Lesson 3.8

ENERGY and FORCE
Power in inductor:

\[ P = I \cdot V = I \cdot L \cdot \frac{dI}{dt} = \frac{d}{dt} \left( \frac{1}{2} L \cdot I^2 \right) \]

Obtain energy in terms of B and H fields

Flux linkage: \[ \Lambda = L \cdot I = B \times \text{Area} \times N \]

Also, \[ I = \frac{\int \vec{H} \cdot d\vec{l}}{N} = \frac{H \cdot \text{length}}{N} \]
Energy =

\[
\frac{1}{2} \cdot L \cdot I^2 = \frac{1}{2} \cdot (L \cdot I) \cdot I = \frac{1}{2} \cdot (B \times \text{Area} \times N) \cdot \left( \frac{H \times \text{Length}}{N} \right)
\]

\[
= \frac{1}{2} \int \vec{B} \cdot \vec{H} \cdot dv = W_m
\]

Energy stored in Magnetic field

Energy Density: (per unit volume)

\[
w_m = \frac{1}{2} \cdot \vec{B} \cdot \vec{H} = \frac{1}{2} \cdot \frac{B^2}{\mu} = \frac{1}{2} \cdot \mu \cdot H^2
\]

Do problem 1a
PROBLEM

For problem 1b:

Difficult to use flux linkage method to compute Inductance
• not all I intersects same flux
• have to add contributions carefully

SIMPLER WAY: \[ \frac{1}{2} L I^2 = W_m \]

known quantities
FORCE

First approach - similar to that for individual particles

For one particle: \[ \vec{F} = q \cdot (\vec{v} \times \vec{B}) \]

For many particles: \[ \frac{\vec{F}}{\text{volume}} = \rho \cdot (\vec{v} \times \vec{B}) = \vec{j} \times \vec{B} \]

Second approach - \( \vec{F} \) does work and changes energy

\[ \text{Work} = \int \vec{F} \cdot d\vec{l} \]

Calculate \( W_m \) for 2 configurations with small \( \delta x \) difference

\[ \Delta W_m = F \cdot \delta x \]

Do Problem 2

\[ F = \frac{\Delta W_m}{\delta x} \]