1. (20)
   a. (5) A source generates a wave on a string. It completes 20 cycles in 30 seconds. The wave is observed to travel 2.8 meters in 5 seconds. What is the wavelength?

   $$T = \frac{cycles}{seconds} = \frac{30}{20} = 1.5\text{s}$$

   $$\upsilon_p = \frac{d}{t} = \frac{2.8\text{m}}{5\text{s}} = 0.56\,{}^{\text{m}}\!/\!_{\text{s}}$$

   $$\lambda = u_p T = 0.84\text{m}$$

   b. For the following expressions determine if the wave is traveling or standing. If the wave is standing, express it in terms of traveling waves. If the wave is traveling, indicate the direction and velocity.
      i. (4) $\cos(\omega t + \phi)\cos(kz)$

      Standing wave. $\frac{1}{2}\left[\cos(\omega t + kz + \phi) + \cos(\omega t - kz + \phi)\right]$

      ii. (2) $\cos(5\omega t - 20ky)$

      Traveling in the +y direction. $\upsilon_p = \dfrac{\omega}{\beta} = \dfrac{5\omega\,{}^{\text{rad}}\!/\!_{\text{s}}}{20k\,{}^{\text{rad}}\!/\!_{\text{m}}} = \dfrac{\omega}{4k}\,{}^{\text{m}}\!/\!_{\text{s}}$

      iii. (3) $\sin(377t + 44z)$

      Traveling in the –z direction. $\upsilon_p = \dfrac{\omega}{\beta} = \dfrac{377\,{}^{\text{rad}}\!/\!_{\text{s}}}{44\,{}^{\text{rad}}\!/\!_{\text{m}}} = 8.568\,{}^{\text{m}}\!/\!_{\text{s}}$

   c. (6) A voltage wave is moving in the –y direction at a velocity of 0.6c. The magnitude is 10 volts and frequency is 1MHz. Write a time domain and a phasor expression for the wave.

   $$\upsilon_p = 0.6c = 1.8\times10^8\,{}^{\text{m}}\!/\!_{\text{s}}$$

   $$f = 10^6\,\text{Hz}$$

   $$\lambda = \frac{\upsilon_p}{f} = \frac{1.8\times10^8\,{}^{\text{m}}\!/\!_{\text{s}}}{10^6\,\text{Hz}} = 1.8\times10^2\,\text{m}$$

   $$\omega = 2\pi f = 2\pi\times10^6\,{}^{\text{rad}}\!/\!_{\text{s}}$$

   $$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{1.8\times10^2\,\text{m}} = \frac{\pi}{90}\,{}^{\text{rad}}\!/\!_{\text{m}} \cong 0.035\,{}^{\text{rad}}\!/\!_{\text{m}}$$

   $$V(y,t) = 10\cos\left(2\pi\times10^6 t + \frac{\pi}{90}y\right)$$

   $$\tilde{V} = 10^{j\left(\frac{\pi}{90}\right)y}$$

2. (25)

a. (3) If a lossless transmission line is terminated in the characteristic impedance there are no standing waves. True or False?

True

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0}{2Z_0} = 0$$

b. (8) For a voltage we have $V = j2e^{-j\frac{2\pi z}{3}}e^{-20z}$. Express this in the time domain using frequency 10MHz.

$$V = \Re\left\{ j2e^{-j\frac{4\pi z}{3}}e^{-20z} \right\} = \Re\left\{ j2e^{j\omega t - \frac{4\pi z}{3}}e^{-20z} \right\} = \Re\left\{ je^{-20z}\left(\cos\left(\omega t - \frac{4\pi}{3}z \right) + j\sin\left(\omega t - \frac{4\pi}{3}z \right) \right) \right\}$$

$$V = -2e^{-20z}\sin\left(\omega t - \frac{4\pi}{3}z \right) = 2e^{-20z}\sin\left(\omega t - \frac{4\pi}{3}z - \pi \right)$$

$$\omega = 2\pi f = 2\pi(10\text{MHz}) = 6.28\times10^{7}\ \text{rad}/\text{s}$$

c. (6) For 2 voltages $v_1 = 126.5\sin(\omega t + 63.4°)$, $v_2 = 44.7\cos(\omega t - 161.5°)$, convert these to complex numbers.

$$v_1 = 126.5\sin(\omega t + 63.4°) = 126.5\cos(\omega t + 63.4° - 90°) = 126.5\cos(\omega t - 26.6°)$$

$$v_1 = 126(\cos(-26.6°) + j\sin(-26.6°))$$

$$v_1 = (112.66 - j56.42)\text{V}$$

$$v_2 = 44.7\sin(\omega t - 161.5°) = 44.7(\cos(-161.5°) + j\sin(-161.5°))$$

$$v_2 = (-42.39 - j14.18)\text{V}$$

d. (6) Write the sum of these voltages as a sine function and then as a cosine function (i.e., $v = A\sin(\omega t - \phi)$).

$$v_3 = v_1 + v_2 = (112.66 - j56.42)\text{V} + (-42.39 - j14.18)\text{V} = (70.27 - j70.6)\text{V}$$

$$v_3 = 99.61e^{-j45.13}\text{V}$$

$$v_3 = 99.61\cos(\omega t - 45.13°)\text{V}$$

$$v_3 = 99.61\sin(\omega t - 45.13° + 90°)\text{V} = 99.61\sin(\omega t + 44.87°)\text{V}$$

e. (2) Give a typical frequency for a cable TV channel?

250MHz

3. (25)
   a. (10)We have a short circuited lossless 50 Ohm transmission line operating at 200MHz. The phase velocity is $0.75c$. If we want the impeadance to be $X = 25\text{Ohms}$ (reactance) what is the shortest length of the line we should use?

$$Z_{IN} = Z_0\left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right) = j25\Omega$$

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{\upsilon_p} = \frac{2\pi(200\text{MHz})}{0.75c} = 5.59\,{}^{\text{rad}}\!/\!_{\text{m}}$$

$$50\Omega\left(\frac{j(50\Omega)\tan \beta l}{50\Omega}\right) = j25\Omega$$

$$j(50\Omega)\tan \beta l = j25\Omega$$

$$l = \frac{\tan^{-1}\left(\frac{1}{2}\right)}{\beta} = 83.02\text{mm}$$

   b. (15) We have a 10m section of transmission line with characteristic impedance of 50 Ohms, phase velocity of 2×10^8 meters/sec, a 26MHz source at $V_S = 100\measuredangle 0°$. The source impedance is 50 Ohms. The line is terminated with an impedance of $Z_L = 100 + j50\Omega$.
      i. (2) Find β.

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{\upsilon_p} = \frac{2\pi(26\text{MHz})}{2\times10^8\,{}^{\text{m}}\!/\!_{\text{s}}} = 0.817\,{}^{\text{rad}}\!/\!_{\text{m}}$$

      ii. (5) Find the reflection coefficients at the load and source.

$$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{50\Omega - 50\Omega}{50\Omega + 50\Omega} = 0$$

$$\Gamma_L = \frac{Z_L - Z_0}{ZL + Z_0} = \frac{(100 + j50)\Omega - 50\Omega}{(100 + j50)\Omega + 50\Omega} = \left(\frac{2}{5} + j\frac{1}{5}\right)\Omega$$

      iii. (8) Find the input impedance at the source end.

$$\tan \beta l = \tan\left[\left(0.817\,{}^{\text{rad}}\!/\!_{\text{m}}\right)(10\text{m})\right] = -3.08$$

$$Z_{IN} = Z_0\left(\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l}\right) = 50\Omega\left(\frac{(100 + j50)\Omega + j(50\Omega)(-3.08)}{50\Omega + j(100 + j50\Omega)(-3.08)}\right)$$

$$Z_{IN} = (19.21 + j3.52)\Omega$$

4. (30) A transmission line is 200 meters long and is short circuited at the load end. The source end is connected to a dc voltage of 100 volts starting at $t = 0$. The internal resistance of the source is $R_G = 100\Omega$. The coax parameters are $C' = 100\,{}^{\text{pF}}\!/\!_{\text{m}}$, $L' = 0.25\,{}^{\mu\text{H}}\!/\!_{\text{m}}$.

   a. (4) Find the characteristic impedance, Z_0.

   $$Z_0 = \sqrt{L'/C'} = \sqrt{0.25\,{}^{\mu\text{H}}\!/\!_{\text{m}} \big/ 100\,{}^{\text{pF}}\!/\!_{\text{m}}} = 50\Omega$$

   b. (4) Find the velocity of propagation, υ_p.

   $$\upsilon_p = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\left(0.25\,{}^{\mu\text{H}}\!/\!_{\text{m}}\right)\left(100\,{}^{\text{pF}}\!/\!_{\text{m}}\right)}} = 200 \times 10^6\,{}^{\text{m}}\!/\!_{\text{s}}$$

   c. (4) Find the transit time, τ.

   $$\tau = \frac{l}{\upsilon_p} = \frac{200\text{m}}{200 \times 10^6\,{}^{\text{m}}\!/\!_{\text{s}}} = 1\mu\text{s}$$

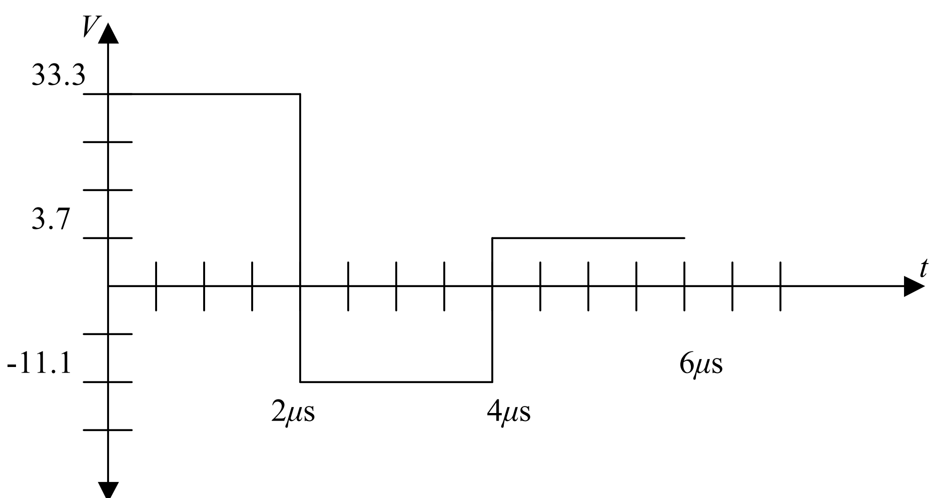
   d. (4) Find the reflection coefficient at the load, Γ_L.

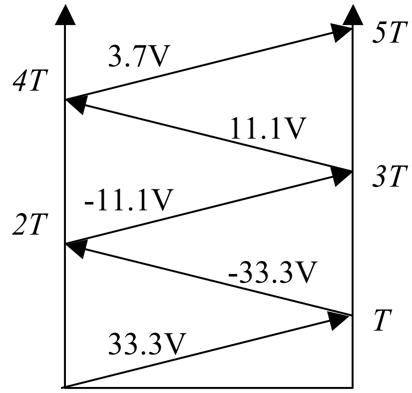
   $$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{0 - 50\Omega}{0 + 50\Omega} = -1$$

   e. (4) Find the reflection coefficient at the source, Γ_S.

   $$\Gamma_S = \frac{Z_S - Z_0}{Z_S + Z_0} = \frac{100\Omega - 50\Omega}{100\Omega + 50\Omega} = -\frac{1}{3}$$

   f. (10) Plot the voltage at the generator end of the line for 5ms.





$$V^+ = \frac{V_S Z_0}{Z_0 + Z_{IN}} = 33.3\text{V}$$

T=6μs