Lesson 1.4

VECTOR CALCULUS - Surface Integrals and Divergence
Surface Integrals look like: \[ \int \vec{A} \cdot d\vec{s} \]

Differs from Lesson 1.2 area integrals because of “dot” product

\( d\vec{s} \) is a vector and points normal to the surface
- the magnitude is the differential area formed from differential lengths of the sides
SURFACE INTEGRALS

\[ \int d\vec{s} = dx \cdot dz \cdot \hat{a}_y \]

Note that all 3 coordinates are involved

\[ \int \vec{A} \cdot d\vec{s} \]

measures flux of \( \vec{A} \) through a surface
Example - FLUID FLOW

For, \( \vec{v} \parallel d\vec{s} \), there is flow through.

But, \( \vec{v} \perp d\vec{s} \), there is no flow.

Hence, \( \vec{v} \cdot d\vec{s} \), measures FLUX.

Example: Let \( y=2, x=0 \) to 3 and \( z = -1 \) to 1

\[
\bar{A} = xy \cdot \hat{a}_x + z^2 \cdot \hat{a}_y, \text{ then, } \int \bar{A} \cdot d\vec{s} = \int_{-1}^{+1} \int_{0}^{3} z^2 \cdot dx \cdot dz = \frac{3}{3} \left[ \frac{z^3}{3} \right]_{-1}^{+1} = 2
\]

Do Problem 1
DIVERGENCE

"Global" quantities

\[ \int \vec{A} \cdot d\vec{s} \]
Measures Flux through any surface

\[ \oint \vec{A} \cdot d\vec{s} \]
Measures Flux through closed surfaces

\[ \nabla \cdot \vec{A} \]
, is a “local” measure of flux property

is related to
DIVERGENCE

Notation: \( \text{div} \; \vec{A} = \nabla \cdot \vec{A} \) \text{ NOT a DOT product but has similar features}

Result is a SCALAR, composed of derivatives like:

\[
\frac{\partial A_r}{\partial r}, \frac{\partial A_\phi}{\partial \phi}
\]

Divergence Theorem:

\[
\oint \vec{A} \cdot d\vec{s} = \int \left( \nabla \cdot \vec{A} \right) \cdot dv
\]

Volume integral on right is volume enclosed by surface on the left