ECSE 2100
Fields & Waves I
Fall 2007
Homework #8 Solutions

1a) Wet soil has the following parameters: $\sigma = 10^{-2}\,{}^{\text{S}}\!/_{\text{m}}$, $\varepsilon_r = 15$. At frequencies 60Hz, 1MHz, 100MHz, and 10GHz, indicate if the soil is a good conductor, good dielectric, or neither.

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{10^{-2}\,{}^{\text{S}}\!/_{\text{m}}}{2\pi f(15)\left(\frac{1}{36\pi}\times 10^{-9}\,{}^{\text{F}}\!/_{\text{m}}\right)} = \frac{1.2\times 10^{7}}{f}$$

60Hz:

$$\frac{\varepsilon''}{\varepsilon'} = \frac{1.2\times 10^{7}}{60\text{Hz}} = 2\times 10^{5} \rightarrow \text{Good Conductor}$$

1MHz:

$$\frac{\varepsilon''}{\varepsilon'} = \frac{1.2\times 10^{7}}{1\text{MHz}} = 12 \rightarrow \text{Neither}$$

100MHz:

$$\frac{\varepsilon''}{\varepsilon'} = \frac{1.2\times 10^{7}}{100\text{MHz}} = 0.12 \rightarrow \text{Neither}$$

1GHz:

$$\frac{\varepsilon''}{\varepsilon'} = \frac{1.2\times 10^{7}}{1\text{GHz}} = 1.2\times 10^{-3} \rightarrow \text{Good Dielectric}$$

1b) For the case of 60Hz, find α, β, and η. How far does the 60Hz wave have to penetrate to attenuate 20dB?

$$\alpha = \sqrt{\pi f\mu\sigma} = \sqrt{\pi(60\text{Hz})\left(4\pi\times 10^{-7}\,{}^{\text{H}}\!/_{\text{m}}\right)\left(10^{-2}\,{}^{\text{S}}\!/_{\text{m}}\right)} = 1.539\times 10^{-3}\,{}^{\text{Np}}\!/_{\text{m}}$$

$$\beta = \alpha = 1.53\times 10^{-3}\,{}^{\text{rad}}\!/_{\text{m}}$$

$$\eta = (1+j)\frac{\alpha}{\sigma} = 0.1539 + j0.1539\Omega = 0.218\angle 45°\Omega$$

$$e^{-\alpha x} = 0.1 \rightarrow x = \frac{\ln(10)}{\alpha} = \frac{\ln(10)}{1.539\times 10^{-3}\,{}^{\text{Np}}\!/_{\text{m}}} = = 1496\text{m}$$

1c) Repeat the last part (1b) for the frequency of 100MHz.

$$\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{10^{-2}\,{}^{\text{S}}\!/_{\text{m}}}{2}\sqrt{\frac{4\pi\times 10^{-7}\,{}^{\text{H}}\!/_{\text{m}}}{15\left(\frac{1}{36\pi}\times 10^{-9}\,{}^{\text{F}}\!/_{\text{m}}\right)}} = 0.4867\,{}^{\text{Np}}\!/_{\text{m}}$$

$$\beta = \omega\sqrt{\mu\varepsilon} = 2\pi(100\text{MHz})\sqrt{\left(4\pi\times 10^{-7}\,{}^{\text{H}}\!/_{\text{m}}\right)\left(15\left(\frac{1}{36\pi}\times 10^{-9}\,{}^{\text{F}}\!/_{\text{m}}\right)\right)} = 8.112\,{}^{\text{rad}}\!/_{\text{m}}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}\angle\tan^{-1}\left(\frac{\alpha}{\beta}\right) = \sqrt{\frac{4\pi\times 10^{-7}\,{}^{\text{H}}\!/_{\text{m}}}{15\left(\frac{1}{36\pi}\times 10^{-9}\,{}^{\text{F}}\!/_{\text{m}}\right)}}\angle\tan^{-1}\left(\frac{0.4867\,{}^{\text{Np}}\!/_{\text{m}}}{8.112\,{}^{\text{rad}}\!/_{\text{m}}}\right) = 97.339\angle 3.4°\Omega = 97.167 + j5.773\Omega$$

$$e^{-\alpha x} = 0.1 \rightarrow x = \frac{\ln(10)}{\alpha} = \frac{\ln(10)}{0.4867\,{}^{\text{Np}}\!/_{\text{m}}} = = 4.73\text{m}$$