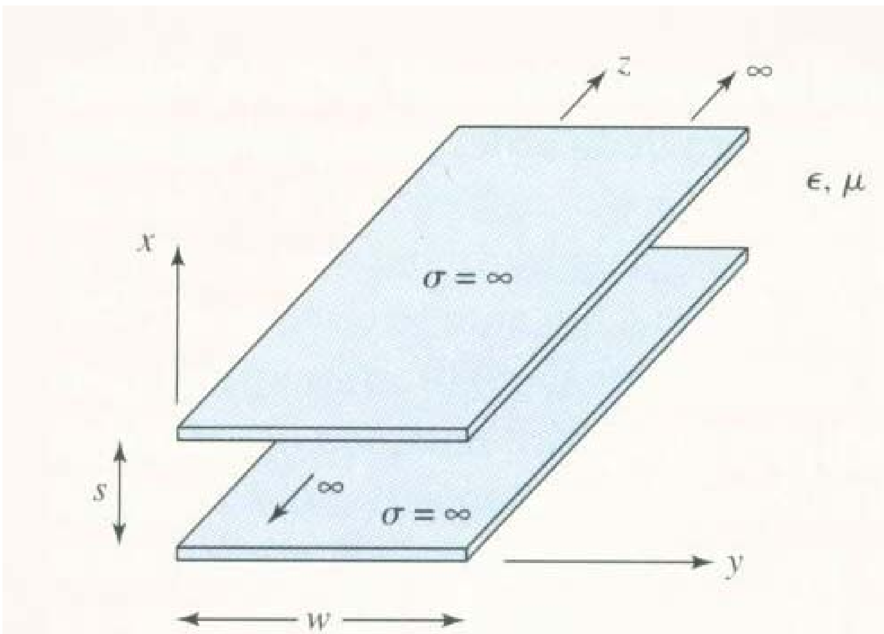
c. From what you know of the wave characteristics, what is the permittivity $\varepsilon = \varepsilon_r \varepsilon_o$? (3)

d. What is the intrinsic impedance of the medium η? (3)

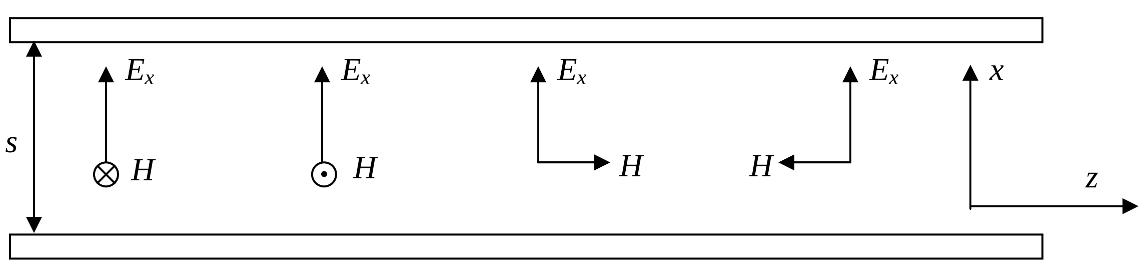
e. Write the phasor form of the magnetic field vector $\vec{H}$? (3)

f. What is the average power density (Poynting Vector) carried by this wave? (4)

Now we want to demonstrate that this wave can propagate in the space between two perfectly conducting plates that form a parallel plate transmission line (also known as a stripline) as shown below. From the given information on the electric field, we know that the electric field vector is as shown.



g. On the diagram below (a side view of the transmission line, also can be described as looking in the negative y-direction), pick out the correct direction for the magnetic field using the information you have given above, by circling the correct configuration. The symbol ⊗ designates into the page while the symbol ⊙ designates out of the page. (2)



h. From the information given on the uniform plane wave above, determine <u>either</u> the phasor form of the voltage *v(z)* or the current *i(z)* on the transmission line. (4)