Fields and Waves I  Lesson 1.3

Gradient, Line integrals, & Curl

Reading assignment
Ulaby, 3-4
Connor and Salon, II-39 → II-44

Software
div_curl_example.m
Maple (check your solutions)

Problem 1 - Line integrals & curl

The magnetic field of a straight wire of radius \( a \) which has a constant current density \( J_0 \), is given by:

\[
B = \mu_0 J_0 \frac{r}{2} a \phi \quad \text{inside the wire (} r < a \}\)
\[
B = \mu_0 J_0 \frac{a^2}{2} \frac{1}{r} a \phi \quad \text{outside the wire (} r > a \). \]

where \( \mu_0 \) and \( J_0 \) are constants.

a. Calculate \( \oint B \cdot dl \) around the 2 paths shown in the figure below. (The drawing shows a cross-sectional view as if the wire had been cut).

b. Calculate \( \nabla \times B \) for both regions.

Problem 2 - Properties of fields with curl

The electric field created by a cylinder of radius \( a \) with constant charge density \( \rho_0 \) is:

\[
E = \rho_0 \frac{r}{2 \varepsilon_0} a_r \quad \text{inside the cylinder (} r < a \) and
\[
E = \rho_0 \frac{a^2}{2 \varepsilon_0 r} a_r \quad \text{outside the cylinder (} r > a \).
\]

where \( \rho_0 \) and \( \varepsilon_0 \) are constants.

a. Verify that \( \oint E \cdot dl = 0 \) on the same paths as above and that \( \nabla \times E = 0 \) for both regions.

b. An illustration of the \( \mathbf{E} \) and \( \mathbf{B} \) fields can be obtained by running \texttt{div_curl_example.m} using matlab. Fig. 1 is the \( \mathbf{B} \) field while Figure 3 is the \( \mathbf{E} \) field. What are the properties of a field with non-zero curl?
Problem 3 - Stokes theorem
Calculate \( \int (\nabla \times \mathbf{B}) \bullet \mathbf{ds} \) over the two surface areas enclosed by each path in Problem 1 (the shaded area). Compare your answer with the results from Problem 1a.

Problem 4 - Gradient
Compute the gradient of the following functions.

a. \( f = 8a^2 \cos \phi + 2rz \) (cylindrical)
b. \( f = a \cos 2\theta / r \) (spherical)

Use the worksheet associated with Problem 2.8.1 in "Visual Electromagnetics for Mathcad" to check your answer. (You may have to use a specific number instead of the variable \( a \)).

c. Calculate \( \nabla \times \nabla f \) for each of the functions above.