Problem Solution #8

Problem 1

$V = V_0 \sin \omega t = 120\sqrt{2} \sin(2\pi \times 60t)$ $\qquad B = B_0 \cos \omega t = 1.8 \cos(2\pi \times 60t)$

$V = -N \dfrac{d\Phi}{dt} = -N \dfrac{dB}{dt} S = NSB_0 \omega \sin \omega t$ $\qquad$ So $\quad N = \dfrac{120\sqrt{2}}{5 \times 10^{-4} \times 1.8 \times 2\pi \times 60} \approx 500(turns)$

Problem 2

$\vec{E} = \dfrac{Q}{2\pi\varepsilon r}\hat{r}$ $\qquad V = \dfrac{Q}{2\pi\varepsilon}\ln(\dfrac{b}{a})$, so $\quad E = \dfrac{V}{r\ln(b/a)} = \dfrac{100\cos(12\pi \times 10^6 t)}{r\ln(5/1)}$

$\vec{D} = \varepsilon\vec{E} = \varepsilon_0\varepsilon_r\vec{E}$ $\qquad A = 2\pi(b-a)$

$I_d = \dfrac{d\vec{D}}{dt} \cdot \vec{A} = 8.85 \times 10^{-12} \times 4 \times \dfrac{-100 \times 12\pi \times 10^6 \sin(12\pi \times 10^6 t)}{r\ln(5/1)} \hat{r} \cdot 2\pi r\hat{r}$

$= 2\pi \times 8.85 \times 10^{-12} \times 4 \times \dfrac{-100 \times 12\pi \times 10^6 \sin(12\pi \times 10^6 t)}{\ln(5/1)} \approx -521\sin(12\pi \times 10^6 t)$ mA

$I_c = I_d \Rightarrow \dfrac{\sigma}{\omega\varepsilon} = 1 \Rightarrow \sigma \approx 1.33 \times 10^{-3}(S/m)$

Problem 3

$\Phi = BA = (L-x)lB_z$

$x = \int v_x dt = \int v_0 \sin \omega t dt = -\dfrac{v_0}{\omega}(\cos \omega t - 1) + x_0$ where x_0 is the position at t=0.

$emf = -\dfrac{d\Phi}{dt} = -\dfrac{d}{dt}(L-x)lB_z = -Ll\dfrac{dB_z}{dt} + l\dfrac{dxB_z}{dt} = -(L-x)l\dfrac{dB_z}{dt} + lB_z\dfrac{dx}{dt}$

$= -\left[L - \left(-\dfrac{v_0}{\omega}(\cos \omega t - 1) + x_0\right)\right] lB_0\omega \cos \omega t + lB_0 \sin \omega t v_0 \sin \omega t$

$= \left[\left(\dfrac{v_0}{\omega} + x_0\right) - L\right] lB_0\omega \cos \omega t - lB_0 v_0 \cos 2\omega t$

$= \dfrac{l\mu_0 NI_0}{s}\left\{\left[\left(\dfrac{v_0}{\omega} + x_0\right) - L\right]\omega \cos \omega t - v_0 \cos 2\omega t\right\}$

Problem 4

Lossless Medium:

$\beta = \omega\sqrt{\mu\varepsilon} \approx 32.5(rad/m)$ $\qquad u = \omega/\beta \approx 1.94 \times 10^8(m/s)$

$\lambda = 2\pi/\beta \approx 0.194(m)$ $\qquad \eta = \sqrt{\dfrac{\mu}{\varepsilon}} \approx 243(\Omega)$

Problem 5

Linear polarization (K>0, in-phase; K<0, out-phase)