Lesson 4.4

TRANSMISSION LINES - INPUT IMPEDANCE
Do Experiment 1

- After Experiment: Cable+Load can look very different than Load
- Characterize V/I as Impedance
Input Impedance

From previous class:

\[ \hat{V}(z) = V^+(z) + V^-(z) = V_m^+ \cdot e^{-j \beta z} \cdot (1 + \Gamma(z)) \]

What about I?

\[ \hat{I}(z) = \frac{V^+(z)}{R_c} - \frac{V^-(z)}{R_c} = \frac{V_m^+}{R_c} \cdot e^{-j \beta z} \cdot (1 - \Gamma(z)) \]

Also,

\[ \Gamma(z) = \Gamma_L \cdot e^{-2j \beta (L-z)} \]
Input Impedance

Form the Ratio:

\[
\frac{\hat{V}(z)}{\hat{I}(z)} = R_c \cdot \frac{1 + \Gamma(z)}{1 - \Gamma(z)}
\]

Book calls this \( Z_{in}(z) \)

We are primarily interested in \( z=0 \) value

• treat connection to rest of circuit as 2 port with,

\[
Z_{in}(z = 0) = R_c \cdot \frac{1 + \Gamma(z = 0)}{1 - \Gamma(z = 0)}
\]
After lots of algebra, one can show:

\[ Z_{in}(z = 0) = R_c \cdot \frac{Z_L + j \cdot R_c \cdot \tan(\beta \cdot L)}{R_c + j \cdot Z_L \cdot \tan(\beta \cdot L)} \]

Special Case example: \( Z_L = 0 \) (short circuit)

\[ Z_{in}(z = 0) = R_c \cdot \frac{0 + j \cdot R_c \cdot \tan(\beta \cdot L)}{R_c + j \cdot 0 \cdot \tan(\beta \cdot L)} = j \cdot R_c \cdot \tan(\beta \cdot L) \]
Input Impedance - SHORT CIRCUIT

\[ Z_{in}(z = 0) = j \cdot R_c \cdot \tan(\beta \cdot L) \]

Can change \( Z_{in} \) by changing these two parameters:
- Fix \( \beta \), vary \( L \) - different effects
- Vary \( \beta \), fix \( L \) - get same effects

Note that \( L \) is the length of the Transmission Line
1a. Open Circuit Case

\[ Z_L = \infty \]

\[ Z_L = \frac{Z_L + j \cdot R_c \cdot \tan(\beta \cdot L)}{R_c + j \cdot Z_L \cdot \tan(\beta \cdot L)} \]

1b. \( Z_L = 93\Omega \) - lots of complex algebra
Problem 1a and 1b

We know $Z_{in}$ ($z=0$) - treat as 2-PORT

$$Power = \frac{1}{2} Re\left\{V_{in} \times I_{in}^*\right\} = \frac{1}{2} Re\left\{\frac{V_{in} \times V_{in}^*}{Z_{in}^*}\right\} = \frac{1}{2} Re\left\{\frac{|V_{in}|^2}{Z_{in}^*}\right\}$$

Do problem 1b - parts 2 & 3
Problem 1a and 1b

In a Lossless Transmission Line, $P_{in}$ flows into the Transmission Line and it is dissipated at the LOAD.

$$P_{in} = \frac{1}{2} \frac{|V_L|^2}{Z_L}$$

**Do Problem 1b - part 4**

$$\hat{V}(z) = V_m^+ \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z))$$

$$\hat{V}(z = 0) = V_{in} = V_m^+ \cdot e^{-j \cdot \beta \cdot z} \cdot (1 + \Gamma(z = 0))$$

$$\Rightarrow V_m^+ = \frac{V_{in}}{1 + \Gamma(0)}$$

Can then plug back and get the full phasor.

**Do Problem 1b - part 5**