We will be looking at:

- sources of magnetic fields ie. currents
- description of resistance

We will consider two parameters:

I - Current (in Amps)

J - Current Density (in Amps/unit area)
Current & Current Density

**Definition:**
\[ I = \frac{\Delta Q}{\Delta t} \]
Charge passing through cross-section in time \( \Delta t \)

**Define:**
\[ \vec{j} = \frac{I}{\text{Area}} \cdot \hat{a}_z \]
cross-section points in direction of current flow

In general,
\[ I = \int \vec{j} \cdot d\vec{s} \]

Wire with current, \( I \)
Current & Current Density

\[ I = \int j \cdot d\vec{s} \]

Example: wire with constant current density

\[ \vec{j} = j_0 \cdot \hat{a}_z \]

\[ I = \int_0^a \int_0^{2\pi} j_0 \cdot \hat{a}_z \cdot r \cdot dr \cdot d\varphi \cdot \hat{a}_z = j_0 \cdot \pi \cdot a^2 \]

\[ d\vec{s}, \ \text{in cylindrical geometry} \]

Problem 1a - Find I in terms of J_0 and invert

\[ \vec{j} = j_0 \cdot \left( \frac{r}{a} \right)^8 \cdot \hat{a}_z \]
Problem 1: Current & Current Density

Current Distribution
Lesson 3.1

Current Density: \( J = J_0 \left( \frac{r}{a} \right)^8 \)
Surface Current Density

Surface Current Density \( J_s \) (in units of A/m)

- conductors often have current flowing in thin sheet (eg. High Freq.)

Example: current flow in a wire

\[
\begin{align*}
\delta \cdot \pi \cdot \delta & \approx a \\
I & \text{Current} \\
\text{flow region} & = \pi \cdot \delta^2 \\
\text{But,} & \delta \xrightarrow{\text{limit}} 0 \Rightarrow j \rightarrow \infty \\
\text{However, define:} & j_s = \frac{I}{\text{Area} / \delta} = \frac{I}{\text{distance}} \\
& = \frac{I}{2 \cdot \pi \cdot a}
\end{align*}
\]
Surface Current Density

\[ j_s = \frac{I}{\text{Area}/\delta} = \frac{I}{\delta} \]

\[ = \frac{I}{2 \cdot \pi \cdot a} \]

Distance over which current is distributed

Current flow region

Do Problem 1b

"Think of cross-section area and throw out \( \delta \)"
We now obtain an alternate expression for current density

Assume all particles move at same $v$

- In $\Delta t$, all particles within $\Delta z = v \cdot \Delta t$ will pass through the right face

\[
\Delta Q = \rho \cdot \Delta z \cdot A = \rho \cdot v \cdot A \cdot \Delta t
\]

\[
\Rightarrow j = \frac{\Delta Q}{A \cdot \Delta t} = \frac{\rho \cdot v \cdot A \cdot \Delta t}{A \cdot \Delta t} = \rho \cdot v
\]

\[
\vec{j} = \rho \cdot \vec{v}
\]
Resistance

- $E$, in wire pushed electrons to the right
- $e^-$ s, collide with lattice ions
- $F_E$, force (from E-field) driving electrons
- $F_{Coll}$, balances $F_E$ at some $v_{average}$

Since $j = \rho \cdot v_{average}$, balance occurs for some value of $j$ for a particular $E$
Ohm’s Law

\[ \vec{j} = \sigma \vec{E} \]

Conductivity - units of S/m or 1/ohm-m
- varies from $10^7$ to $10^{-15}$

Good conductor eg. Cu
Good insulator

\[ \vec{j} = \sigma \cdot \vec{E} \], is Fields and Waves version of Ohm’s Law
Ohm’s Law

\[ I = j \cdot A = \sigma \cdot E \cdot A \]

\[ V = -\int \vec{E} \cdot d\vec{l} = E \cdot l \]

\[ \frac{V}{I} = \frac{E \cdot l}{\sigma \cdot E \cdot A} = \frac{l}{\sigma \cdot A} = R \]

Do problem 2a and 2b.....note for 2b the effective area is an annulus
Ohm’s Law

\[ R = \frac{l}{\sigma \cdot A} \]

Valid if only \( j \) and \( A \) are constant

What if they are not? Compute \( V \) and \( I \) separately and form \( V/I \)

Example: Disk with Radial Current

Look at wedge (sketch)

\( I \) is the same at the inner and outer part
Ohm’s Law - alternative calculation

$I$ is the same at the inner and outer part of disk

\[
j = \frac{I}{\text{Area}} = -\frac{I}{2 \cdot \pi \cdot r \cdot t} \cdot \hat{a}_r
\]

\[
\vec{E} = \frac{j}{\sigma} = -\frac{I}{2 \cdot \pi \cdot r \cdot \sigma \cdot t} \cdot \hat{a}_r
\]

Compute potential difference between inner and outer part of disk:

\[
V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{l} = I \int_a^b \frac{dr}{2 \cdot \pi \cdot \sigma \cdot t} = I \frac{1}{2 \cdot \pi \cdot \sigma \cdot t} \ln\left(\frac{b}{a}\right)
\]

\[\therefore R = \frac{V}{I} = \frac{\ln(b/a)}{2 \cdot \pi \cdot \sigma \cdot t}\]

Do problem 2c