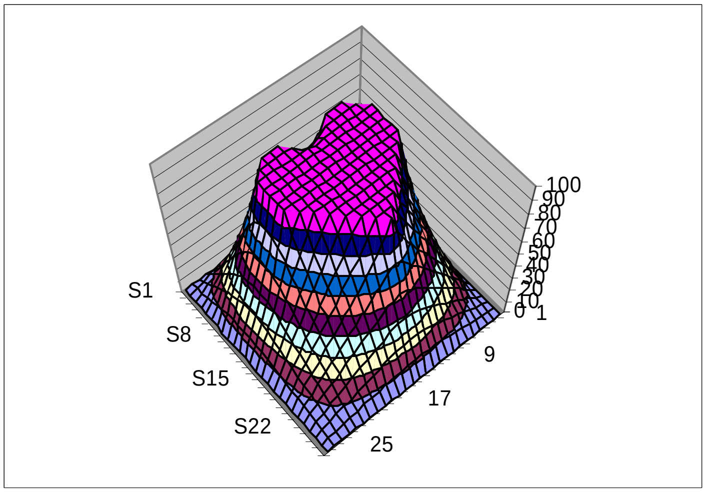
Quiz 1

TRANSMISSION LINES



Name _____Solution____________
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Section ______________________

Multiple Choice

1. (12 Pts) ____________

2. (5 Pts) ____________

3. (5 Pts) ____________

4. (10 Pts) ____________

5. (8 Pts) ____________

Regular Questions

6. (14 Pts) ____________

7. (16 Pts) _Extra Credit_

8. (18 Pts) ____________

9. (12 Pts) ____________

Total

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Notes:
1. In the multiple choice questions, each question may have more than one correct answer; circle all of them.
2. For multiple choice questions, you may add some comments to justify your answer.
3. Make sure your calculator is set to perform trigonometric functions in radians & not degrees.

Some Useful Equations:

Wavelength

$\lambda f = u$ where u is the propagation velocity; $\beta = \dfrac{2\pi}{\lambda}$

Low Loss Lines

$$Z_o = \sqrt{\frac{r + j\omega l}{g + j\omega c}} \approx \sqrt{\frac{r + j\omega l}{j\omega c}} \approx \sqrt{\frac{j\omega l}{j\omega c}}\sqrt{1 + \frac{r}{j\omega l}} \approx \sqrt{\frac{l}{c}}\left(1 - j\frac{r}{2\omega l}\right)$$

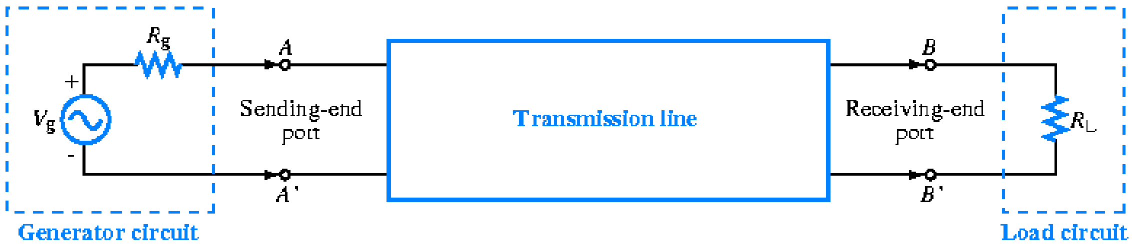
$$\gamma = \alpha + j\beta = \sqrt{(r + j\omega l)(g + j\omega c)} \approx \sqrt{(r + j\omega l)(j\omega c)} \approx \sqrt{(j\omega l)(j\omega c)}\sqrt{1 + \frac{r}{j\omega l}}$$

$$j\beta \approx j\omega\sqrt{lc} \text{ and } \alpha \approx \omega\sqrt{lc}\left(\frac{r}{2\omega l}\right) = \frac{r}{2Z_o}$$

MULTIPLE CHOICE AND SHORT ANSWER QUESTIONS

1. Specific Length Transmission Lines (12 points)

A lossless transmission line configuration with a characteristic impedance of Z_o and propagation velocity *u* is shown generically as:



Determine the input impedance Z_{in} for the following conditions:

a) Transmission line length equals $\lambda/2$

1. Load impedance $Z_L = Z_o$ $\qquad Z_{in} = Z_L = Z_o$
2. Load impedance $Z_L = 0$ $\qquad Z_{in} = Z_L = 0$
3. Load impedance $Z_L \to \infty$ $\qquad Z_{in} = Z_L \to \infty$

b) Transmission line length equals $\lambda/4$

1. Load impedance $Z_L = Z_o$ $\qquad Z_{in} = Z_L = Z_o$
2. Load impedance $Z_L = 0$ $\qquad Z_{in} = \dfrac{Z_o^{\,2}}{Z_L} \to \infty$
3. Load impedance $Z_L \to \infty$ $\qquad Z_{in} = \dfrac{Z_o^{\,2}}{Z_L} = 0$

Ans: For a half wavelength line, the input impedance equals the load impedance or $Z_{in} = Z_L$. *For a quarter wavelength line, the input impedance is given by* $Z_{in} = \dfrac{Z_o^{\,2}}{Z_L}$

2. Wave Equation (5 points)

A wave equation is given by the following expression:

$$\frac{\partial^2}{\partial z^2}G - a^2\frac{\partial^2}{\partial t^2}G = 0$$

The propagation velocity for this wave is

a. $\sqrt{a}$
b. a
c. a^2
d. $1/a^2$
e. $1/a$
f. $1/\sqrt{a}$

This looks just like the wave equation we have used for transmission lines $\frac{\partial^2 v(z)}{\partial z^2} = lc\frac{\partial^2 v(z)}{\partial t^2}$ *for which* $u = \frac{1}{\sqrt{lc}}$. *Thus, the velocity above should be* $u = \frac{1}{\sqrt{a^2}} = \frac{1}{a}$.

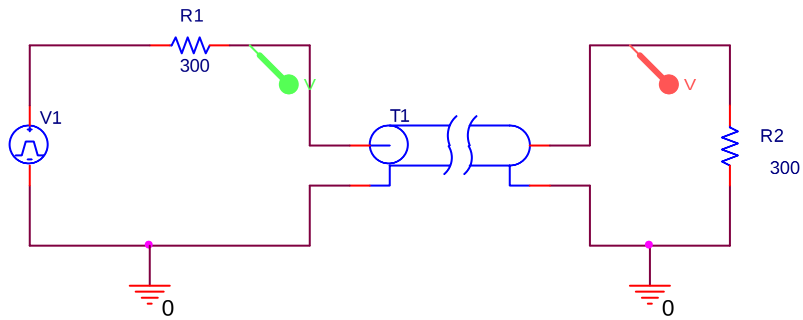
3. Low Loss Transmission Line (5 points)

Someone tries to build a transmission line in salt water for which $\sigma = 5$ and $\varepsilon = 80\varepsilon_o$. The capacitance per unit length of the line is 100pF per meter. For which of the following frequencies can the line be considered to be low loss? Assume that all of the line loss is in the sea water and, thus, you can neglect the resistance per unit length of the lines *r*. Circle the correct answers.

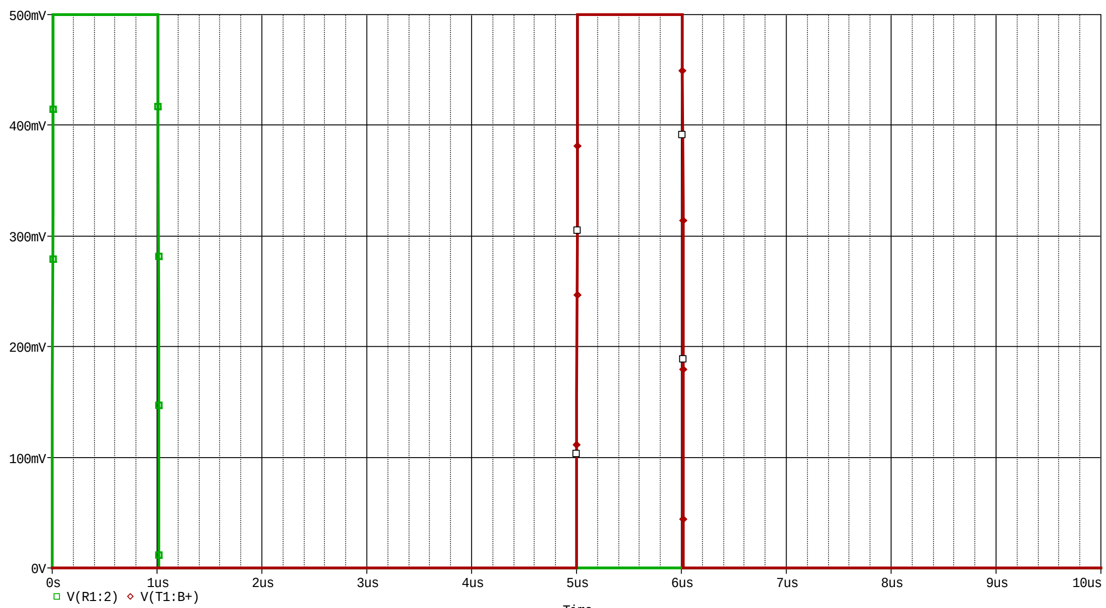
a. 1 Hz
b. 10 Hz
c. 100 Hz
d. 1 kHz
e. 10 kHz
f. 100 kHz
g. 1 MHz
h. 10 MHz
i. 100 MHz
j. 1 GHz
k. 10 GHz
l. 100 GHz
m. 1 THz
n. 10 THz
o. 100 THz

For low loss $r \ll \omega l$ *and/or* $g \ll \omega c$. *The first is satisfied trivially since we can neglect the resistance of the wires. For the latter,* $\frac{\sigma}{\varepsilon}c = g \ll \omega c$ *or* $f \gg \frac{\sigma}{2\pi\varepsilon} = \frac{9}{8}x10^9$

4. Pulses on a Matched Transmission Lines (10 points)



Assume that a pulsed source is matched to its line and load. That is $R_g = R_L = Z_o = 300$ *Ohms*. The propagation velocity on the line is $u = 2x10^8$ *m/s*. A 1 microsecond voltage pulse is launched on the line. The following voltages are observed at the input and output of the line:



a. What is the length of the line?

The delay gives us the line length $d = uT = 2x10^8 \cdot 5x10^{-6} = 10x100 = 1000m$

b. What is the source voltage V1?

The source voltage $V1 = \dfrac{300+300}{300} 0.5 = 1V$

5. Valentines (8 points)

Fill in the blanks or true-false (2 pts each).

a. Roses are red, violets are blue, if $\Gamma_L = \frac{1}{3}$, the *SWR* = *_2?*

b. From an Engineer's Valentine (Unknown Author)

I was alone and all was dark
Beneath me and above
My life was full of volts and amps
But not the spark of love

But now that you are here with me
My heart is overjoyed
You've turned the square of my heart
Into a *sinusoid*?

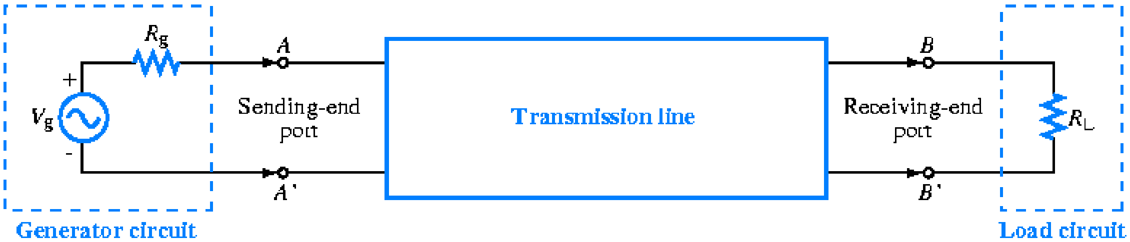
c. True or false: There are 10 types of people in the world, those who understand binary and those who don't.

d. True or false: I wish Valentine candy hearts looked like this: 

REGULAR QUESTIONS

6. Sinusoidal Voltages on a Matched Lossless Transmission Line (14 points)

A lossless transmission line is properly matched to its source and load, $R_g = R_L = Z_o = 50$ Ohms. The propagation velocity on the line is $u = 1.5x10^8\ {}^{m}\!/_{s}$.



a. What is the reflection coefficient Γ_L at the load? (2 pts)

The reflection coefficient is zero. $\Gamma_L = \dfrac{Z_L - Z_o}{Z_L + Z_o} = \dfrac{0}{2Z_0} = 0$

b. What is the standing wave ratio *SWR*? (2 pts)

The SWR is 1 for a matched load. $SWR = \dfrac{1+|\Gamma_L|}{1-|\Gamma_L|} = \dfrac{1}{1} = 1$

c. For a frequency $f = 100x10^6\,Hz$, what is the propagation constant β? (2 pts)

The propagation constant is given by $\beta = \dfrac{\omega}{u} = \dfrac{2\pi 10^8}{1.5x10^8} = \dfrac{2\pi}{1.5}$

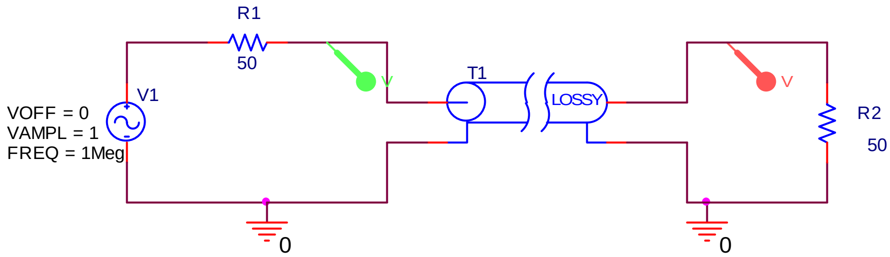
d. What is the wavelength λ? (2 pts)

The wavelength is $\lambda = \dfrac{2\pi}{\beta} = 1.5$

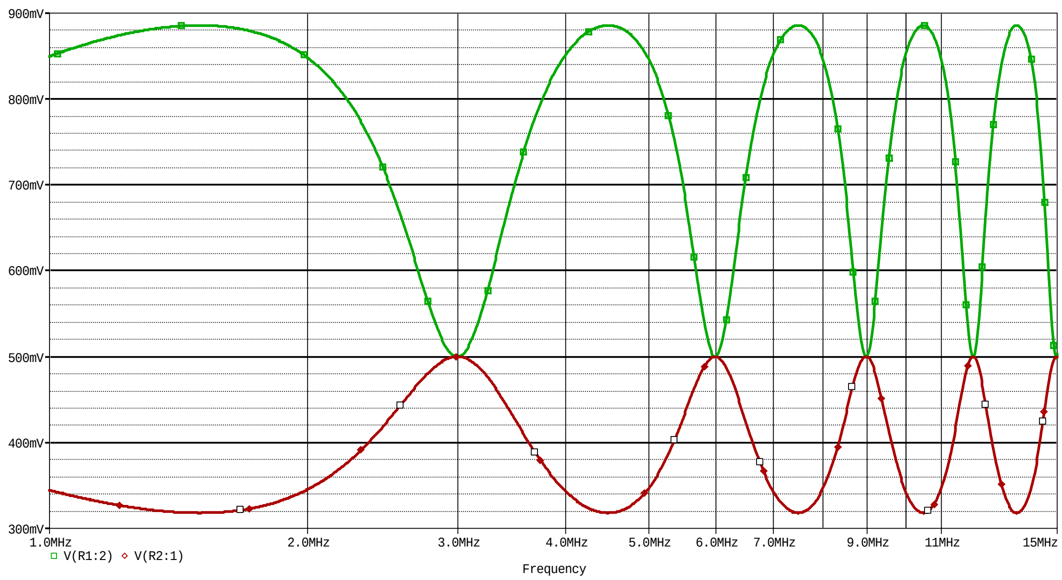
e. Assuming that position z = *0m* at the load, write the voltage and current as a function of position on the line in phasor form. (6 pts)

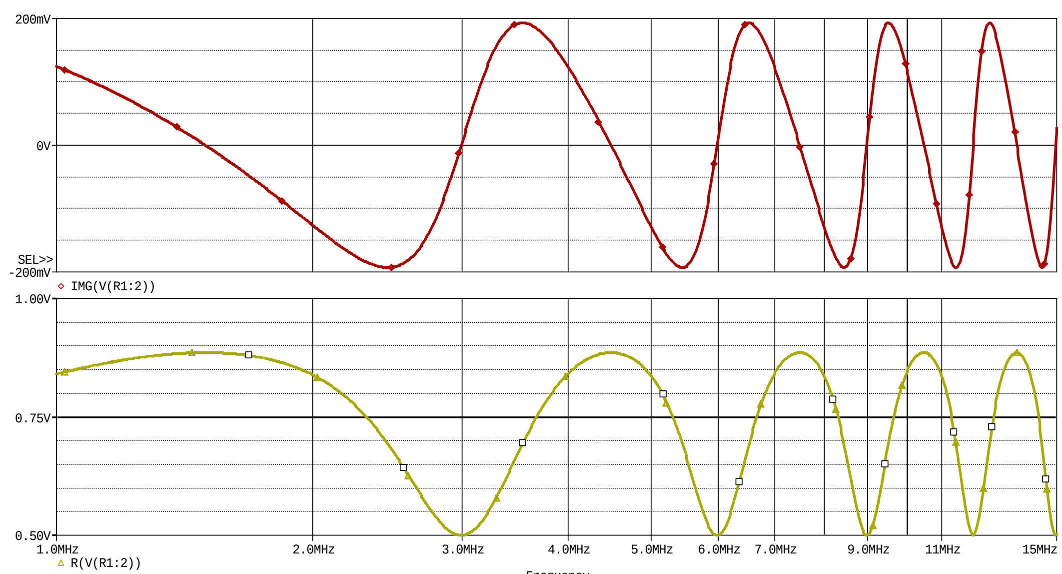
There are several acceptable ways that are, at least, almost correct. The main issue is to show only a positive traveling wave such as $v(z) = V^+ e^{-j\beta z}$ *and* $i(z) = \dfrac{V^+}{Z_o} e^{-j\beta z}$

Determining Unknown Transmission Line Properties (16 points)

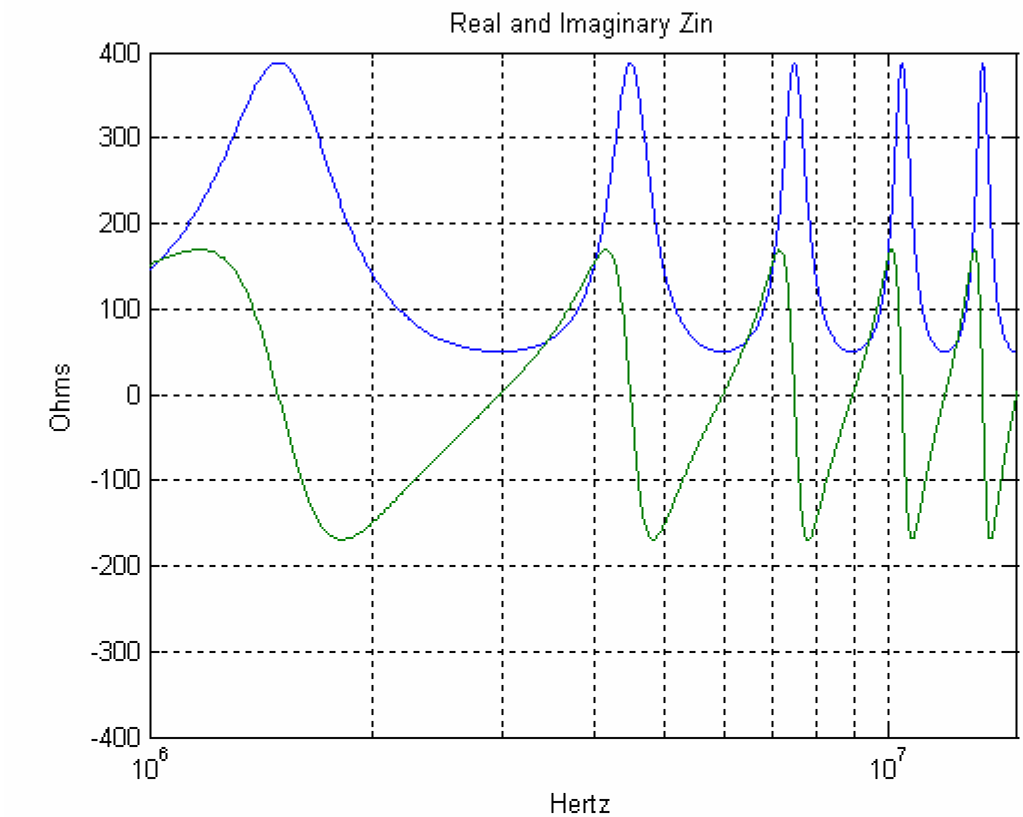


Assume we have a transmission line for which we know nothing, except its length (24 meters). We set up the standard configuration shown above and find the following voltages at the input and output ends as a function of frequency:





We also use a network analyzer to determine the real and imaginary parts of the input impedance Z_{in} as a function of frequency for the same range.



From this information, determine the characteristic impedance Z_o, the propagation velocity u, the capacitance per unit length c and the inductance per unit length l.
From the Z_{in} plot, note that the input impedance is real and about 390 Ohms at f=1.5MHz, 4.5MHz, 7.5MHz. It is real and 50 Ohms at 3MHz, 6MHz, 9MHz. Thus the line must be a half wavelength at 3MHz or $\lambda = 48$. At this frequency, then,

$\beta = \frac{2\pi}{\lambda} = \frac{2\pi}{48}$. $u = \frac{\omega}{\beta} = \frac{2\pi 3x10^6}{2\pi} 48 = 1.44x10^8$. At 1.5MHz, $Z_{in} = \frac{Z_o^{\ 2}}{Z_L} = 390$ *or* Z_o = *140 Ohms. The inductance and capacitance per unit length can be found from* $\beta = \omega\sqrt{lc}$ *and* $Z_o = \sqrt{\frac{l}{c}}$. *Combining these expressions we get* $\beta Z_o = \omega l$ *and* $\frac{\beta}{Z_o} = \omega c$. *Finally l is a little less than 1 micro Henry and c is 50pF both per meter.*

Z_o __140___

u _1.44x10^8

l ___1 $^{\mu H}/_{m}$

c _50 $^{pF}/_{m}$–

8. Standing Waves (18 points)

A lossless transmission line is driven by a sinusoidal voltage source with a frequency of 50MHz. Four different loads are connected to the line and the standing wave pattern is determined (maybe by miracle, since this is hard to do) as shown on the next two pages. The loads are a short circuit, 25 Ohms, 400 Ohms and an open circuit.

a. First, indicate which plot corresponds to each load impedance. (4 pts)

1. Z_L=*400* 2. Z_L=*short* 3. Z_L=*open* 4. Z_L=*25*

b. Next, determine the wavelength on the line λ. (2 pts)

One half wavelength is 2.5 meters, thus $\lambda = 5$

c. Determine the propagation constant β. (2 pts)

The propagation constant $\beta = \frac{2\pi}{\lambda} = 0.4\pi$

d. Determine the propagation velocity *u*. (2 pts)

The propagation velocity is $u = \frac{\omega}{\beta} = \frac{2\pi 5x10^7}{0.4\pi} = 2.5x10^8$

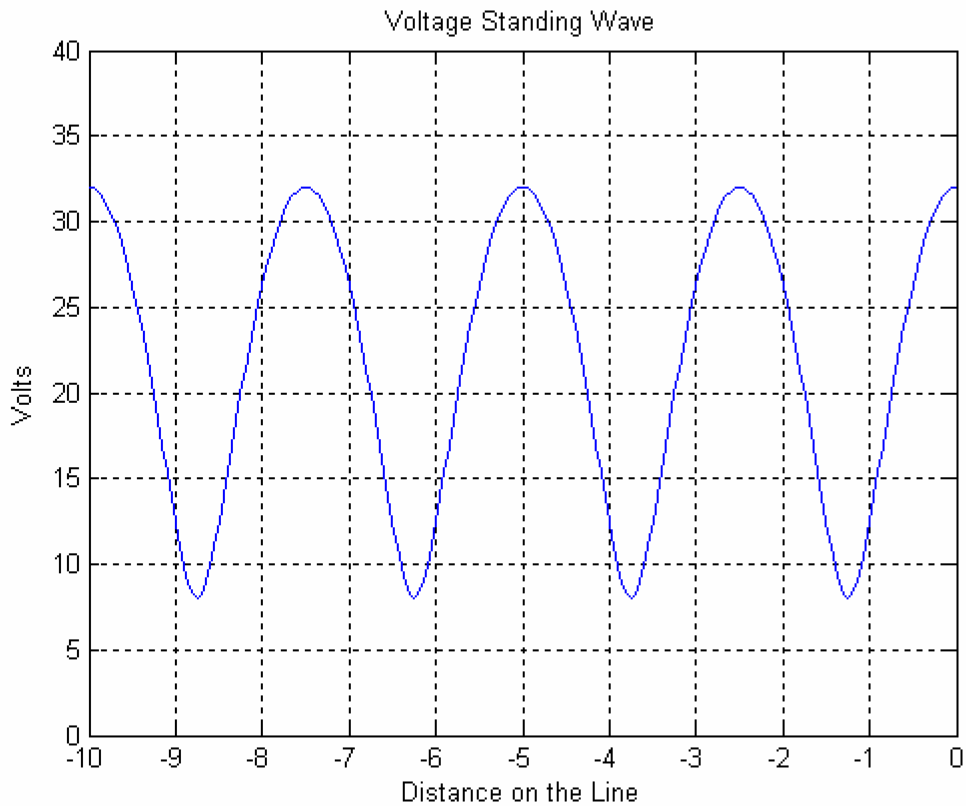
e. Determine the characteristic impedance of the line Z_o. (4 pts)

The characteristic impedance can be determined from one of the finite load impedances. For Z_L*=400, we have that* $\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_0} = \frac{400 - Z_o}{400 + Z_0} = \frac{12}{20} = \frac{3}{5}$ *or*

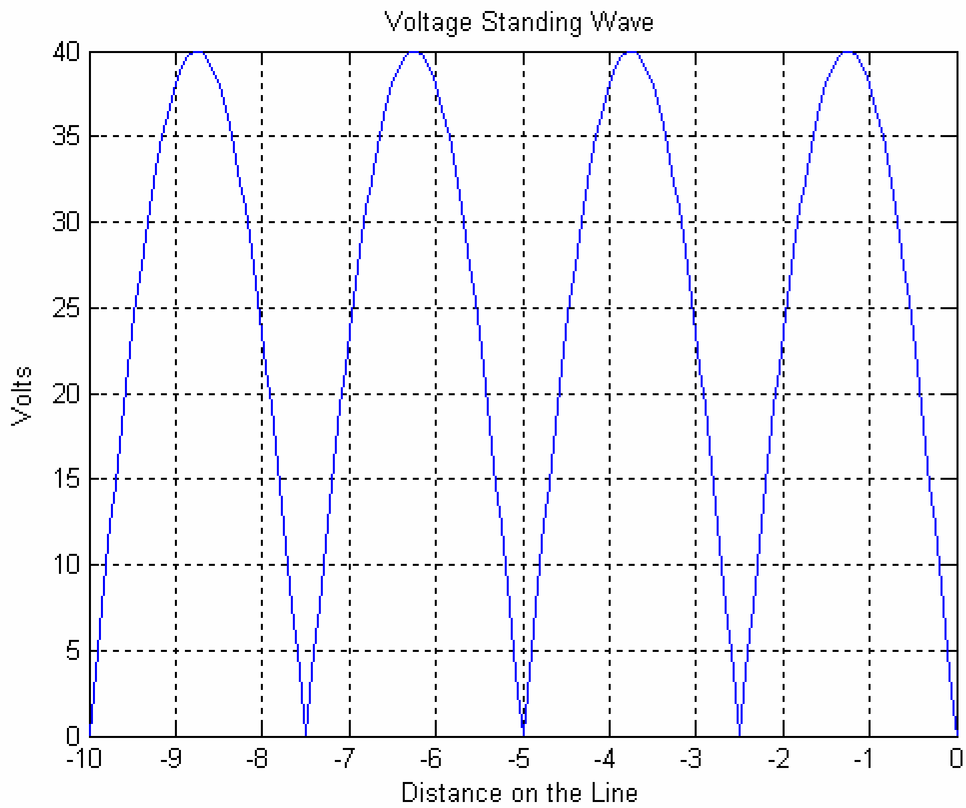
$2000 - 5Z_o = 1200 + 3Z_o$ *or finally* $8Z_o = 800$ *and* $Z_o = 100$. *The same could be done with the 25 Ohm load.*

f. Finally, if the current standing wave patterns were displayed instead of voltage standing wave patterns, indicated which plot would correspond to each load. (4 pts)

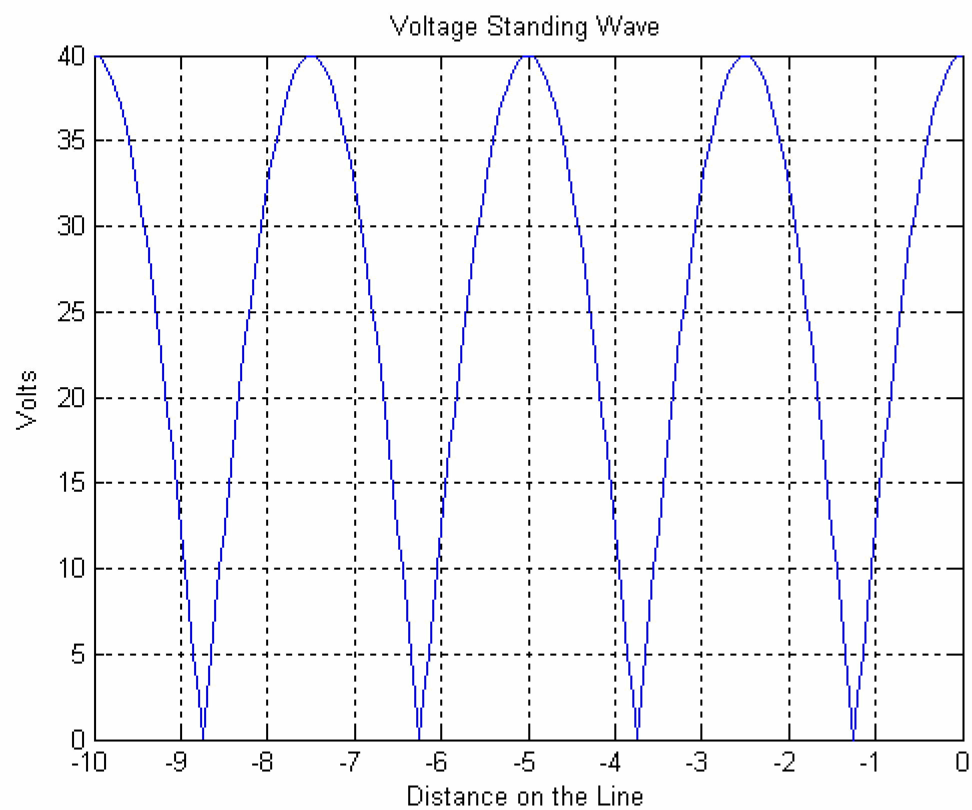
1. Z_L=*25* 2. Z_L=*open* 3. Z_L=*short* 4. Z_L=*400*


Voltage Standing Wave
Volts
Distance on the Line
40
35
30
25
20
15
10
5
0
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

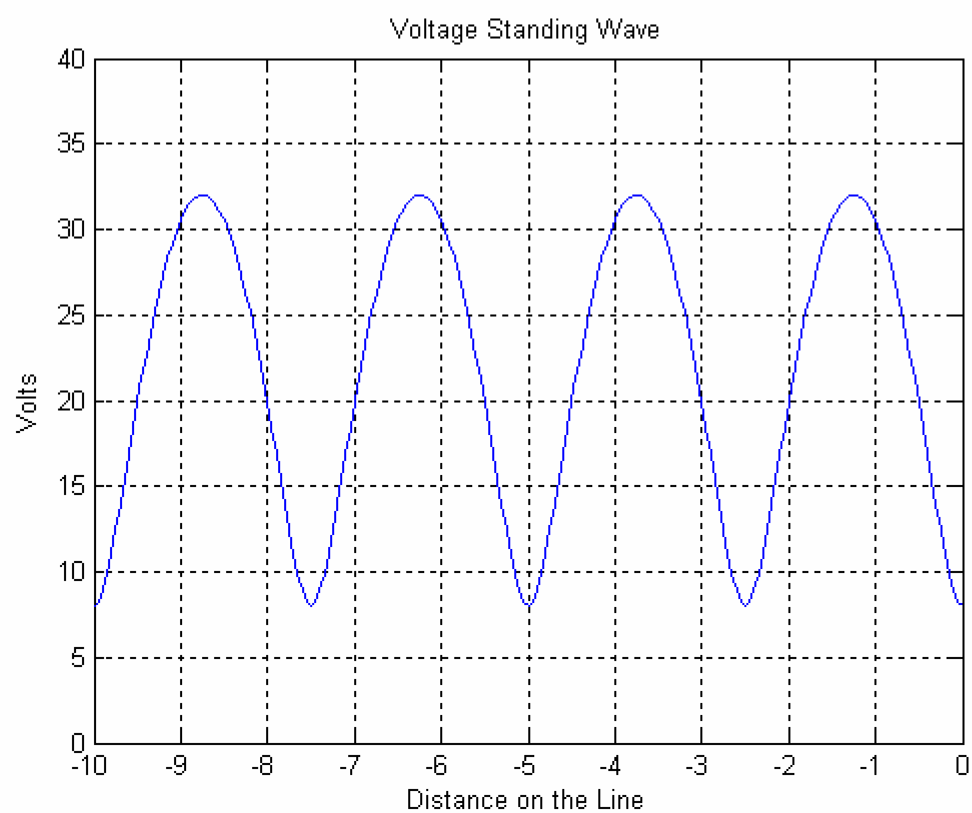
1.


Voltage Standing Wave
Volts
Distance on the Line
40
35
30
25
20
15
10
5
0
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

2.

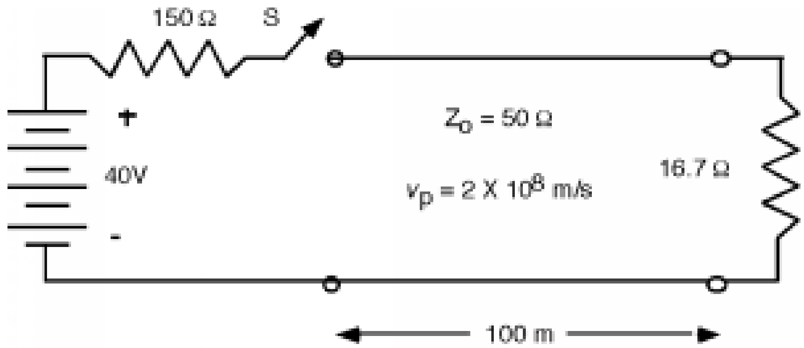

Voltage Standing Wave
Volts
Distance on the Line
40
35
30
25
20
15
10
5
0
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

3.


Voltage Standing Wave
Volts
Distance on the Line
40
35
30
25
20
15
10
5
0
-10
-9
-8
-7
-6
-5
-4
-3
-2
-1
0

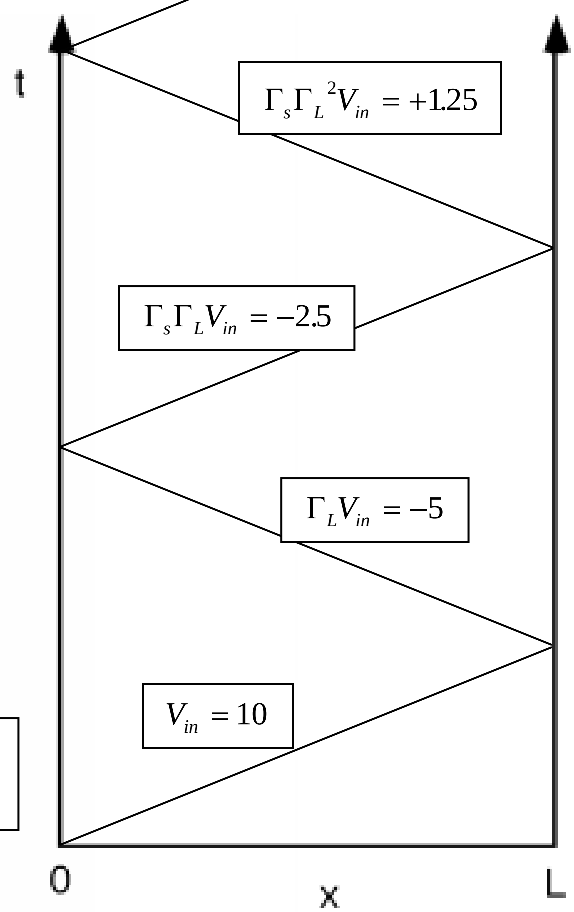
4.

9. Transients on Transmission Lines (12 points)



A 40V DC source with a 150 Ohm internal impedance is connected to a 50 Ohm transmission line with a 16.7 Ohm load. The length of the line is 100 meters and the propagation speed is $2x10^8$ m/s.

a. Generate the bounce diagram



$$\Gamma_s{}^2\Gamma_L{}^2 V_{in} = +0.625$$

$$\Gamma_s\Gamma_L{}^2 V_{in} = +1.25$$

$$\Gamma_s\Gamma_L V_{in} = -2.5$$

$$\Gamma_L V_{in} = -5$$

$$T = \frac{100}{2x10^8} = 500ns$$

$$V_{in} = 10$$

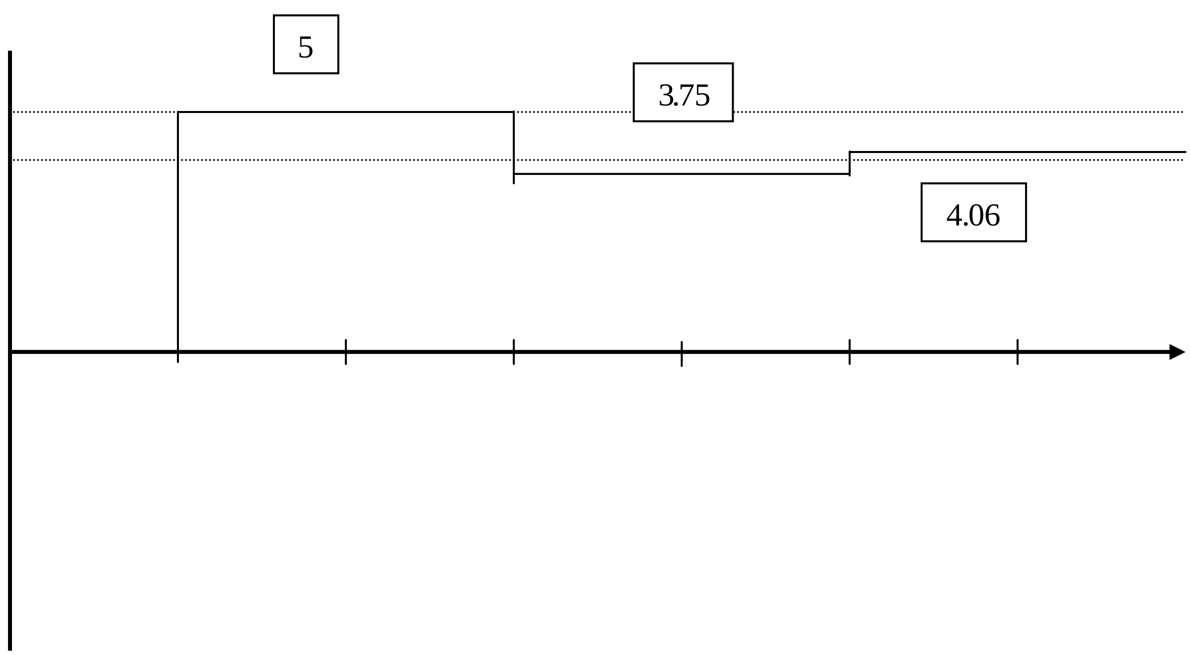
$$V_{in} = \frac{Z_o}{Z_o + Z_s} V_s = \frac{50}{50+150} 40 = 10$$

$$\Gamma_s = \frac{Z_s - Z_o}{Z_s + Z_o} = \frac{150-50}{150+50} = +0.5$$

$$\Gamma_L = \frac{Z_L - Z_o}{Z_L + Z_o} = \frac{16.7-50}{16.7+50} = -0.5$$

b. Determine and plot the voltage observed at the load as a function of time. Indicate the value the voltage will eventually reach if we wait long enough (time goes to infinity).

For very large times, the transmission line will be fully charged up so that it will appear to have no effect and behave in the way we have assumed connections behave when we have done DC circuit analysis in the past. Thus the voltage will be what one sees across a 16.7 Ohm resistor connected to the 40 Volt source with an internal impedance of 150 Ohms or $V = \frac{16.7}{150+16.7}40 = 4$



Quiz 1

Using PSpice to solve this problem produces the following plot: